ν - masses

and mixings

from symmetry

Standard - Model

neutrino masses $\hat{=}$

dimension five operators

$$\frac{1}{M} \nu_L \nu_L \phi \phi$$

non-renormalizable

suppressed by heavy mass scale M

physics beyond the SM

Weinberg

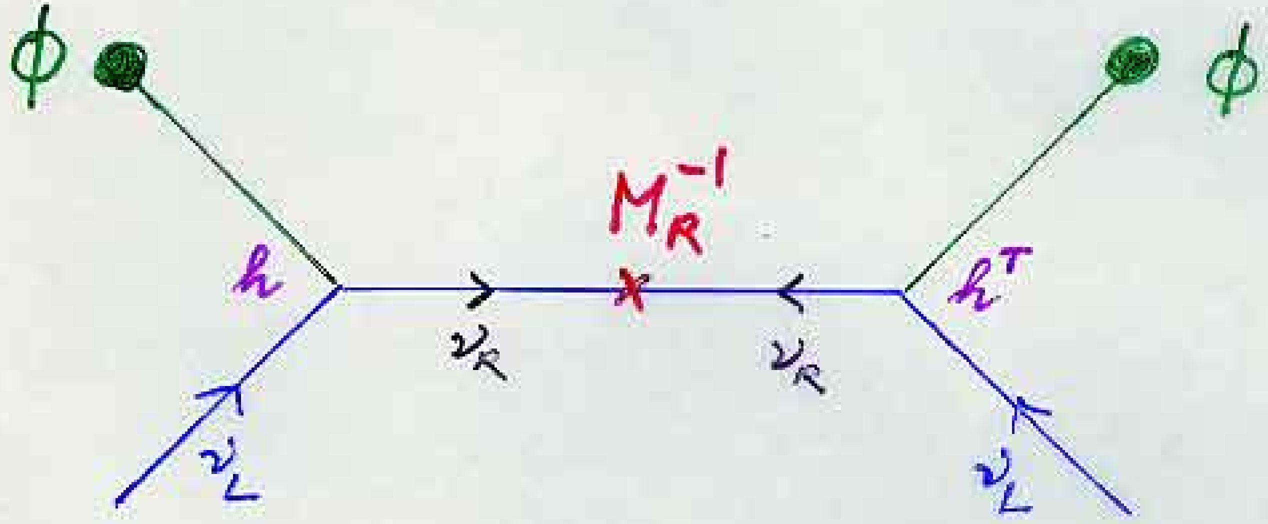
Barbieri, Ellis, Gaillard

Witten

see - saw

Yanagida

Gell-Mann, Ramond, Slanski

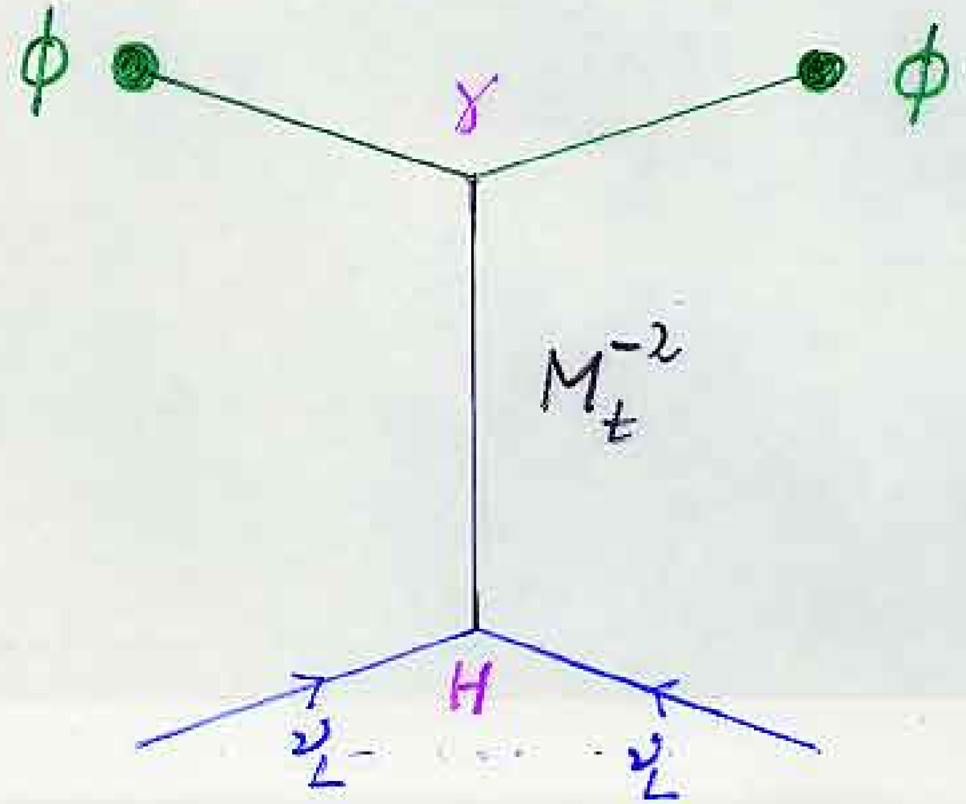


ν_R : singlet neutrino

for SO(10) etc. : $M_R \sim M_{B-L}$

induced triplet

Magg, CW



$\gamma \sim M, \quad M_t^2 \sim M^2 \quad \Rightarrow \quad M^{-1}$

t : $SU(2)_L$-triplet , $Y = -2$

for SO(10) etc : $\gamma \sim M_{B-L}$, often $M_t \sim M_{B-L}$

other picture :

mass matrix for ν_L and ν_L^c

$$\begin{array}{ccc} \nu_L^c & \nu_L & \\ \left(\begin{matrix} A & B \\ B & C \end{matrix}\right) & & \begin{matrix} \nu_L^c \\ \nu_L \end{matrix} \end{array}$$

ν_L^c : singlet neutrino

number could be $\neq 3$

$A \sim M$

$B \sim \langle \varphi_0 \rangle$ (Dirac mass)

for $C = 0$: eigenvalues A, B^2/A

see-saw mechanism

Yanagida, Gell-Mann, Ramond, Slansky

but : $C = 0$ not natural

Barbieri et al

nevertheless : $C \sim \langle \varphi_0 \rangle^2 / M$

