Theoretical Review of Rare B Decays

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DESY, Hamburg

1. Interest in Rare B Decays
2. The Effective Hamiltonian
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4. Inclusive $B \to X_s\ell^+\ell^-$ in NNLO in SM
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WIN 2002, Christchurch, January 21 –26, 2002

Interest in Rare B-Decays

- Rare B Decays ($b \to s\gamma$, $b \to d\gamma$, $b \to s\ell^+\ell^-$, $b \to d\ell^+\ell^-$,...) & Particle -Antiparticle Mixings (B^0 - $\overline{B^0}$, B_s^0 - $\overline{B_s^0}$,...) represent Flavour-Changing-Neutral-Current (FCNC) Processes

- In SM, No Tree-Level FCNC Processes allowed;
  Induced FCNC transitions require Loops (Penguins, Boxes);
  Governed by GIM Mechanism $\Longrightarrow$ Sensitivity to Higher scales (m_t,...)

- Provide valuable information on the top quark couplings
  $\Longrightarrow$ V_{td}, V_{ts}, V_{tb} & Test CKM unitarity

- A Laboratory for QCD Technology (HQET, Lattice-QCD, QCD-SR)

- May reveal New Physics, such as Supersymmetry

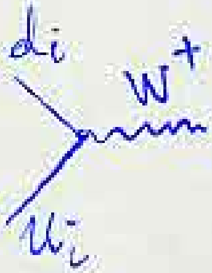
- Of great topical interest for the past, present and planned experiments (CLEO, LEP, Tevatron, B Factories, BTEV, LHC)

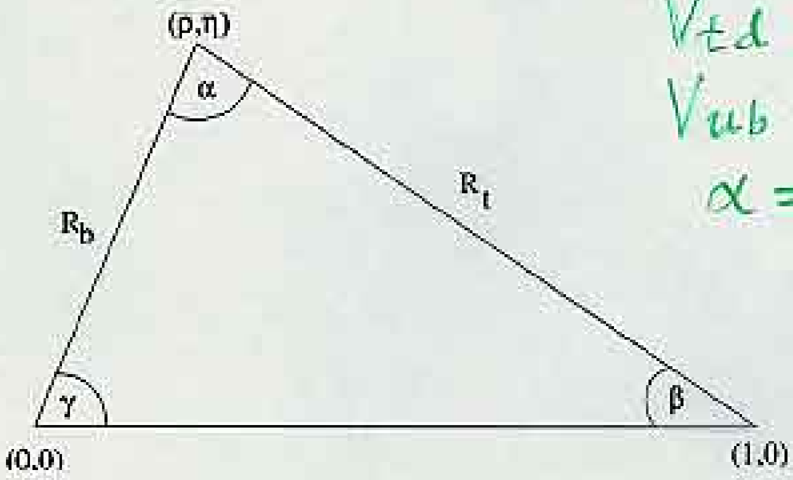
Cabibbo-Kobayashi-Maskawa

The Unitarity Triangle

Wolfenstein parametetrization of the CKM-Matrix

$$
V = \begin{array}{c c}
 & \begin{array}{ccc} d & s & b \end{array} \\
\begin{array}{c} u \\ c \\ t \end{array} & \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}
\end{array}
$$





