Comparison of LMA and LOW Solar Solution Predictions in an SO(10) SUSY GUT Model

Model is particularly useful in that it is quantitatively predictive, can explain the LMA or LOW solution, and can be used to assess the need for a neutrino factory.

Albright, Barr
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Albright, Geer
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- Based on a minimum set of Higgs fields which solves the doublet-triplet splitting problem.

  $\mathbf{45_H}$,

  $\mathbf{16_H}$, $\overline{\mathbf{16}}_\mathbf{H}$, $\mathbf{16'_H}$, $\overline{\mathbf{16'}}_\mathbf{H}$,

  $\mathbf{10_H}$, $\mathbf{10'_H}$, ...,

  $\mathbf{1_H}$'s,

  where

  $\langle \mathbf{45_H} \rangle_{B-L}$, $\langle 1(\mathbf{16_H}) \rangle$, $\langle 1(\overline{\mathbf{16}}_\mathbf{H}) \rangle$ break $SO(10) \to$ SM;

  $v_u = \langle 5(\mathbf{10_H}) \rangle$, $v_d = \langle \overline{5}(\mathbf{10_H}) \rangle \cos\gamma + \langle \overline{5}(\mathbf{16'_H}) \rangle \sin\gamma$

  break EW symmetry.

  Higgs superpotential exhibits the $U(1) \times Z_2 \times Z_2$ flavor symmetry.

- Matter Superfields

  $\mathbf{16_1}$, $\mathbf{16_2}$, $\mathbf{16_3}$; $\mathbf{16}$, $\overline{\mathbf{16}}$, $\mathbf{16'}$, $\overline{\mathbf{16'}}$

  $\mathbf{10_1}$, $\mathbf{10_2}$

  $\mathbf{1}$'s, ...

  where all but the $\mathbf{16_i}$, $i = 1, 2, 3$ get superheavy and are integrated out.

- **Dirac Mass Matrices**

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \qquad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D,$$

where

$$\begin{array}{llll} M_U & \simeq 113 \text{ GeV}, & M_D & \simeq 1 \text{ GeV}, \\ \sigma & = 1.78, & \epsilon & = 0.145, \\ \delta & = 0.0086, & \delta' & = 0.0079, \\ \phi & = 54^o, & \eta & = 8 \times 10^{-6} \end{array}$$

are input parameters defined at the GUT scale to fit the low scale observables after evolution downward from Λ_G.

- Above textures were obtained by imposing the Georgi-Jarlskog relations at Λ_G:

  $$m_s^0 \simeq m_\mu^0/3, \quad m_d^0 \simeq 3m_e^0$$

  and Yukawa coupling unification with $\tan\beta \sim 5$.

- Froggatt-Nielsen diagrams illustrate various contributions and features:

  "1" obtained from $\mathbf{16_3 \cdot 16_3 \cdot 10_H}$ vertices.

  "ϵ" obtained from $\langle \mathbf{45_H} \rangle_{B-L}$ suppression.

  "σ" obtained from the $\mathbf{16_2 \cdot 16_H \cdot 16'_H \cdot 16_3}$ effective operator which contributes only to D and L in a lop-sided fashion.

– All 9 quark and charged lepton masses plus the 3 CKM angles and 1 phase are well-fitted with the 8 input parameters (after evolution from the GUT scale):

$$
\begin{aligned}
m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV} \\
m_u(1 \text{ GeV}) &= 4.5 \text{ MeV}, & m_\mu &= 105.7 \text{ MeV} \\
V_{us} &= 0.220, & m_e &= 0.511 \text{ MeV} \\
V_{cb} &= 0.0395, & \delta_{CP} &= 64^\circ
\end{aligned}
$$

which lead to the following predictions:

$$
\begin{aligned}
m_b(m_b) &= 4.25 \text{ GeV}, & m_c(m_c) &= 1.23 \text{ GeV} \\
m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & m_d(1 \text{ MeV}) &= 7.9 \text{ MeV} \\
|V_{ub}/V_{cb}| &= 0.080, & \sin 2\beta &= 0.64.
\end{aligned}
$$

– $U^\dagger U$, $D^\dagger D$, and $N^\dagger N$ are diagonalized with small LH rotations, while $L^\dagger L$ is diagonalized by a large LH rotation.

