Non-leptonic Two-Body B Decays

(Analyzing $B \to \pi\pi$, πK Modes) in Perturbative QCD

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representing the PQCD group

- Introduction
- Formalism of Perturbative QCD (PQCD)
- Numerical Results of $B \to \pi\pi$, πK Decays and Comparison with Experiments
- Summary

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Introduction

Non-leptonic B decays:

- Study of CP violation – direct CP violation
- CKM angle measurement
- Test of standard model
- Signal of new physics
- … …

$\phi_2(\alpha) \to B \to \pi\pi, \pi\rho$

$\phi_1(\beta) \to B \to J/\psi K_S, K\phi$

$\phi_3(\gamma) \to B \to K\pi,\ B_s \to K\rho$

The calculation of non-leptonic B decays requires:

- Electroweak theory for the quark decay
- Perturbative calculation of radiative corrections
- Non-perturbative calculation of hadronization
- The prove of factorization theorem

Naïve factorization approach

Decay matrix element can be separated to two parts:

- Short distance Wilson coefficients
- Hadronic parameters:

  form factors and decay constants

- $< \pi\pi|H_{\mathrm{eff}}|B> = C_i <\pi|V\text{-}A|0> <\pi|V\text{-}A|B>$

  $= C_i\, f_\pi\, F^{B\to\pi}$

four quark operator

II Effective Hamiltonian (short distance)

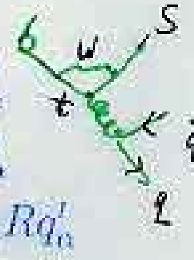
1 Operators

The effective Hamiltonian for the charmless non-leptonic B decays is shown as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{uq}^{*}\left(C_1 O_1^u + C_2 O_2^u\right) + V_{cb}V_{cq}^{*}\left(C_1 O_1^c + C_2 O_2^c\right) - V_{tb}V_{tq}^{*}\left(\sum_{i=3}^{10} C_i O_i + C_g O_g\right)\right] \quad , \qquad (1)$$

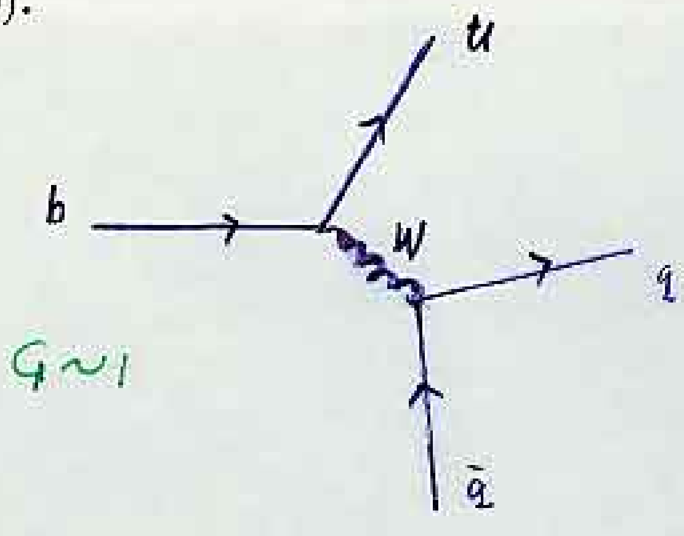
where $q = d, s$ and V_{CKM} denotes the CKM factors, and the operators are ($s \to d$ for $b \to d$ transition):

$$\begin{array}{ll}
O_1^{(q)} = (\bar{s}_\alpha \gamma^\mu L q_\alpha) \cdot \bar{q}_\beta \gamma_\mu L b_\beta & O_2^{(q)} = \bar{s}_\alpha \gamma^\mu L q_\beta \cdot \bar{q}_\beta \gamma_\mu L b_\alpha \\
O_3 = (\bar{s}_\alpha \gamma^\mu L b_\alpha) \cdot \Sigma_{q'} \bar{q}'_\beta \gamma_\mu L q'_\beta & \underline{O_4 = \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \Sigma_{q'} \bar{q}'_\beta \gamma_\mu L q'_\alpha} \\
O_5 = (\bar{s}_\alpha \gamma^\mu L b_\alpha) \cdot \Sigma_{q'} \bar{q}'_\beta \gamma_\mu R q'_\beta & \underline{O_6 = \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \Sigma_{q'} \bar{q}'_\beta \gamma_\mu R q'_\alpha} \\
O_7 = (\frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha) \cdot \Sigma_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R q'_\beta & O_8 = \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \Sigma_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R q'_\alpha \\
O_9 = (\frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha) \cdot \Sigma_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu L q'_\beta & O_{10} = \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \Sigma_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu L q'_\alpha \\
O_g = (g_s/16\pi^2)\, m_b\, \bar{s} \sigma^{\mu\nu} R \frac{\lambda^A}{2} b G^A_{\mu\nu} &
\end{array} \qquad (2)$$



where L and R are the left- and right-handed projection operators. $q' = (u, d, s, c, b)$.

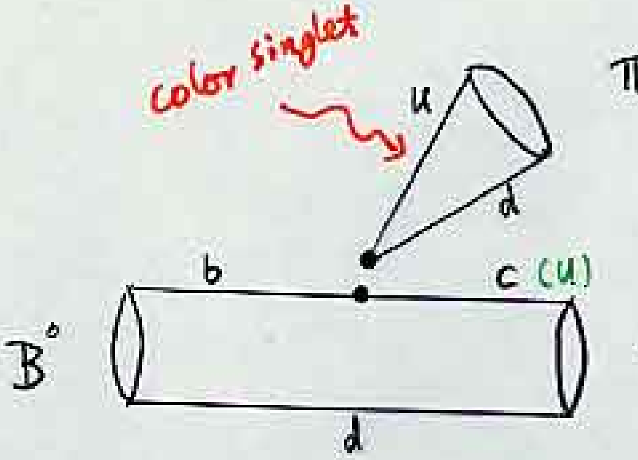
The operators O_1, O_2 are current operators.