$V_{td} = |V_{td}|\, e^{-i\beta}$

$V_{ub} = |V_{ub}|\, e^{-i\gamma}$

$\alpha = \pi - \gamma - \beta$

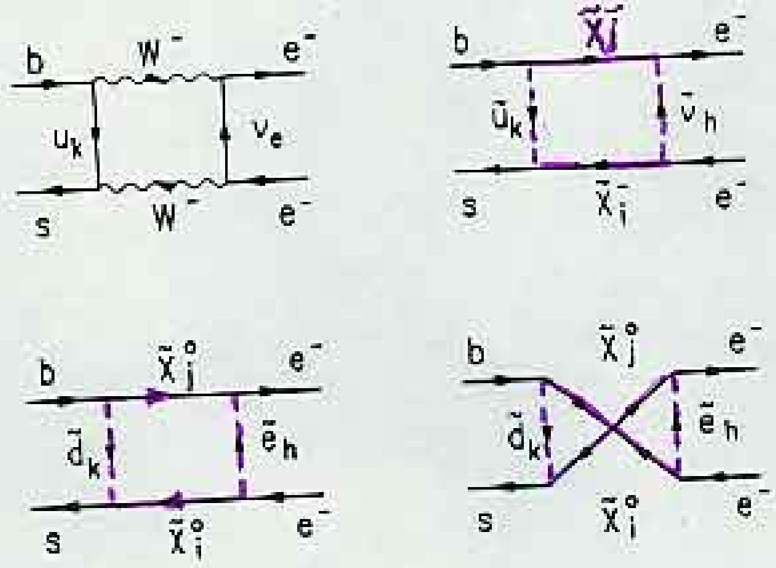
$\boxed{\lambda \simeq 0.22}$

$$R_b = \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = |\bar\rho - i\bar\eta|$$

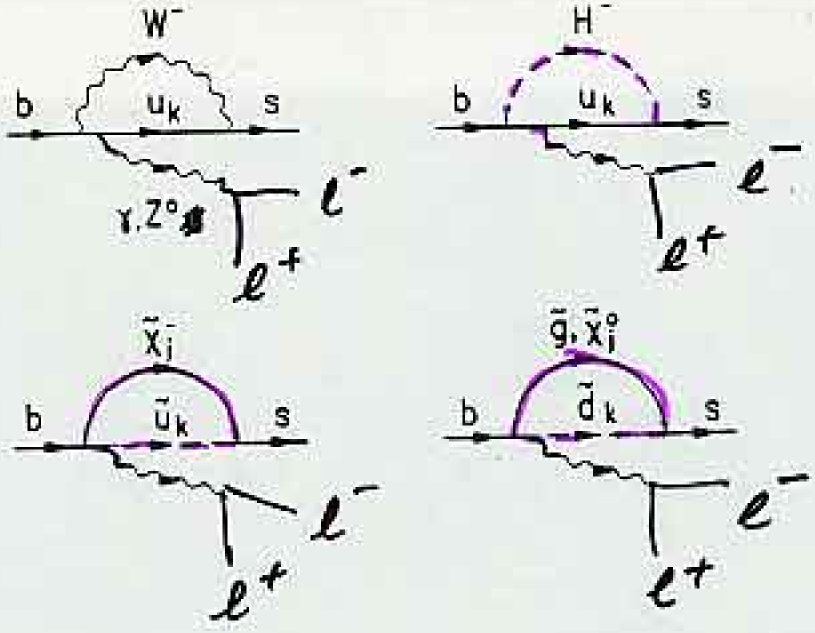
$$R_t = \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = |1 - \bar\rho - i\bar\eta|$$

$$\bar\rho = \rho(1 - \frac{\lambda^2}{2}), \quad \bar\eta = \eta(1 - \frac{\lambda^2}{2})$$

$b \to s \ell^+ \ell^-$ in SUSY



$b \to s \gamma$ in SUSY



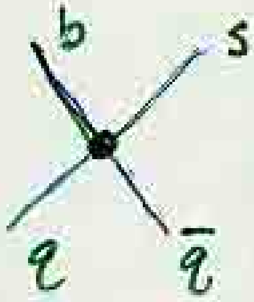
Effective Hamiltonian

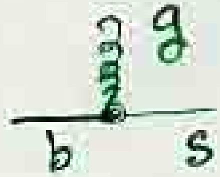
$$\mathcal{H}_{eff}(b \to s + \ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

$O_{1,\ldots,6}$: 4-Fermi operators, O_8: b-s-gluon vertex, effects only through operator mixings

LD

$$\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu c_{L\beta}) \\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})(\bar{c}_{L\beta}\gamma^\mu c_{L\alpha}) \\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}) \\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}) \\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}) \\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}) \\
O_8 &= \frac{g}{16\pi^2} \bar{s}_\alpha T^a_{\alpha\beta} \sigma_{\mu\nu}(m_b R + m_s L) b_\beta G^{a\mu\nu}
\end{aligned}$$

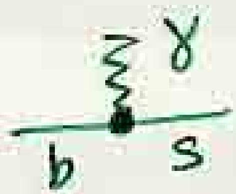


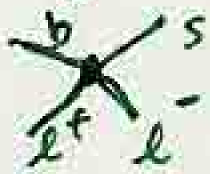


Dominant Operators:

SD

$$\begin{aligned}
O_7 &= \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu}(m_b R + m_s L) b_\alpha F^{\mu\nu}, \\
O_9 &= \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell}\gamma_\mu\gamma_5 \ell\,.
\end{aligned}$$





