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- Introduction
- Recent results
- Miss conceptions
- Issue of sign
- Where do we go from here
- Helicity analysis
- pQCD

# Keys to probing new physics

- K-K and B-B mixing
- CP violation
- Loop induced rare decays



$$K \! \Rightarrow \! \pi \pi \! \Rightarrow \! \overline{K}$$

#### Mixing probes 2<sup>nd</sup> order Weak effects

$$\begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} = \begin{pmatrix} M & -\frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & 0 \end{pmatrix}$$
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = \pm \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$$
$$\left| K_{\pm} \right\rangle = \frac{1}{\sqrt{2}} \left\| K_0 \right\rangle \pm \left| \overline{K_0} \right\rangle$$
 CP eigenstate

No matter how small the off diagonal elements are, the solution is same

#### Dynamics of double pendulum



#### Sensitive microscope for the coupling



# Quantum mechanics tells us that To make it hermitian $H = c h + c* h^{\dagger}$

 $CP h CP^{\dagger} = h^{\dagger}$ 

# $CP H CP^{\dagger} = c h^{\dagger} + c^{*}h$ $\neq H \quad unless c = c^{*}$

So, the phase is crucial in CPV

$$\begin{pmatrix} 0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & 0 \end{pmatrix} \begin{pmatrix} p \\ \pm q \end{pmatrix}$$
  
=  $\pm \sqrt{M_{12} - \frac{i}{2}\Gamma_{12}} M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} \begin{pmatrix} p \\ \pm q \end{pmatrix}$ 

$$\begin{array}{l} CP \quad \left| p \middle| K_0 \right\rangle + q \middle| \overline{K_0} \right\rangle = \left| p \middle| \overline{K_0} \right\rangle + q \middle| K_0 \rangle \\ \\ \hline \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad \text{complex} \\ \\ \text{It is sensitive to the hidden phase! Not CP eigenstate} \end{array}$$

It must come from new physics - sheds new light toward understanding quark masses!



#### Mass CPV

#### Amplitude CPV



 $(P \lor V) |^2 \neq |A(\overline{P} \to F_{CP})|^2$ 

S  

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W$$

#### Phases of the KM matrix

#### 3x3 unitary matrix has 9 real parameters

$$q_i \rightarrow e^{i\phi_i}q_i$$

For 6 quarks, there are 5 phases We xcan adjust

1 CPV phase

KM matrix has 4 parameters 3 rotation angle





#### $M_{12}$ becomes complex

#### A(K $\rightarrow \pi\pi$ ) amplitude becomes complex







#### Penguins and me in Cape Town, Jan., 1999



## Experimental result

$$\frac{\varepsilon'}{\varepsilon} = \begin{cases} (15.3 \pm 2.6) \times 10^{-4} & NA48\\ (20.7 \pm 2.8) \times 10^{-4} & KTeV \end{cases}$$

# Theory:

$$\frac{\varepsilon'}{\varepsilon} = 13 \times \operatorname{Im}(V_{ts}V_{td}^*) \left(\frac{110MeV}{m_s(2GeV)}\right)^2 \left[ B_6^{(1/2)}(1-\Omega_{IB}) - 1.4B_8^{(3/2)} \left[\frac{m_t}{165GeV}\right]^{2.5} \right]$$
QCD penguin EW penguin

Matrix elements are poorly known And they cancel! So, for now there is no constraint

Experimental value can be understood with B's =1

# Second candidate for $K^+ \rightarrow \pi^+ + \nu + \overline{\nu}$

1)



 $\rho_0 \ge 1.5$ 

# Mixing and CPV in B decays







# Gold plated decays





# CPV in B decays





$$V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*} + V_{tb}V_{td}^{*} = 0$$