This accounts for the fact that $V_{cb} = (U_U^\dagger U_D)_{cb}$ is small while $U_{\mu 3} = (U_L^\dagger U_\nu)_{\mu 3}$ is large for any reasonable M_R.

- **Right-Handed Majorana Matrix**

  The type of $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ solar neutrino mixing is determined by the texture of M_R, since the solar and atmospheric mixings are essentially decoupled in this model.

  - **LMA Solar Neutrino Solution**

    This solution requires fine-tuning of M_R and a nearly hierarchical texture:

$$M_R = \begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R \text{ having no texture zeros.}$$

    With $a = 1,\ b = 2$ and $\Lambda_R = 2.5 \times 10^{14}$ GeV,

$$M_\nu = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R \text{ with 3 texture zeros}$$

    leads to

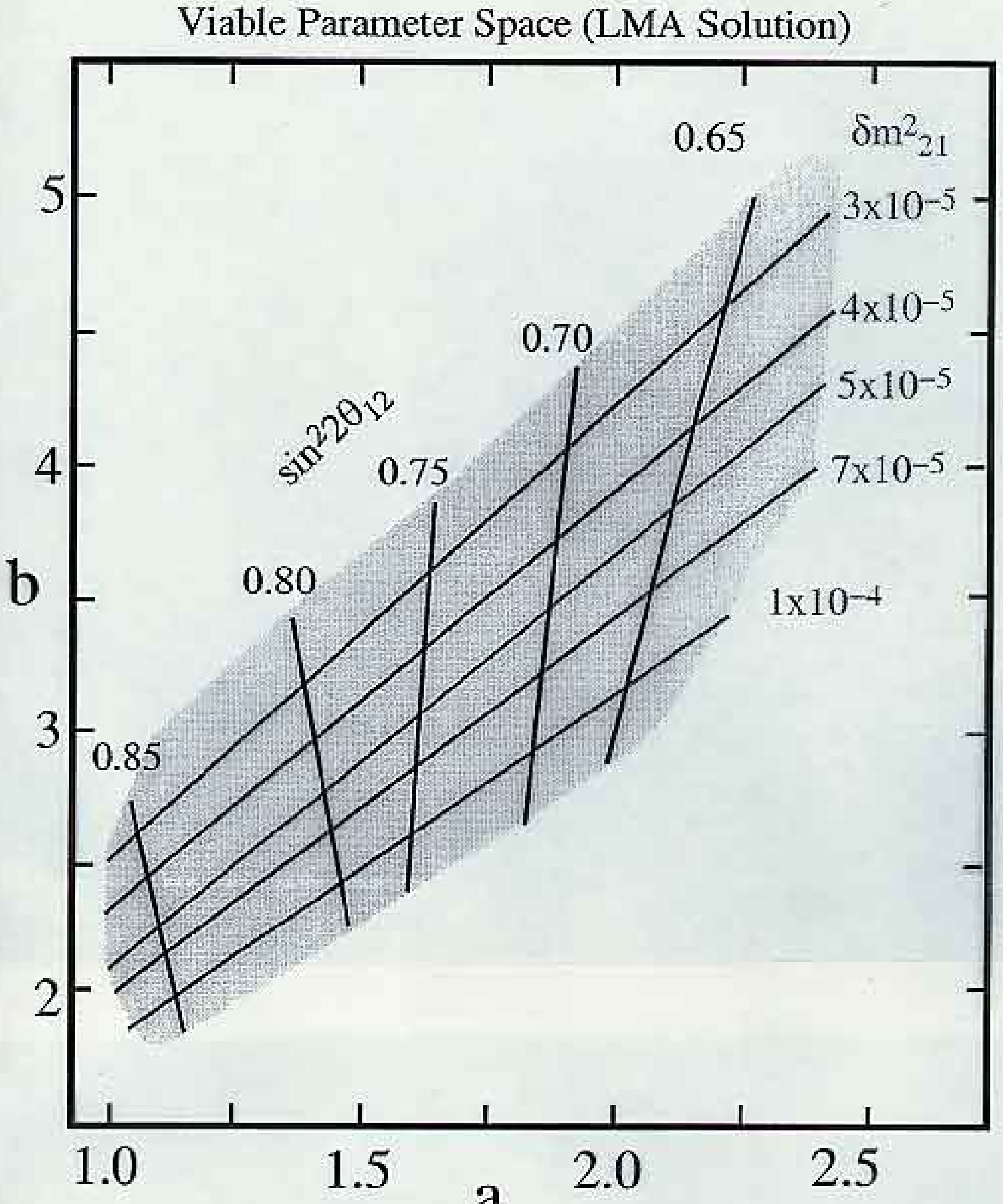
$$M_1 = M_2 = 2.8 \times 10^8 \text{ GeV}, \qquad M_3 = 2.5 \times 10^{14} \text{ GeV},$$
$$\Delta m^2_{32} = 3.2 \times 10^{-3} \text{ eV}^2, \qquad \sin^2 2\theta_{\rm atm} = 0.994,$$
$$\Delta m^2_{21} = 6.5 \times 10^{-5} \text{ eV}^2, \qquad \sin^2 2\theta_{\rm sol} = 0.88,$$
$$U_{e3} = -0.014, \qquad \sin^2 2\theta_{\rm reac} = 0.0008.$$