Example:

$$B^0 \to \pi^+ D^- \; (\pi^-)$$

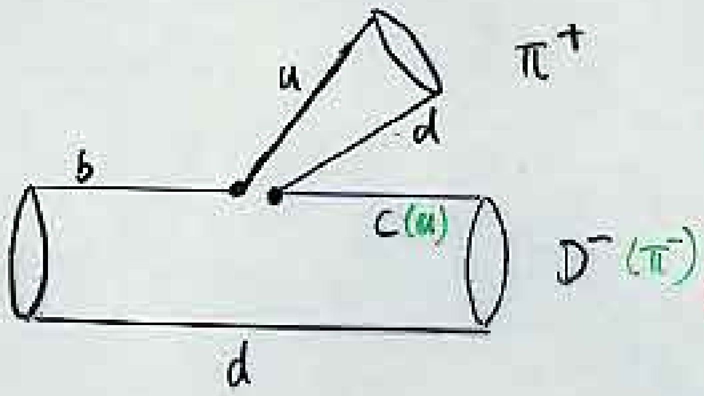
①



Operator O_1

$C_1 \sim 1$

②



Operator O_2

$C_2 \sim -0.26$

Matrix element $\propto$ $C_1 + \frac{1}{3} C_2 \sim 1$

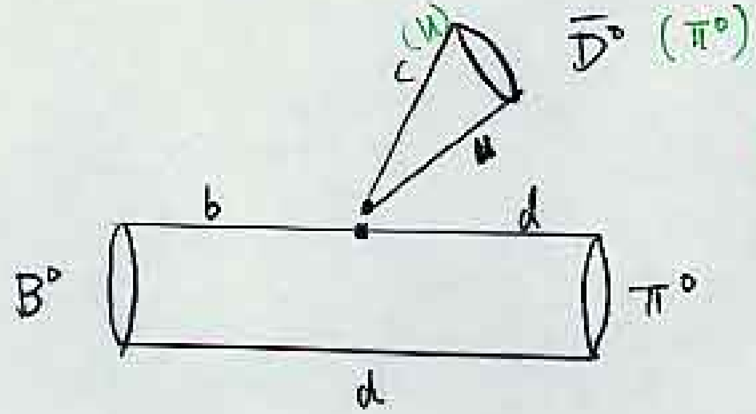
Diagram ① is dominant

② is color suppressed

$$B^0 \to \pi^0 \underset{\overline{J}}{\overline{D}^0} (\pi^0)$$

recent Belle, CLEO measurement

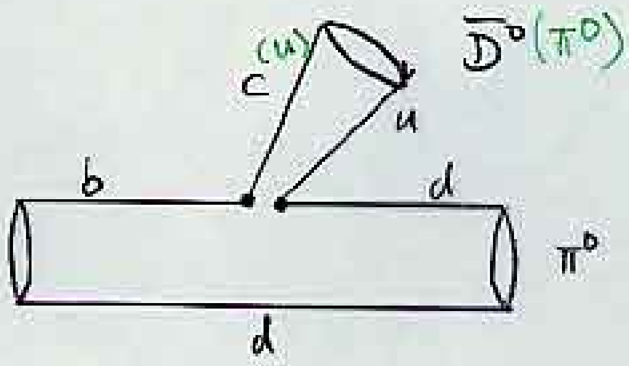
①



operator O_2

$C_2 \sim -0.26$

②



operator O_1

$C_1 \sim 1$

Diagram ① : Wilson coefficient is suppressed

② is color suppressed

① & ② comparable

$\mathcal{M} \propto C_2 + \frac{1}{3} C_1$ small.

Generalized Factorization Approach

$$\mathcal{M}(B^0 \to \pi^+ D^-) \propto C_1 + \frac{1}{N_c^{eff}} C_2 = a_1$$

$$\mathcal{M}(B^0 \to \pi^0 \bar{D}^0) \propto C_2 + \frac{1}{N_c^{eff}} C_1 = a_2$$

$$N_c^{eff} \approx 2$$

Ali, Kramer, Lü

Cheng, Tseng, Yang

can explain $B^0 \to \pi^+ D^-$, $B^+ \to \pi^+ \bar{D}^0$

(a_1) $\quad$ $(a_1 + a_2)$

Class I decay

Although Factorization Approach successfully describe the branching ratios of most D and B meson hadronic Decays,

theoretical improvements are needed:

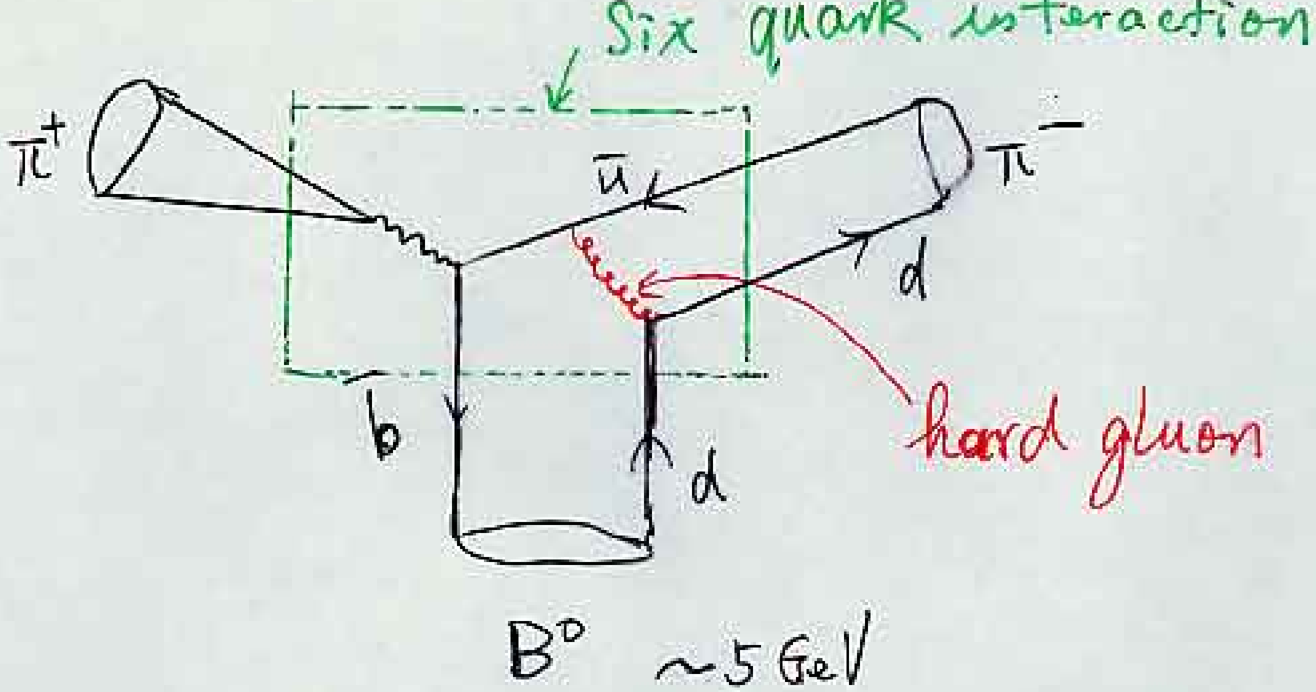
1. Size of non-factorizable contribution
2. knowledge of form factors
3. annihilation type diagrams
4. strong phases

Approaches:

1. QCD Factorization (BBNS)
2. Perturbative QCD

Perturbative QCD Approach (PQCD)

in non-leptonic B Decays



The spectator quark (light quark) of B meson is at rest, needs a hard gluon to kick it strongly to form a fast moving pion.

$$\text{Amplitude} = \underbrace{(\text{Six quark operator})}_{\text{hard part}} \otimes \underbrace{(\text{wave functions})}_{\text{Soft part}}$$

Factorization formula

$$A = C(t) \times H(t) \times \underline{\Phi}(t) \times \exp\left[-S - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s)\right]$$

$C(t)$: Wilson Coefficients of 4-quark operators

$H(t)$: Hard part ~ six quark operator

$\underline{\Phi}(t)$: Wave functions of meson

S : Sudakov factor

Only $H(t)$ is process (channel) dependent

calculable in perturbation theory

ϕ_π, ϕ_K ··· light meson Distribution Amplitude

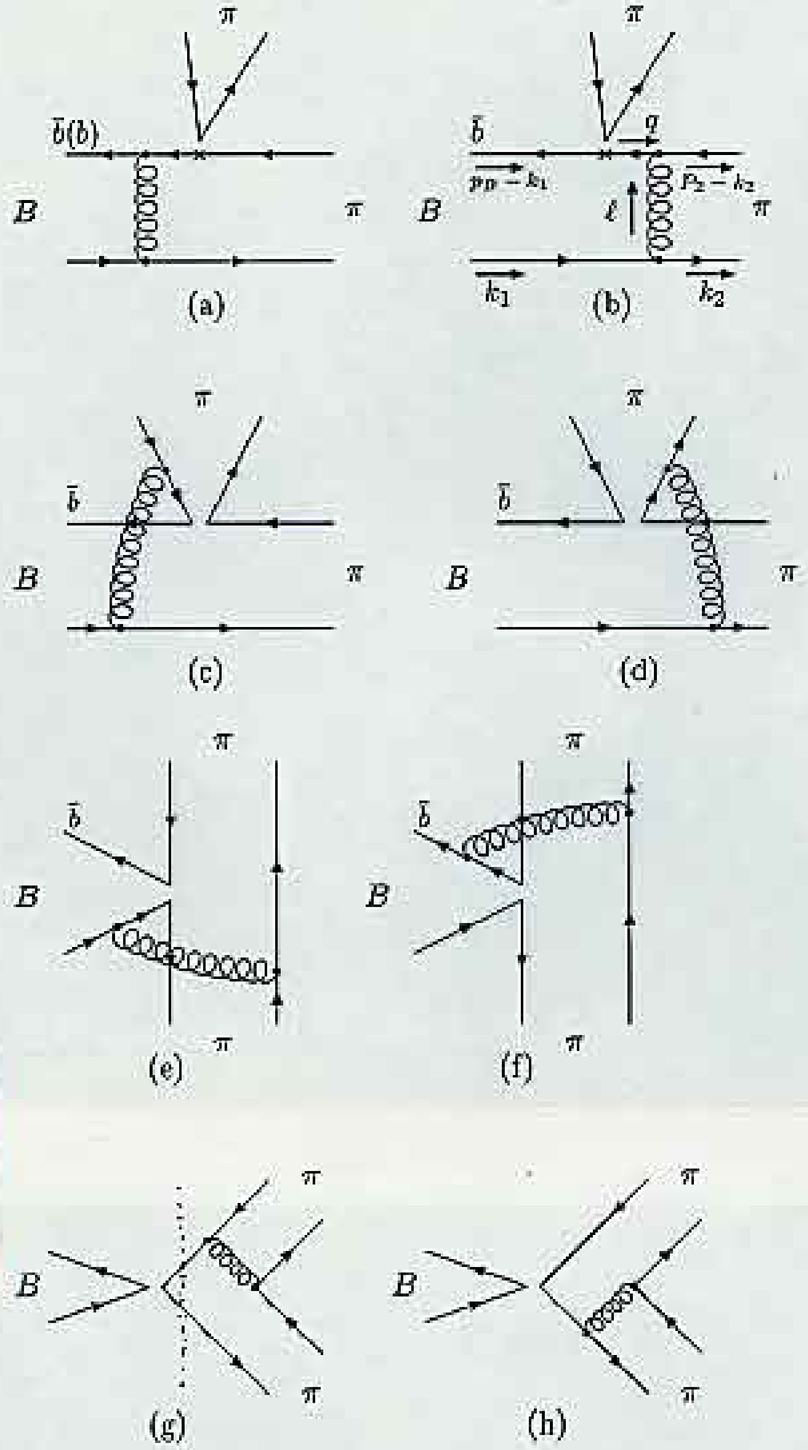
known from QCD Sum rules.