Magg, CW

$$\boxed{M_\nu = M_N^{(D)} \underset{\substack{\uparrow \\ \nu_R\text{-} \\ \text{contrib. (B)}}}{M_R^{-1}} M_N^{(D)T} + \underset{\substack{\uparrow \\ \text{triplet} \\ \text{contrib. (A)}}}{M_L^{(M)}}}$$

CW, NuPhB 187 (1981) 343

SO(10) : often

$$M_L^{(M)} = c \; M_R$$

$$c \sim M_Z^2 / M_c^2 \sim O(10^{-23}) \; !$$

$$M_N^{(D)} = M_U + O\left(\frac{M_c^2}{M_X^2}\right)$$

- $M^{(D)}_{N\,33} = m_t \; !$
- for 1,+2. generation : similar scale pattern

$\underline{\text{if}}$ M_R^{-1} would be unit matrix

and $M_L^{(M)}$ negligible $\Rightarrow m_{\nu_i} \sim m_{u_i}^2$

no reason to expect this behavior;

strongly disfavored in SO(10)

cw '81

in general:

some „generation structure"

expected for M_R

example: models which explain all orders

of magnitude of charged fermion

masses only in terms of a

generation symmetry $U(1)_G$

$\Rightarrow$ large ratios of eigenvalues of M_R typical

J. Bijnens, CW

* orders of magnitude of (neutral) lepton masses can be understood from symmetries :

  - singlets $\sim M_{B-L}$
  - charged leptons $\sim M_L$
  - neutrinos $\sim M_L^2 / M_{B-L}$

* no good reason for existence of light sterile neutrinos !