# Discovery of CP violation in B decay

# $\sin(2\phi_1) = \begin{cases} (0.99 \pm 0.14 \pm 0.06)(Belle)\\ (0.59 \pm 0.14 \pm 0.05)(Babar) \end{cases}$

## **GLOBAL FIT: RESULTS**

#### <u>Global fit including sin28:</u>





$$\overline{\xi}_{td}^2 = sign((1-\rho)^2 - \eta^2) |\xi_{td}^2|$$

$$\frac{\Gamma(\overline{B}(t) \to \psi K_S) - \Gamma(B(t) \to \psi K_S)}{\Gamma(\overline{B}(t) \to \psi K_S) + \Gamma(B(t) \to \psi K_S)} = \sin(2\phi_1)\sin(\Delta M t)$$



## Correct allowed region on the $\rho-\eta$ plane





 $\Gamma(\overline{B}(t_B) \to [\mu^- \pi^+ \overline{\nu}]_{\overline{K}} \psi) \propto [A + B + C + D]$   $\Gamma(\overline{B}(t_B) \to [\mu^+ \pi^- \nu]_K \psi) \propto [A - B + C - D]$   $\Gamma(B(t_B) \to [\mu^- \pi^+ \overline{\nu}]_{\overline{K}} \psi) \propto [A - B - C - D]$  $\Gamma(B(t_B) \to [\mu^+ \pi^- \nu]_K \psi) \propto [A + B - C + D]$ 

$$A = \frac{1}{4}e^{-\Gamma_{S}t_{K}-\Gamma_{B}t_{B}}[1+e^{\Delta\Gamma t_{K}}]$$

$$B = \frac{1}{2}e^{-\Gamma_{S}t_{K}-\Gamma_{B}t_{B}}e^{\Delta\Gamma t_{K}}\cos(\Delta M_{B}t_{B})\cos(\Delta M_{K}t_{K})$$

$$C = \frac{1}{4}e^{-\Gamma_{S}t_{K}-\Gamma_{B}t_{B}}[1-e^{\Delta\Gamma t_{K}}]\sin(\Delta M_{B}t_{B})\sin(2\phi_{1})$$

$$D = \frac{1}{2}e^{-\Gamma_{S}t_{K}-\Gamma_{B}t_{B}}e^{\Delta\Gamma t_{K}}\sin(\Delta M_{B}t_{B})\cos(2\phi_{1})\sin(\Delta M_{K}t_{K})$$

# Detailed simulation is needed to understand the best strategy

$$\frac{\Gamma(B(t_B) \to \psi[\mu^- \overline{\nu} \pi^+]_{\overline{K}(t_K)}) - \Gamma(B(t_B) \to \psi[\mu^+ \nu \pi^-]_{K(t_K)})}{\Gamma(\overline{B}(t_B) \to \psi K_S) + \Gamma(B(t_B) \to \psi K_S)} = \frac{\Gamma(K_S \to \mu \nu \pi)}{\Gamma(K_S \to \pi^+ \pi^-)} e^{\frac{1}{2}\Delta\Gamma t_K} [\cos(\Delta M_B t_B) \cos(\Delta M_K t_K) + \sin(\Delta M_B t_B) \sin(\Delta M_K t_K) \cos(2\phi_1)]$$



$$\frac{\Gamma(\overline{B}(t) \to \pi\pi) - \Gamma(B(t) \to \pi\pi)}{\Gamma(\overline{B}(t) \to \pi\pi) + \Gamma(B(t) \to \pi\pi)} = \operatorname{Im}\left(\frac{q}{p}\frac{A(\overline{B} \to \pi\pi)}{A(B \to \pi\pi)}\right)\sin(\Delta M t)$$

$$(p \ A(B \to \pi\pi))$$
  $(M_{12} \ Te^{i(-\phi_T + \delta_T)} + Pe^{i(-\phi_P + \delta_P)})$ 