$$\phi_\pi(x) = \frac{f_\pi}{2\sqrt{6}} x(1-x) \left[1 + a_2 C_2^{1/2}(1-2x) + a_4 C_4^{1/2}(1-2x)\right]$$

ϕ_B is unknow. choose quark model.

parameter determined from

semi-leptonotic deccay $F^{B\to\pi}(0) = 0.30 \pm 0.03$



← Factorizable

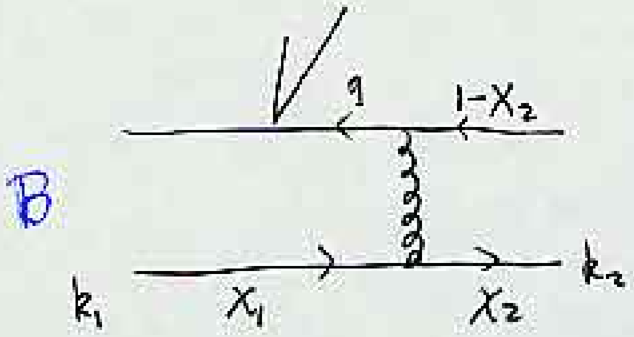
(Form Factor)

Non-factorizable

imaginary part provide strong phase

Annihilation

Figure 3:



$x_i \sim 0 - 1$

gluon: $\frac{1}{q_g^2} = \frac{1}{(k_1 - k_2)^2} = \frac{1}{-x_1 x_2 m_B^2 - (k_1^T - k_2^T)^2}$

quark: $\frac{1}{q_q^2} = \frac{1}{(P_\pi - k_1)^2} = \frac{1}{-x_1 m_B^2 - k_1^{T\,2}}$

$(k_1^2 = k_2^2 = 0 \; , \; P_\pi^2 = 0)$

previous PQCD, $k_i^T = 0 \Rightarrow$

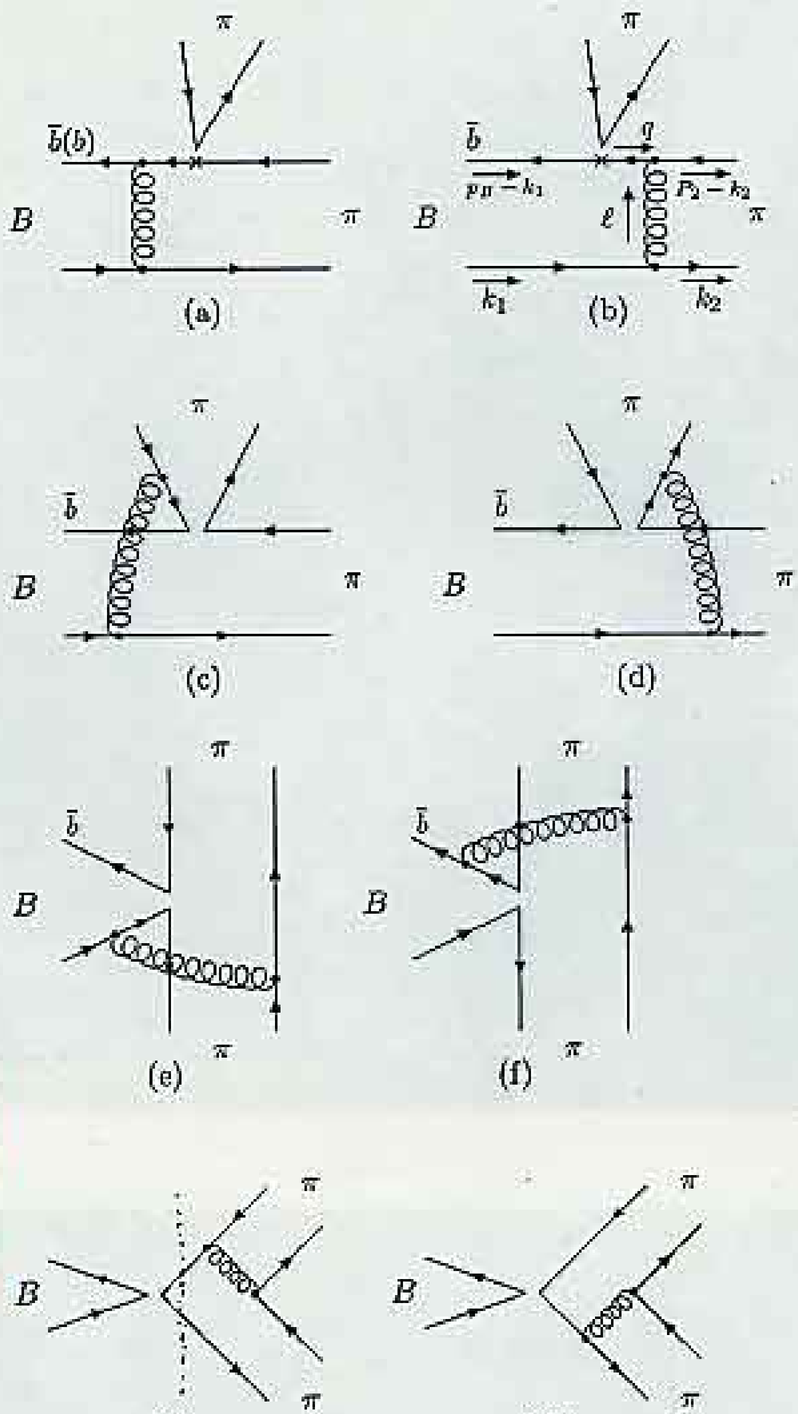
$$\frac{1}{q_g^2 q_q^2} = \frac{1}{x_1^2 x_2 m_B^2}$$