Generation symmetry

e.g. $SU(5) \times U(1)_G$

$$\overline{10}\ \left.\begin{array}{l} u \\ d \\ u^c \\ e^c \end{array}\right\} \quad 0,\ 1,\ 2$$

$$5\ \left.\begin{array}{l} \nu \\ e \\ d^c \end{array}\right\} \quad 1,\ 1,\ 2$$

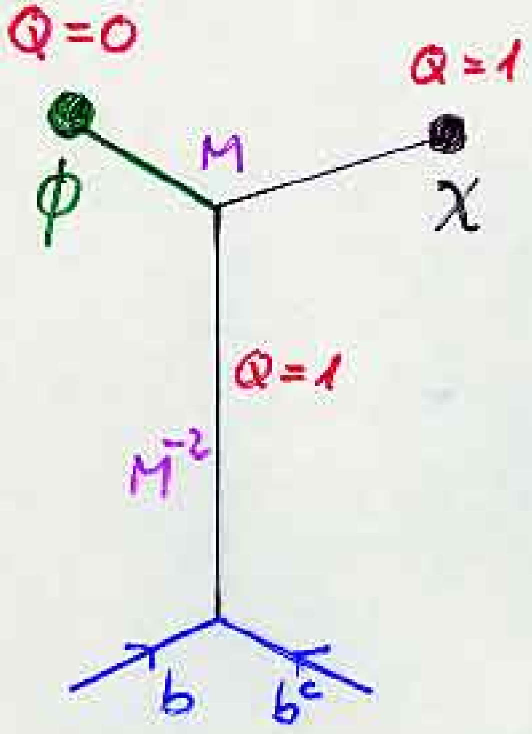
Bijnens, CW '87

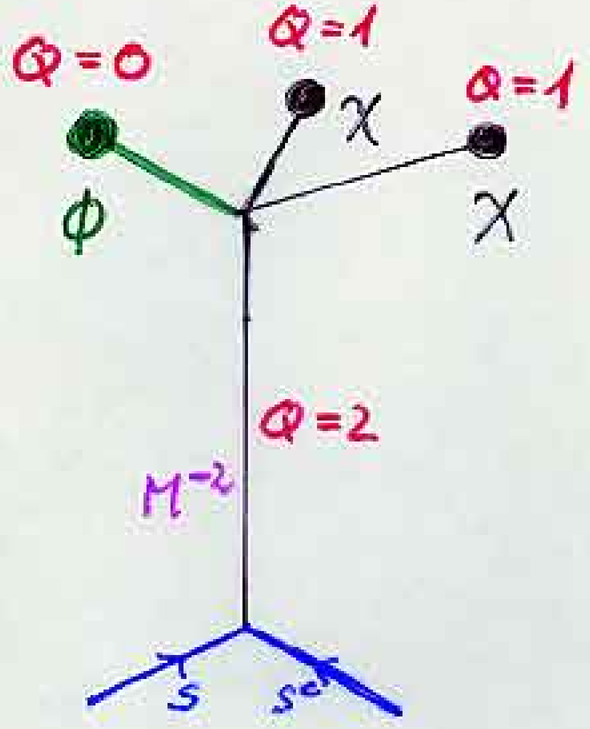
$$M_U : \begin{array}{c} \\ t^c \\ c^c \\ u^c \end{array} \begin{array}{c} \begin{array}{ccc} t & c & u \end{array} \\ \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \end{array} \qquad M_D : \begin{array}{c} \\ b^c \\ s^c \\ d^c \end{array} \begin{array}{c} \begin{array}{ccc} b & s & d \end{array} \\ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \end{array}$$

$$M_L : \begin{array}{c} \\ \tau^c \\ \mu^c \\ e^c \end{array} \begin{array}{c} \begin{array}{ccc} \tau & \mu & e \end{array} \\ \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 3 & 4 \end{pmatrix} \end{array} \qquad M_\nu^{(D)} : \begin{array}{c} \\ \nu_3^c \\ \nu_2^c \\ \nu_1^c \end{array} \begin{array}{c} \begin{array}{ccc} \nu_\tau & \nu_\mu & \nu_e \end{array} \\ \begin{pmatrix} a & a & b \\ c & c & d \\ e & e & f \end{pmatrix} \end{array}$$

mass entries $\sim \lambda^q$

Leading Higgs doublet : $Q=0 \Rightarrow$





$$\sim \frac{\chi}{M} \sim \lambda \qquad\qquad \sim \left(\frac{\chi}{M}\right)^2 \sim \lambda^2$$