    to be compared wit the present S-K best-fit point in the LMA region:

$$\Delta m^2_{21} = 7 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{\rm sol} = 0.87$$

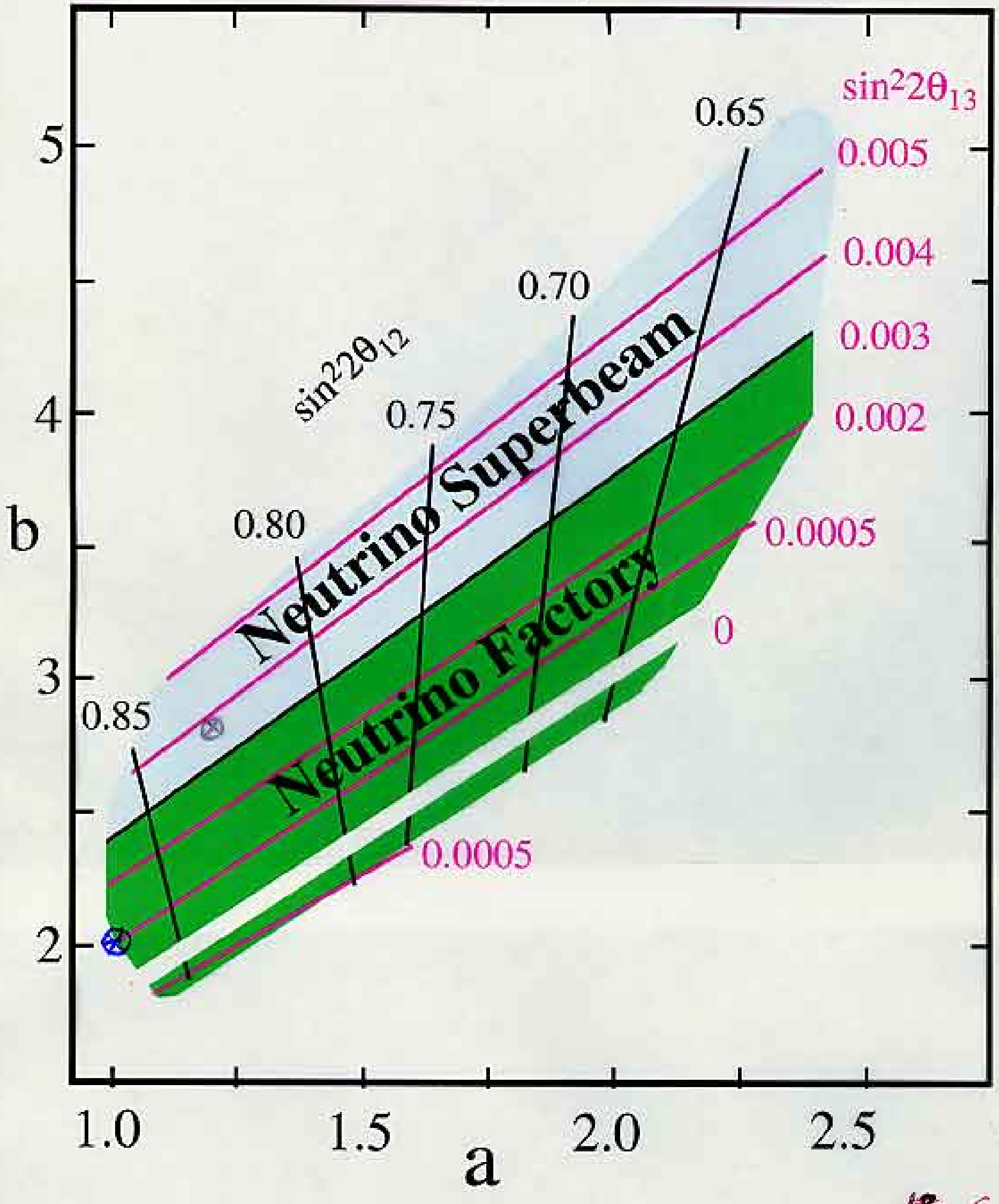
    In fact, the whole presently-allowed LMA region can be covered with