$\phi \sim x(1-x)$

endpoint singularity at $x_1 = 0$

dominant contribution from endpoint

Sudakov factor exponetially suppress endpoint contribution !



← Factorizable

(Form Factor)

Non-factorizable

imaginary part provide strong phase

Annihilation

Figure 3:

Power Counting

emission : annihilation : nonfactorizable

$$= 1 : \frac{2m_0}{m_B} : \frac{\bar{\Lambda}}{m_B}$$

only for $(V-A)(V+A)$ operators O_5, O_6

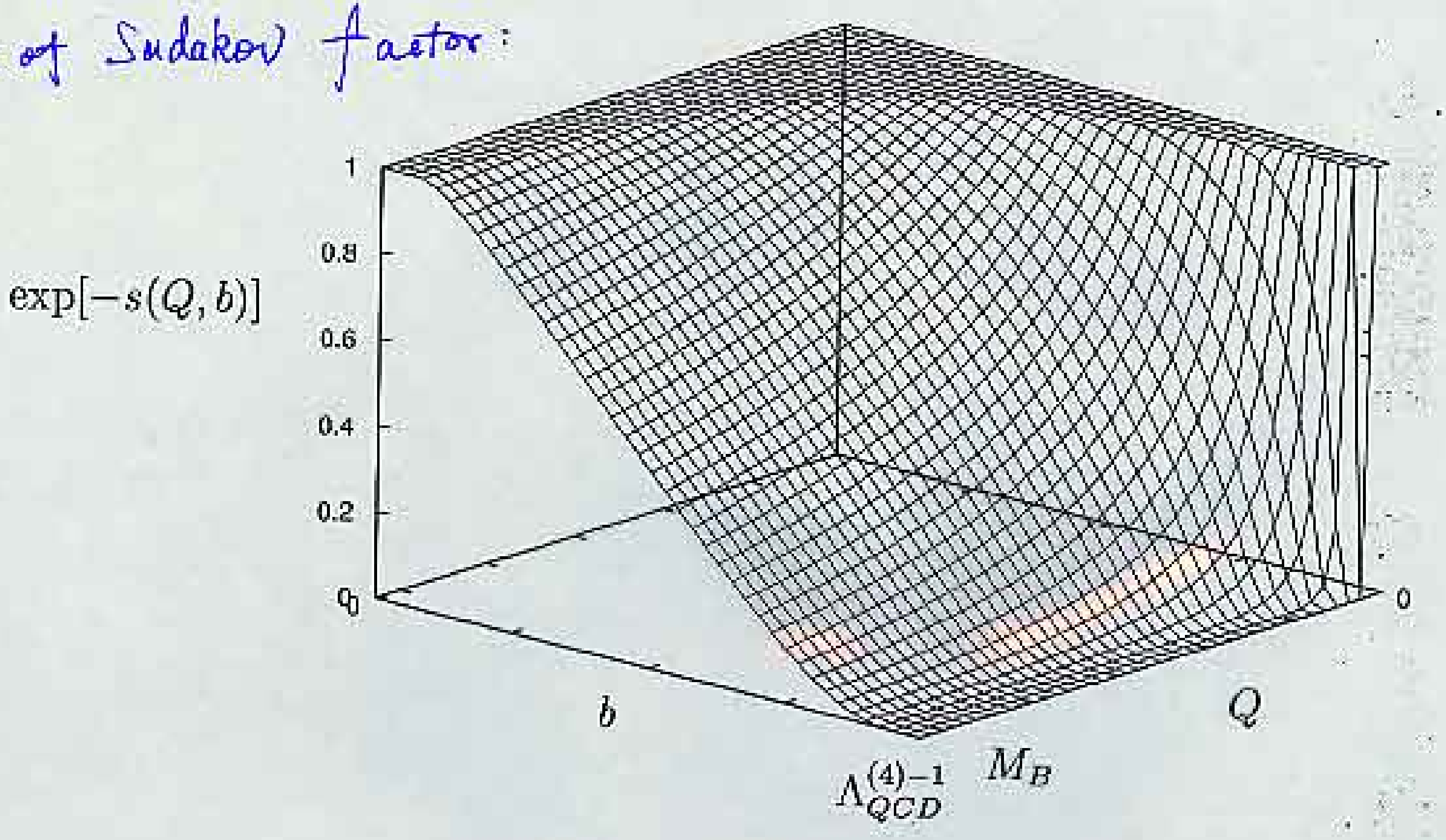
$$m_0 = \frac{m_\pi^2}{(m_u+m_d)} \sim 1.2-1.5 \text{ GeV}$$