Wilson Coefficients

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \cdots$$

Estimates of $\mathcal{B}(B \to X_s + \gamma)$ & $|V_{ts}|$

- Lowest order (One-Loop) contribution to $b \to s + \gamma$

$$\mathcal{M}(b \to s+\gamma) = \frac{G_F}{\sqrt{2}}\frac{e}{2\pi^2}\sum_{i=u,c,t} \lambda_i F_2(x_i) q^\mu \epsilon^\mu \bar{s}\sigma_{\mu\nu}\left(m_b R + m_s L\right) b$$

$$L = (1-\gamma_5)/2; \quad R = (1+\gamma_5)/2; \quad x_i = m_i^2/m_W^2$$

- Inami-Lim Function

$$F_2(x) = \frac{x}{24(x-1)^4}\left[6x(3x-2)\ln x - (x-1)(8x^2+5x-7)\right]$$

- CKM Factors

$$\lambda_i \equiv V_{ib}V_{is}^*; \quad \text{CKM Unitarity:} \quad \sum_{i=u,c,t} \lambda_i = 0$$

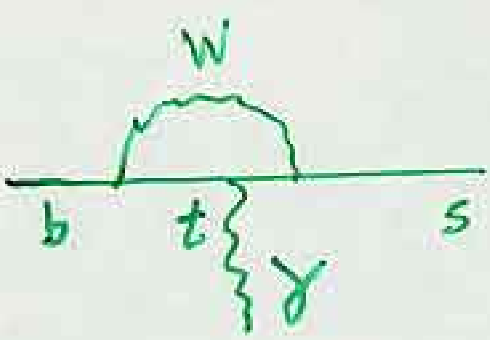
- Since $\lambda_u/\lambda_c \ll 1$, CKM Unitarity implies $\lambda_c \simeq -\lambda_t$
$\Longrightarrow$

$\lambda_t = V_{tb}V_{ts}^*$
$\simeq V_{ts}^*$

$$\begin{aligned}\mathcal{M}(b \to s+\gamma) &= \frac{G_F}{\sqrt{2}}\frac{e}{2\pi^2}\lambda_t \\ &\times \left(F_2(x_t) - F_2(x_c)\right) q^\mu \epsilon^\mu \bar{s}\sigma_{\mu\nu}\left(m_b R + m_s L\right) b\end{aligned}$$

$\Longrightarrow \Gamma(b \to s+\gamma)$ depends on m_t and $\lambda_t = V_{tb}V_{ts}^*$

- Since m_t known, $\mathcal{B}(b \to s+\gamma)$ measures the CKM ratio $|\lambda_t/V_{cb}|$



$+ \cdots$

$\mathcal{B}(B \to X_s\gamma)$ in LO & NLO

- A truly cooperative effort by several groups!

- LO Anomalous Dimension Matrix [Ciuchini et al.; Cella et al.; Misiak]
- NLO Anomalous Dimension Matrix [Chetyrkin,Misiak,Münz]
- NLO Virtual Corrections in ME [Greub,Hurth,Wyler]
- Matching Conditions [Adil,Yao; Greub,Hurth; Buras,Kwiatkowski,Pott]
- Bremsstrahlung Corrections [Greub, A.A.; Pott]
- E_γ-spectrum [Greub;A.A.]
- Scale dependence, E_γ-spectrum [Neubert, Kagan]

$$\mathcal{B}(B \to X_s\gamma) = [\frac{\Gamma(B \to \gamma + X_s)}{\Gamma_{SL}}]^{th}\,\mathcal{B}(B \to X\ell\nu_\ell)$$

- SM (pole mass): $\mathcal{B}(B \to X_s\gamma)$
$= [(3.35 \pm 0.30) \times 10^{-4}](|V_{ts}^* V_{tb}/V_{cb}|/0.976)^2$

- SM ($\overline{MS}$ mass): $\mathcal{B}(B \to X_s\gamma)$
$= [(3.73 \pm 0.30) \times 10^{-4}](|V_{ts}^* V_{tb}/V_{cb}|/0.976)^2$