# Two clean possibilities:

$$\operatorname{Im}\left(\frac{M_{12}^{*}}{M_{12}}\frac{Te^{i(\phi_{T}+\delta_{T})}+Pe^{i(\phi_{P}+\delta_{P})}}{Te^{i(-\phi_{T}+\delta_{T})}+Pe^{i(-\phi_{P}+\delta_{P})}}\right)$$

$$\begin{cases} \operatorname{Im}\left(\frac{M_{12}^{*}}{M_{12}}\frac{e^{i\phi}}{e^{-i\phi}}\right) \\ \operatorname{Im}\left(\frac{M_{12}^{*}}{M_{12}}\frac{e^{i\phi_{T}}}{e^{-i\phi_{T}}}\right) \end{cases}$$

$$f \phi = \phi_P = \phi_T$$

If P=0

# Penguins are larger than we thought

#### $Br(B \to K\pi) \ge Br(B \to \pi\pi)$

Implies that

$$\frac{P}{T} = \mathcal{O}(\lambda) \approx 0.23$$

# Isospin Analysis



$$asym = \left|\frac{1+\overline{z}}{1+z}\right| \sin\left[2\phi_2 + \arg\left(\frac{1+\overline{z}}{1+z}\right)\right]$$
$$z = \frac{\sqrt{2}A_0}{A_2} \qquad \overline{z} = \frac{\sqrt{2}\overline{A}_0}{\overline{A}_2}$$

$$\frac{\Delta \phi_2}{\phi_2} = \sqrt{\frac{(0.1\sigma_{+-})^2 + (0.16\sigma_{00})^2 + (0.5\sigma_{+0})^2}{(1.26\sigma_{+-})^2 + (0.28\sigma_{00})^2 + (0.11\sigma_{asym})^2}}$$

$$\sigma_{+-} = \frac{\Delta A(B \to \pi^+ \pi^-)}{A(B \to \pi^+ \pi^-)}$$
$$\overline{\sigma}_{+-} = \frac{\Delta A(\overline{B} \to \pi^+ \pi^-)}{A(\overline{B} \to \pi^+ \pi^-)}$$

To get 1% error on  $\phi_2$ 

$$\sigma_{+-} = 4\%$$
  $\sigma_{00} = 2.5\%$   $\sigma_{+0} = .8\%$   
 $\overline{\sigma}_{+-} = .3\%$   $\overline{\sigma}_{00} = 1.4\%$   $\sigma_{asym} = 4\%$ 

$$\left(\frac{1.26}{.28}\right)^2 \approx 20 \ge \frac{Br(B \to \pi^+ \pi^-)}{Br(B \to \pi^0 \pi^0)}$$

Need  $4.4 \times 10^5$   $B \rightarrow \pi^+ \pi^-$ With  $Br(B \rightarrow \pi^+ \pi^-) = 4.3 \times 10^{-6}$  $10^{11}B's$ 

Luminosity 10<sup>37</sup>

# where do we go from here

It is often implied: any asymmetry which does not lead to a clean determination of the KM element is not worth measuring. This is a wrong attitude! Just keep at it. We will learn how to extract new physics as we go along.

 $\frac{Br(B^- \to D_{1,2}K^-) - Br(B^+ \to D_{1,2}K^+)}{Br(B^- \to D_{1,2}K^-) + Br(B^+ \to D_{1,2}K^+)}$ asym =





$$asym = -\frac{2ab\sin\phi_3\sin(\delta'-\delta)}{a^2 + b^2 + 2ab\cos\phi_3\cos(\delta'-\delta)}$$

# B→K\*+D\* helicity analysis



# $B \rightarrow K^*D^*$ has 3 helicity amplitudes: 00, +-, -+

$$6 - a_{\lambda} e^{i\delta_{\lambda}^{a}}$$
$$6 - b_{\lambda} e^{i\delta_{\lambda}^{b}}$$
$$2 - \phi_{a}, \phi_{b}$$

$$A_{\lambda} = Amp(B \to f)_{\lambda} = a_{\lambda}e^{i\delta_{\lambda}^{a}}e^{i\phi_{a}}$$
$$A'_{\lambda} = Amp(\overline{B} \to f)_{\lambda} = b_{\lambda}e^{i\delta_{\lambda}^{b}}e^{i\phi_{b}}$$
$$\overline{A}'_{\lambda} = Amp(B \to \overline{f})_{\lambda} = b_{\lambda}e^{i\delta_{\lambda}^{b}}e^{-i\phi_{b}}$$
$$\overline{A}_{\lambda} = Amp(\overline{B} \to \overline{f})_{\lambda} = a_{\lambda}e^{i\delta_{\lambda}^{a}}e^{-i\phi_{a}}$$