In the limit $m_B \to \infty$, emission dominant

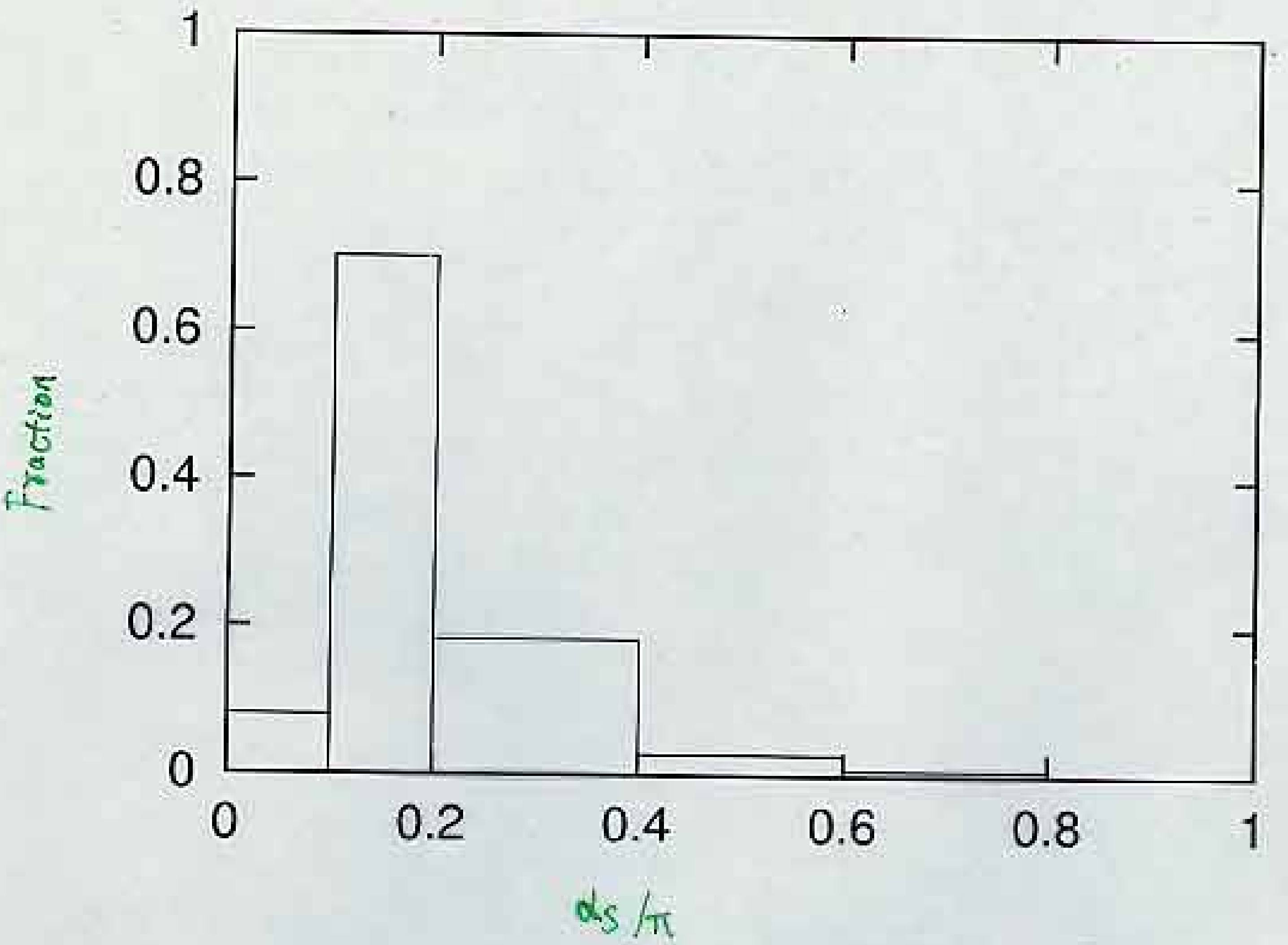
factorization works

$m_B \sim 5$ GeV, annihilation for O_5, O_6 not very small

Effect of Sudakov factor:



Vanishing at large b (small k_T^2)



Contributions as a function of α_s/π

Most contribution comes from $\alpha_s/\pi \sim 0.2$
perturbative region

Numerical Results :

$B \to \pi\pi$, πK Branching Ratios

agree well with exp.

Decay Channel	CLEO	BELLE	BABAR	World Av.	PQCD
$\pi^+\pi^-$	$4.3^{+1.6}_{-1.4} \pm 0.5$	$5.6^{+2.3}_{-2.0} \pm 0.4$	$4.1 \pm 1.0 \pm 0.7$	4.4 ± 0.9	$7.0^{+2.0}_{-1.5}$
$\pi^+\pi^0$	$5.6^{+2.6}_{-2.3} \pm 1.7$	< 13.4	< 9.6	–	$3.7^{+1.3}_{-1.1}$
$\pi^0\pi^0$	< 5.7	–	–	–	0.3 ± 0.1
$K^0\pi^\pm$	$18.2^{+4.6}_{-4.0} \pm 1.6$	$13.7^{+5.7+1.9}_{-4.8-1.8}$	$18.2^{+3.3}_{-3.0} \pm 2.0$	17.3 ± 2.7	$16.4^{+3.3}_{-2.7}$
$K^\pm\pi^\mp$	$17.2^{+2.5}_{-2.4} \pm 1.2$	$19.3^{+3.4+1.5}_{-3.2-0.6}$	$16.7 \pm 1.6 \pm 1.3$	17.3 ± 1.5	$15.5^{+3.1}_{-2.5}$
$K^\pm\pi^0$	$11.6^{+3.0+1.4}_{-2.7-1.3}$	$16.3^{+3.5+1.6}_{-3.3-1.8}$	$10.8^{+2.1}_{-1.9} \pm 1.0$	12.1 ± 1.7	$9.1^{+1.9}_{-1.5}$
$K^0\pi^0$	$14.6^{+5.9+2.4}_{-5.1-3.3}$	$16.0^{+7.2+2.5}_{-5.9-2.7}$	$8.2^{+3.1}_{-2.7} \pm 1.2$	10.4 ± 2.7	8.6 ± 0.3

Table 2: Branching ratios of $B \to \pi\pi$ and $K\pi$ decays with $\phi_3 = 80^0$, $R_b = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV. Unit is 10^{-6}.