- Expt.(LP '01): $\mathcal{B}(B \to X_s\gamma) = [(3.22 \pm 0.40) \times 10^{-4}$
$\Longrightarrow \quad |\frac{V_{ts}^* V_{tb}}{V_{cb}}| = 0.96 \pm 0.075$
[cf. Unitarity fits: $|\frac{V_{ts}^* V_{tb}}{V_{cb}}| = 0.976 \pm 0.010$]
- Using the present measurements

$$|V_{cb}| = 0.04 \pm 0.002, \quad |V_{tb}| \simeq 1.0$$

$\Longrightarrow \quad |V_{ts}| = 0.038 \pm 0.003$

$B \to X_s \gamma$

(CLEO) hep-ex/0108032

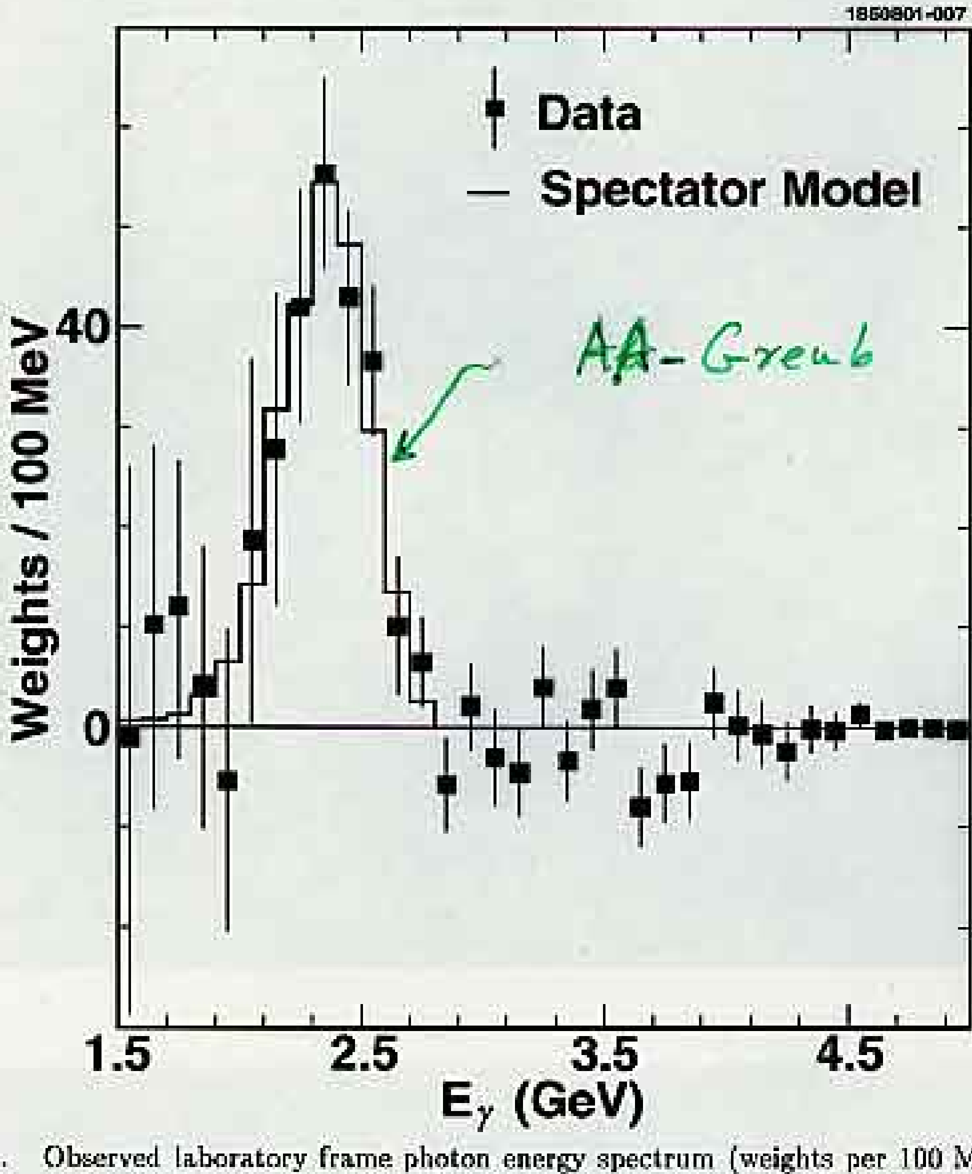
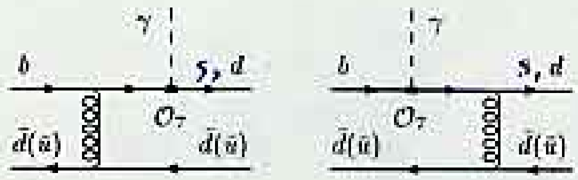