$$1.0 \lesssim a \lesssim 2.5, \quad 1.8 \lesssim b \lesssim 5.2$$


Viable Parameter Space (LMA Solution)
b
a
1.0
1.5
2.0
2.5
2
3
4
5
$\sin^2 2\theta_{12}$
0.85
0.80
0.75
0.70
0.65
δm^2_{21}
3x10^{-5}
4x10^{-5}
5x10^{-5}
7x10^{-5}
1x10^{-4}

S. Geer, C.H. Albright

Viable Parameter Space (LMA Solution)



– **LOW Solar Neutrino Solution**

This solution requires no fine-tuning of M_R:

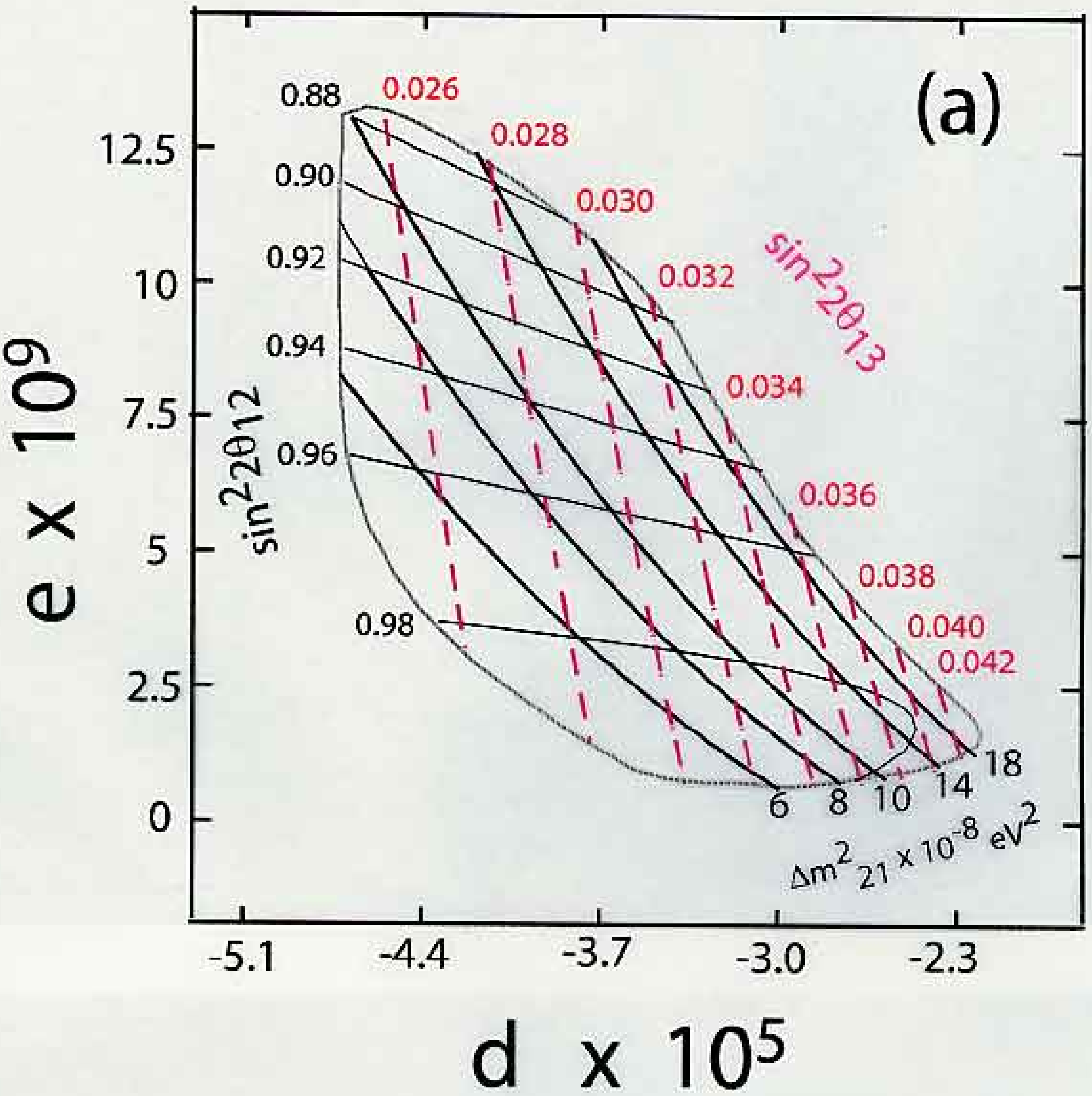
$$M_R = \begin{pmatrix} e & d & 0 \\ d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R$$