Ratios of $B \to \pi\pi$, $B \to \pi K$

Quantity	CLEO	BBNS	PQCD
$\dfrac{Br(B \to \pi^+\pi^-)}{Br(\pi^\pm K^\mp)}$	0.25 ± 0.10	$0.5-1.9$	$0.30-0.69$
$\dfrac{Br(\pi^\pm K^\mp)}{2Br(\pi^0 K^0)}$	0.59 ± 0.27	$0.9-1.4$	$0.78-1.05$

Need a $\phi_3(\gamma) > 90^\circ$ to agree with exp. for $\dfrac{Br(\pi^+\pi^-)}{Br(\pi^\pm K^\mp)}$ in BBNS.

However, in PQCD, it is not necessary.

$B \to \phi K$: may distinguish PQCD & BBNS

10×10^{-6}	PQCD	11	BELLE
4×10^{-6}	BBNS	7-8	BABAR
		5	CLEO

Direct CP Asymmetry :
as a function of CKM angle
(Sanda, Ukai)

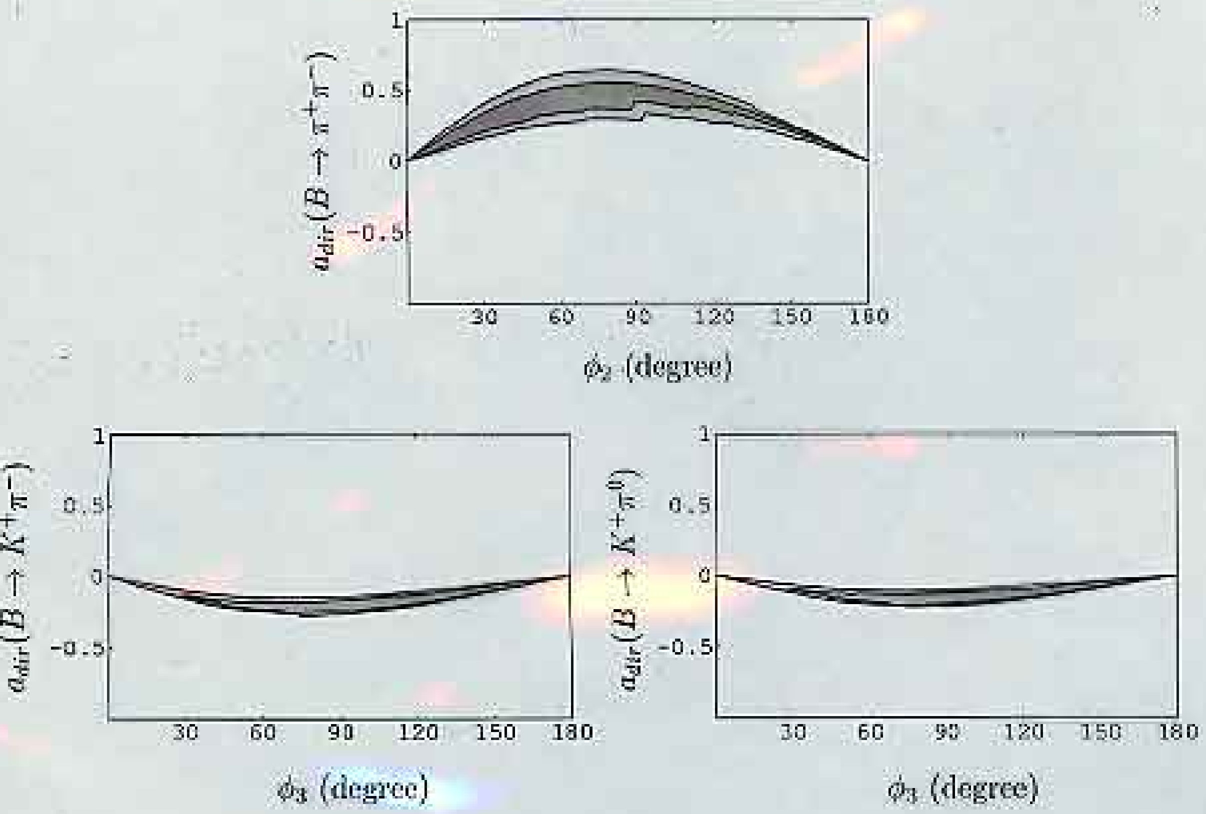


Figure 1: Direct CP asymmetry for $B \to \pi\pi$, $K\pi$ decay modes. The central value of KM factors[?, ?] gives the darker shaded regions, and the lighter shaded regions include the error of KM factors. $a_{dir}(\pi^+\pi^0)$ is almost zero for any ϕ_2. $a_{dir}(K^0\pi^+)$ is almost zero for any ϕ_3. $a_{dir}(K^0\pi^0)$ becomes maximum at $\phi_3 = 90°$, $a_{dir}(K^0\pi^0) = -0.04$.