FIG. 2. Observed laboratory frame photon energy spectrum (weights per 100 MeV) for On minus scaled Off minus B backgrounds, the putative $b \to s\gamma$ plus $b \to d\gamma$ signal. No corrections have been applied for resolution or efficiency. Also shown is the spectrum from Monte Carlo simulation of the Ali-Greub spectator model with parameters $\langle m_b \rangle = 4.690$ GeV, $P_F = 410$ MeV/c, a good fit to the data.

Hard Spectator Contributions in $B \to (K^*, \rho)\gamma$

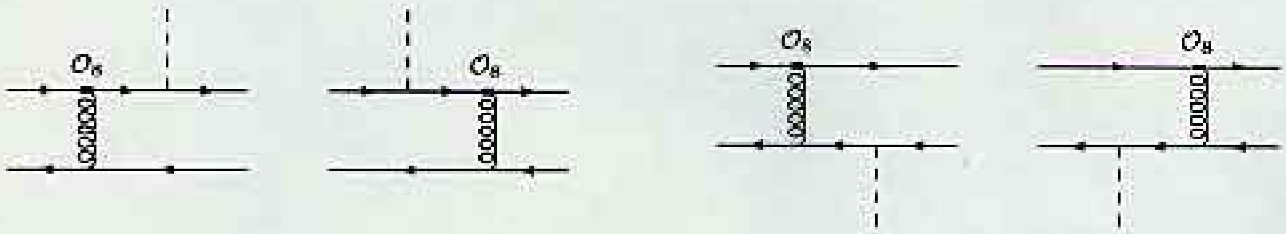
- Spectator corrections due to $\mathcal{O}_7$

$\bar{B}^0, \bar{B}^-$

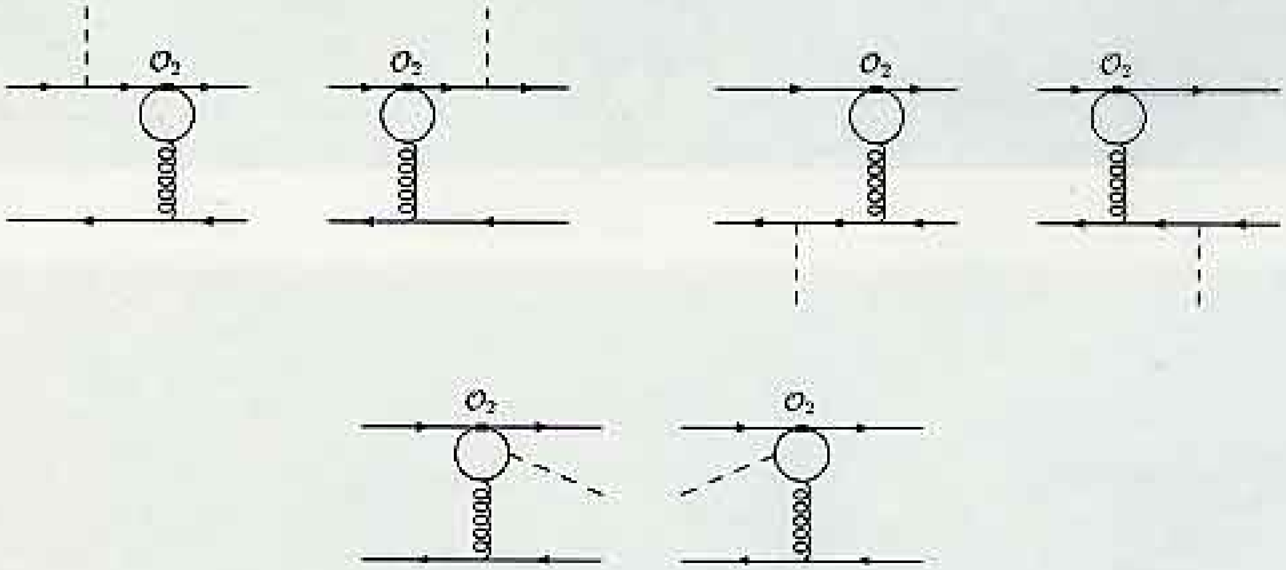


K^* u ρ

- Spectator corrections due to $\mathcal{O}_8$



- Spectator corrections due to $\mathcal{O}_2$



$B \to \rho\gamma$ Decay

[Parkhomenko,A.A.; Bosch, Buchalla]

$$\frac{\overline{\mathcal{B}}_{\rm th}(B \to \rho\gamma)}{\overline{\mathcal{B}}_{\rm th}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \left[1 + \Delta R(\rho/K^*)\right]$$