having 3 texture zeros.

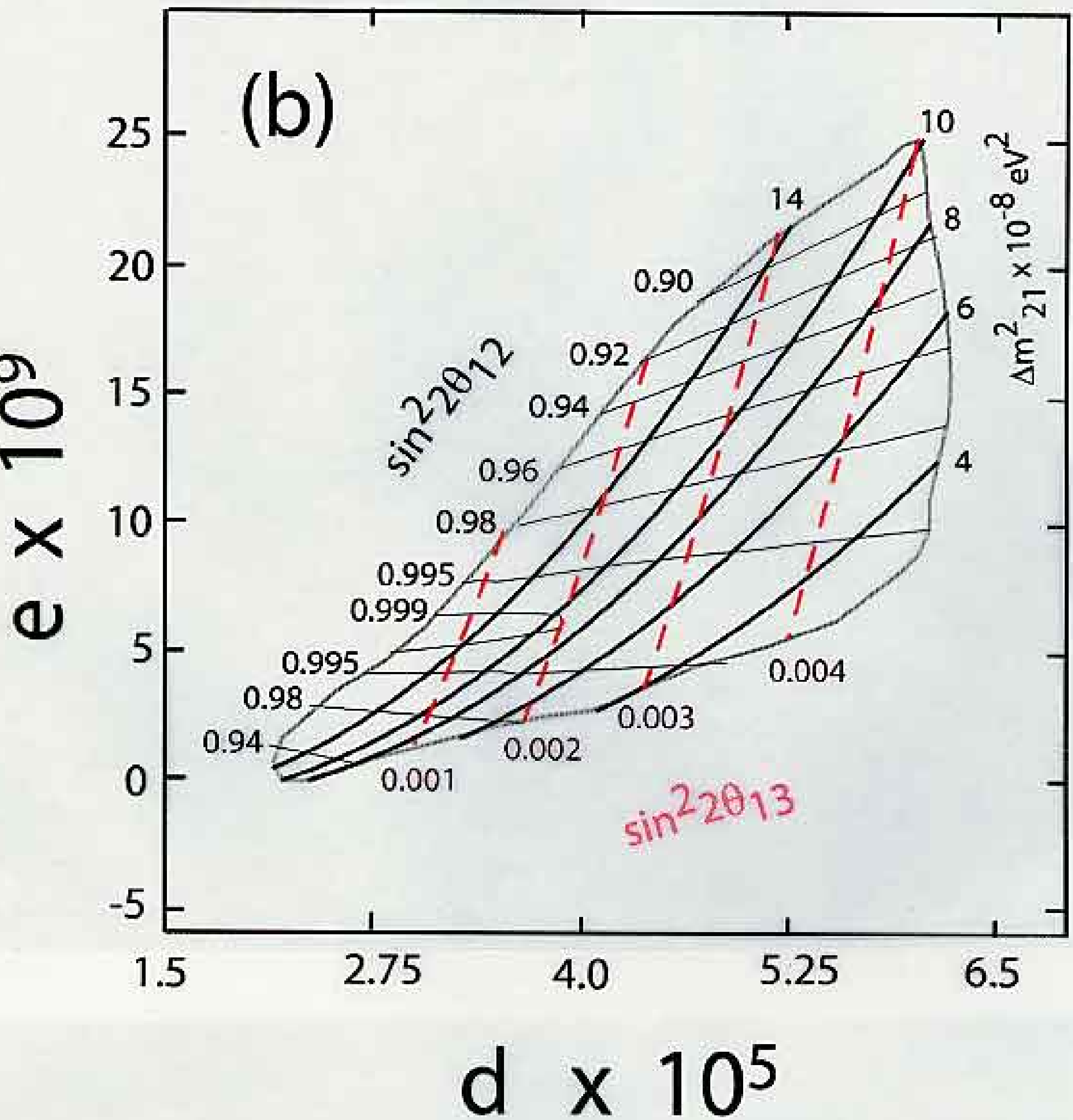
Seesaw mechanism then gives

$$M_\nu = \begin{pmatrix} 0 & 0 & -\epsilon\eta/d \\ 0 & \epsilon^2 & \epsilon \\ -\epsilon\eta/d & \epsilon & 1-\epsilon^2 e/d \end{pmatrix} M_U^2/\Lambda_R$$

with 2 texture zeros.
Here d can be negative or positive and leads to two regions in parameter space which cover the allowed LOW solar solution region.



(a)
e x 10^9
d x 10^5
12.5
10
7.5
5
2.5
0
-5.1
-4.4
-3.7
-3.0
-2.3
sin^2 2θ12
0.88
0.90
0.92
0.94
0.96
0.98
sin^2 2θ13
0.026
0.028
0.030
0.032
0.034
0.036
0.038
0.040
0.042
6
8
10
14
18
Δm^2 21 x 10^-8 eV^2



(b)
e x 10^9
d x 10^5
25
20
15
10
5
0
-5
1.5
2.75
4.0
5.25
6.5
$\sin^2 2\theta_{12}$
0.90
0.92
0.94
0.96
0.98
0.995
0.999
0.995
0.98
0.94
$\sin^2 2\theta_{13}$
0.001
0.002
0.003
0.004
$\Delta m^2_{21} \times 10^{-8} eV^2$
14
10
8
6
4

LMA Solutions $\Delta m^2_{32} = 3.2 \times 10^{-3}\, eV^2$