$\hat{=}$ Frogatt-Nielsen mechanism

(heavy fermion exchange)

Higher dimensional theories :

doublets, heavy fermions with all Q in KK tower our example :

realistic masses and mixings

for quarks and charged leptons

$$Q(\nu_\tau) = Q(\nu_\mu)$$

predicts $\nu_\tau - \nu_\mu$ mixing $O(1)$

!

1'07

maximal neutrino mixing by symmetry

$\nu_\mu - \nu_\tau$ - mixing:

$$\sin^2 2\vartheta = 1 - \varepsilon$$

$\varepsilon \neq 0 \quad \hat{=}$ breaking of generation symmetry

$\Rightarrow$ natural small parameter, e.g.

$$\varepsilon \lesssim 0.01$$

* ε plays the role of the small mixing angle for quarks!

* „neutrino mismatch" favors induced triplet mechanism

Maximal $\nu_\mu - \nu_\tau$ mixing

- SO(3) generation symmetry

$\begin{pmatrix} \nu \\ l \end{pmatrix}$ in $\underline{3}$; $l^c : \underline{1} + \underline{2}$

$q = \begin{pmatrix} u \\ d \end{pmatrix} : \underline{1} + \underline{2}$, $u^c : \underline{1} + \underline{2}$, $d^c : \underline{3}$

consistent with

$$SU(5) \times U(1)_{B-L} \times SO(3)_G \; (\times U(1)_G)$$

- Leading weak doublet : $H_1 : \underline{1}$

contributes only to m_t :

$$\sim t t^c H_1^*$$

typically $m_t \approx M_W$

▶ Leading B-L violating field: s: $\underline{1}$

$(B-L = -2)$

▶ induced triplet is also $\underline{1}$!

$$\mathcal{L} = h_t \, t s^* H_1^* H_1^* + m_t^2 t^* t$$

$$\Rightarrow \langle t \rangle = \frac{h_t}{m_t^2} \langle s \rangle \langle H_1 \rangle^2$$

$$\sim M_W^2 M_{B-L} / m_t^2$$

$\nu_i \nu_i t^* \quad \Rightarrow$

degenerate mass for all three neutrinos

note: if there are triplets in several reps. of $SO(3)_G$, i.e. $\underline{1}, \underline{3}, \underline{5}, \dots$

* only singlet gets induced VEV in leading order !
* only $\underline{1}$ and $\underline{5}$ contribute to M_ν

limit of unbroken $SO(3)_G \times U(1)_G$:

weak symmetry breaking by
Higgs doublet :

* only top quark acquires mass

$$m_t \approx M_W$$

* three degenerate neutrino masses

$$m_\nu \approx M_W^2 / M_{B-L}$$

e.g. $M_{B-L} = 10^{14}$ GeV

$\Rightarrow m_\nu \approx 0.1$ eV

$SO(3)_G$ - breaking

consider $U(1)$ subgroup acting in $\nu_2 - \nu_3$ plane

generator : I_{3G}

▶ breaking by v : $\underline{3}$

with $I_{3G} = +1$

	τ	μ	e	ν_τ	ν_μ	ν_e	τ^c	μ^c	e^c
I_{3G}:	-1	$+1$	0	-1	$+1$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$

▶ induced weak doublet : H_2 : $\underline{3}$

$$\gamma H_1^* H_2 v^* + m_2^2 H_2^* H_2 \Rightarrow$$

$$\langle H_2 \rangle = \frac{\gamma \langle v \rangle}{m_2^2} \langle H_1 \rangle = \lambda_0 \langle H_1 \rangle$$

$$\lambda_0 \ll 1$$

$$\langle H_2 \rangle : \quad I_{3G} = 1$$

$\Rightarrow$ mass term for τ , $\sim \tau \tau^c H_2$

$$\frac{m_\tau}{m_t} \sim \lambda_0 \quad !$$

This singles out the I_{36} eigenstates as basis for the charged leptons.