14 parameters



$$|A(B \to K^* D^*)|^2 \propto$$
  
$$\sum_{\lambda \sigma} g_{\lambda}(\theta_1, \theta_2, \phi) g_{\sigma}(\theta_1, \theta_2, \phi) [\Lambda_{\lambda \sigma} + \Sigma_{\lambda \sigma} \cos(\Delta M t) - \Xi_{\lambda \sigma} \sin(\Delta M t)]$$



#### 6x3=18 observables

$$6 - a_{\lambda} e^{i\delta_{\lambda}^{a}}$$

$$6 - b_{\lambda} e^{i\delta_{\lambda}^{b}}$$

$$2 - \phi_{a}, \phi_{b}$$
14 parameters

4 parameters

#### Nonleptonic 2 body

$$B \rightarrow \pi^{+}\pi^{-}, \pi^{0}\pi^{0}, \eta'\eta', \eta\eta', \pi^{0}\eta', \pi^{0}\eta \\ K_{S}\pi^{0}, K_{S}\eta', K_{S}\eta, \rho^{0}K_{S}, \phi K_{S}, \\ K^{0}\bar{K}^{0}, K^{;}K^{-}, K^{*0}\bar{K}^{*0}, K^{*+}K^{*-} \\ \rho^{0}\pi^{0}, \omega\pi^{0}, \rho^{0}\eta, \rho^{0}\eta', \omega\eta, \omega\eta', \\ \phi\pi^{0}, \phi\eta, \phi\eta', \rho^{+}\rho^{-}, \omega\omega, \rho^{0}\omega, \rho^{0}\phi', \end{cases}$$

Over 70 decay modes

# History of pQCD approach

- Brodsky Lepage PR D22,2157(80)
- Isgar Llewellynsmith NPB317,526(89)
- Botts Sterman NP B325, 62(89)
- Li and his collaborators
- Kroll Eur.Phys.J.C12,99(00)
- Li, Keum, AIS hep-ph/0004173 PR

#### Feynman's Mistake? Pion form factor



F (Q <sup>2</sup>)

Depends on wee dynamics Cannot be computed by perturbative QCD

#### Feynman's reasoning – Naive QCD



$$\langle \pi(p_2) | J_{\mu}(0) | \pi(p_1) \rangle = g^2 \int \frac{d^4 k_1 d^4 k_2 d^4 x_1 d^4 x_2}{(2\pi)^4 (2\pi)^4} e^{-ik_2 y} \langle \pi | \overline{u}_{\gamma}(0) d_{\beta}(y) | 0 \rangle$$
$$\times e^{ik_1 x} \langle 0 | \overline{d}_{\alpha}(x) u_{\delta}(0) | \pi \rangle T_H^{\gamma\beta;\alpha\delta}$$

$$T_H^{\gamma\beta;\alpha\delta} = [\gamma_\sigma]^{\alpha\beta} \frac{1}{(k_2 - k_1)^2} \left[ \gamma_\mu \frac{\not\!\!\!\!/ P - \not\!\!\!\!/ k_2}{(P - k_2)^2} \gamma^\sigma \right]^{\gamma\delta} \,.$$

$$(k_2 - k_1)^2 \propto x_1 x_{22}$$

#### Infrared singularity! Infrared singularity!

Isgar Llewellynsmith NPB317,526(89) Isgar Llewellynsmith

#### Feynman says small x and small $k_{\perp}$ dominates

This is not so in QCD When quark and anti-quark are far

apart in space they radiate gluons

Small  $x \Rightarrow$  large longtudinal separation

#### Small $k_{\perp} \Rightarrow$ large transverse separation

Probability of not emitting any gluon: Sudakov factor suppress these regions



# Sudakov Factor in QED





# Remember!

Even if you emit one gluon, you don't get  $B - > \pi\pi$ 

#### PQCD approach to pion form factor



#### Pion form factor





Sudakov factor

# $B \to \pi~$ transition form factor

$$\int \frac{d^4y}{(2\pi)^4} e^{ik_1y} \langle 0|\bar{b}_{\beta}(0)d_{\gamma}(y)|B(P_1)\rangle = -\frac{i}{\sqrt{2N_c}} \left[ (\not P_1 + M_B)\gamma_5(\phi_B(k_1) + (\not h_+ - \not h_-)\bar{\phi}_B(k_1)) \right]_{\gamma\beta}$$