a	b	$\Delta m^2_{21} (eV^2)$	$\tan^2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$
1.0	2.0	6.5×10^{-5}	0.49	0.994	0.0008
1.7	2.7	10.9×10^{-5}	0.32	0.996	0.00008
1.7	3.4	4.0×10^{-5}	0.33	0.992	0.0033
2.2	3.5	8.8×10^{-5}	0.24	0.996	0.0008

LOW Solutions $\Delta m^2_{32} = 3.0 \times 10^{-3}\, eV^2$

d	e	Δm^2_{21}	$\tan^2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$
-4.2×10^{-5}	10.0×10^{-9}	1.20×10^{-7}	0.56	0.911	0.028
-3.6×10^{-5}	3.0×10^{-9}	0.64×10^{-7}	0.86	0.898	0.030
3.6×10^{-5}	5.0×10^{-9}	0.98×10^{-7}	1.00	0.914	0.0016
5.0×10^{-5}	13.0×10^{-9}	0.85×10^{-7}	0.70	0.918	0.0033

SUMMARY

- This $SO(10)$ SUSY GUT model can apparently explain the observed atmospheric and LMA or LOW solar neutrino oscillation data within the (3 active, 0 sterile) neutrino framework.
- No strong preference for either particular solar neutrino solution.
- Neutrino Factory is essentially required in order to determine U_{e3} and test the $\sin^2 2\theta_{13}$ predictions.