In leading order: mass eigenstates e, μ, τ have fixed I_{36}

$$
\begin{array}{cc|ccc|}
 & & 0 & 1 & -1 \\
 & & e & \mu & \tau \\
\hline
-\frac{1}{2} & e^c & -\frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\
\frac{1}{2} & \mu^c & \frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\
0 & \tau^c & 0 & 1 & -1
\end{array}
\qquad\qquad
\begin{array}{cc|ccc|}
 & & 0 & 1 & -1 \\
 & & \nu_e & \nu_\mu & \nu_\tau \\
\hline
0 & \nu_e & 0 & 1 & -1 \\
1 & \nu_\mu & 1 & 2 & 0 \\
-1 & \nu_\tau & -1 & 0 & -2
\end{array}
$$

leading neutrino mass matrix:

$$
M_\nu^{(1)} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
$$

$\Rightarrow$ maximal $\nu_\mu - \nu_\tau$ -mixing!

mismatch between basis of I_{3c}-eigenstates,

i.e. (e, μ, τ), $(\nu_e, \nu_\mu, \nu_\tau)$

and standard $SO(3)$-basis (ν_1, ν_2, ν_3)

in leading order:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -i \\ 0 & 1 & i \end{pmatrix}$$

$$\vartheta = \frac{\pi}{4}$$

simple explanation for neutrino mismatch!

see - saw - contribution

in leading order :

$M_\nu \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ dictated by $SO(3)$-symmetry

Leading B-L violating VEV: S : $\underline{1}$

contributes to all singlet neutrino masses

if N in irreducible representation of $SO(3)_G$

$\Rightarrow$ degenerate mass for all N, $M_R \sim M_{B-L}$

(i) $N : \underline{3} \Rightarrow M_N^{(D)} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ or 0, $M_R \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\Rightarrow M_N^{(D)} M_R^{-1} M_N^{(D)T} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

see-saw contr. can be of same order as triplet-contr.

(ii) $N : \underline{1}, \underline{2}, \underline{4}, \underline{5}$ etc : $M_N^{(D)} = 0$

induced triplet dominates

next to leading entry in M_ν :

from $\underline{5}$ of $SO(3)_G$

$\phi_{ij} = \phi_{ji}$, $\mathrm{Tr}\,\phi = 0$

* $\langle \phi_{ij} \rangle$ depends on next to leading vevs with $B-L \neq 0$

* interesting possibility : $\phi_{ij} \sim (\lambda_8)_{ij}$

$\Rightarrow$ mass split between ν_3 and ν_1, ν_2

$\hat{=}\ \Delta m_a^2$ for atmospheric neutrinos

$\Rightarrow$ mixing remains maximal ($\sin^2 2\vartheta = 1$)

$\Rightarrow$ ν_1 and ν_2 remain degenerate in next to leading order :

$$m_b, m_c, m_\tau \sim \lambda_0 m_t$$

$$m_{\nu_3} \neq m_{\nu_{1,2}} \quad , \quad m_{\nu_2} = m_{\nu_1}$$

no mixing in quark sector

maximal mixing between ν_μ and ν_τ

conclusions

* Size of neutrino masses hints at physics at high mass scale, $M_{B-L} \sim 10^{14}$ GeV

* Difference in structure for masses and mixings of neutrinos as compared to quarks can be understood by generation symmetry

* Maximal $\nu_\mu - \nu_\tau$ mixing would hint to non-abelian generation symmetry

  also induced triplet mechanism may be favored

* typical prediction : $\sin^2 2\vartheta = 1 - \epsilon$

  $\epsilon < 0.05$