The diagram which produces strong interaction phase -> CP violation





#### I can't make it small. Non-leading effects?

final state	exp. $data(10^{-5})$	min.	max
$\pi^+\pi^-$	$0.43^{+0.16}_{-0.14} \pm 0.05$ (CLEO)	0.67	1.14
	$0.56^{+0.23}_{-0.00} \pm 0.04$ (BELLE)	0.67	1.14
	$0.41 \pm 0.10 \pm 0.07$ (BABAR)	0.67	1.14
$\pi^0 \pi^0$	10 10	0.010	0.028
$\pi^+\pi^0$		0.44	0.50
$K^+\pi^-$	$1.72^{+0.25}_{-0.24} \pm 0.12$ (CLEO)	1.26	2.64
	1.93 +0.34 +0.15 -0.32 -0.06 (BELLE)	1.26	2.64
	$1.67 \pm 0.16 \pm 0.13$ (BABAR)	1.26	2.64
$K^+\pi^0$	1.16 +0.30 +0.14 -0.27 -0.13 (CLEO)	0.88	1.70
	1.63 +0.35 +0.16 (BELLE)	0.88	1.70
	$1.08^{+0.21}_{-0.19} \pm 0.10(BABAR)$	0.88	1.70
$K^0\pi^+$	$1.82^{+0.46}_{-0.40} \pm 0.16$ (CLEO)	2.03	2.06
	1.37 +0.57 +0.19 0.18 (BELLE)	2.03	2.06
	$1.82^{+0.33}_{-0.30} \pm 0.20$ (BABAR)	2.03	2.06
$K^0\pi^0$	1.46 <sup>+0.59</sup> +0.24(CLEO)	0.74	0.77
	1.60 +0.72 +0.25 (BELLE)	0.74	0.77
	$0.82^{+0.31}_{-0.27} \pm 0.12$ (BABAR)	0.74	0.77
$\pi^+  ho^-$	$2.76^{+0.84}_{-0.74} \pm 0.42$ (CLEO)	2.39	3.37
	$2.89 \pm 5.4 \pm 0.43$ (BABAR)	2.39	3.37
$\pi^+ \rho^0$	$1.04^{+0.33}_{-0.24} \pm 0.21$ (CLEO)	0.48	0.59
$\pi^0 \rho^+$	-0.54	0.64	1.01
$\pi^0 \rho^0$		0.008	0.011
$\pi^+\omega$	$1.13^{+0.33}_{-0.29} \pm 0.14$ (CLEO)	0.43	0.81
	$0.66^{+0.21}_{-0.18} \pm 0.07(BABAR)$	0.43	0.81
$\pi^0 \omega$		0.010	0.028
$\phi K^+$	$0.55^{+0.21}_{-0.18} \pm 0.06$ (CLEO)	1.01	
	$1.06^{+0.21}_{-0.19} \pm 0.22$ (BELLE)	1.01	
	$0.77^{+0.16}_{-0.14} \pm 0.08(BABAR)$	1.01	
$\phi K^0$	$0.87^{+0.38}_{-0.30} \pm 0.15$ (BELLE)	0.943	
	$0.81^{+0.31}_{-0.25} \pm 0.08(BABAR)$	0.943	

# Summary

- Dreams become reality over 20 yers.
- What used to be a dream is being realized in rare K decays.
- Detecting large CPV in B decay has been a dream for a long time.
- Lets hope that our dreams in B decays will also become reality.
- $K_L \rightarrow \pi^0 v \overline{v}$  will be measured eventually.