Mixing induced CP asymmetry of $B \to \pi^+\pi^-$

(Sanda, Ukai)

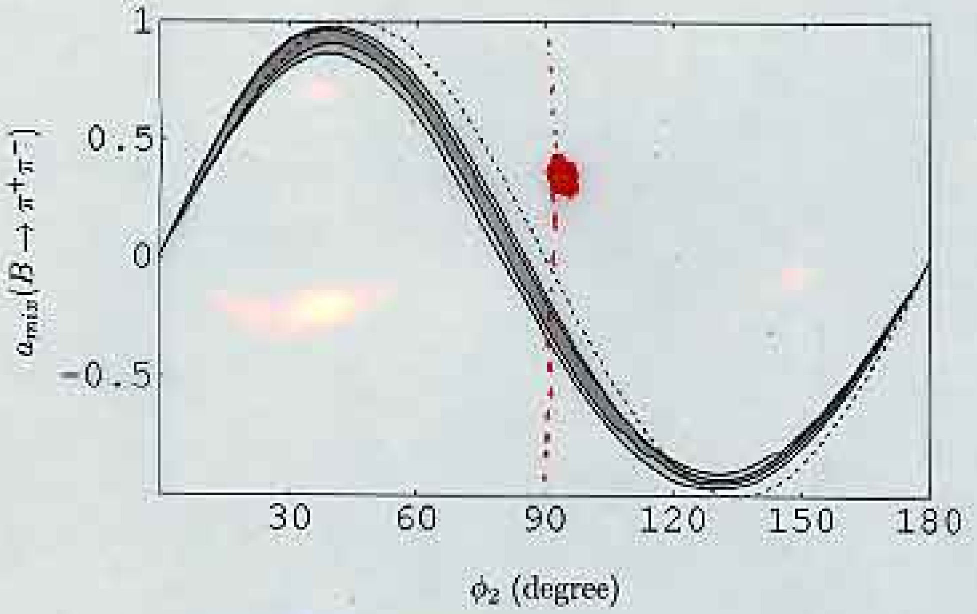


Figure 2: Mixing induced CP asymmetry for $B \to \pi^+\pi^-$. The difference from the dotted line($\sin 2\phi_2$) shows sizeable penguin pollution.

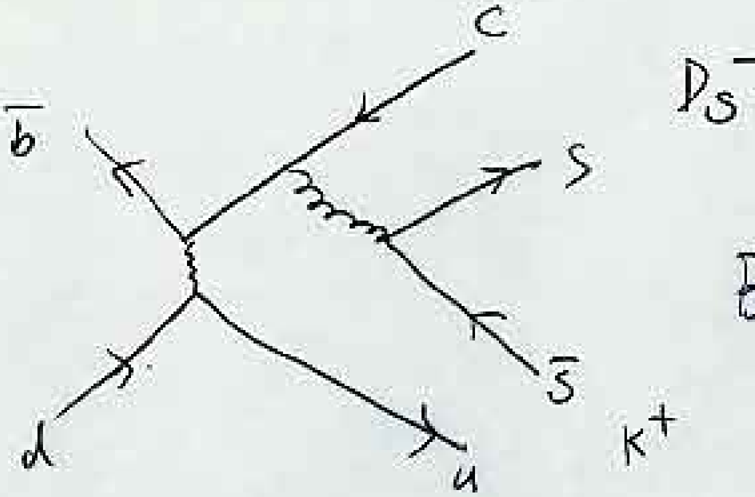
Comparison with experiments

$A_{CP}(\%)$	Experiment (CLEO,BELLE,BABAR)	Theory PQCD	BBNS	CPMPS ($\lvert A_{CP}\rvert$)
π^+K^-	-4.8 ± 6.8 (BaBar) $-7 \pm 8 \pm 2$	$-12.9 \sim -21.9$	5 ± 9	17 ± 6
π^0K^-	-9.6 ± 11.9	$-10.0 \sim -17.3$	7 ± 9	18 ± 6
$\pi^-\bar{K}^0$	-4.7 ± 13.9	$-0.6 \sim -1.5$	1 ± 1	3 ± 3
$\pi^+\pi^-$	-25 ± 48	$15.0 \sim 20.0$	-6 ± 12	58 ± 29

Table 5: CP-asymmetry in $B \to K\pi, \pi\pi$ decays with $\phi_3 = 40^0 \sim 90^0$, $R_b = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV.

$B^0 \to D_s^- K^+$

pure Annihilation

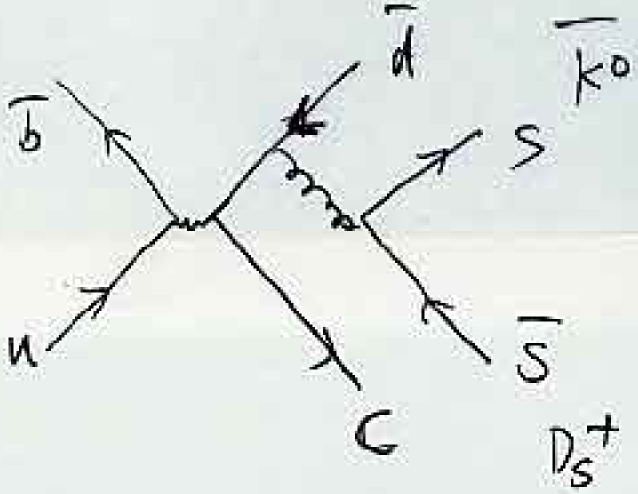


$B_r \sim 10^{-5}$

$\downarrow$

measurable at B factories

$B^+ \to D_s^+ \bar{K}^0$



$B_r \sim 10^{-8}$

C.D. L & Ukai

Summary

- Perturbative QCD approach is a self-consistent theory for non-leptonic two-body B decays..
- The non-factorizable and annihilation type diagrams are important for non-leptonic B decays, especially for CP asymmetry.
- There are large CP asymmetries predicted in the PQCD approach.
- They will be tested by experiments soon.