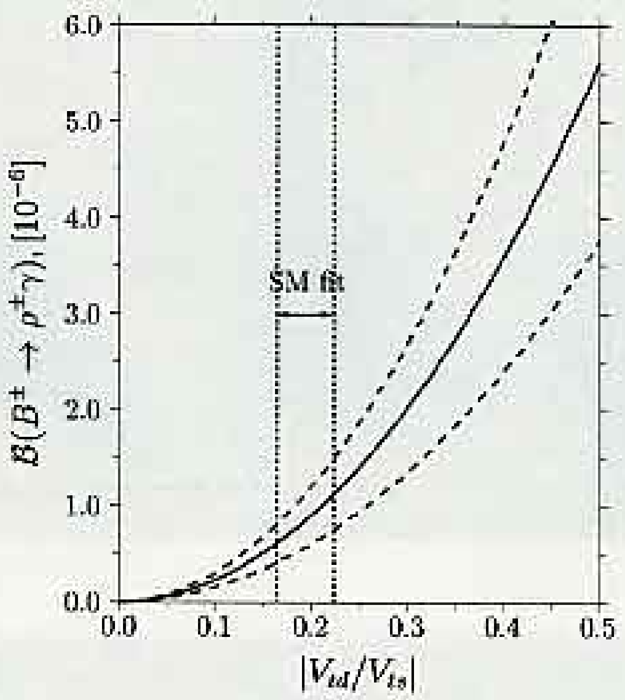
$S_\rho = 1$ for $B^\pm$
$= 1/2$ for B^0

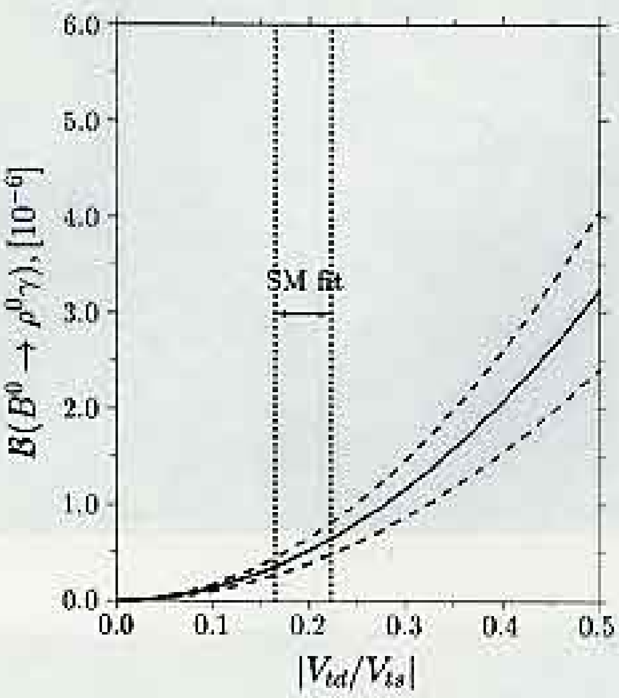
$$\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.76 \pm 0.06$$

[Braun, Simma, A.A. '94]

$$\Delta R(\rho^\pm/K^{*\pm}) = 0.003 \pm 0.159$$

[Parkhomenko,A.A. '01]





$$\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = (0.85 \pm 0.30[\rm th] \pm 0.10[\rm exp]) \times 10^{-6}$$

$$\bar{\mathcal{B}}_{\rm th}(\bar{B}^0 \to \rho^0\gamma) = (0.49 \pm 0.17[\rm th] \pm 0.04[\rm exp]) \times 10^{-6}$$

$\mathcal{B}(B^0 \to \omega\gamma) \simeq \mathcal{B}(B^0 \to \rho^0\gamma)$

$B \to K^*\gamma$ in the LEET-Approach

- Consider $B \to V\gamma^*$; In general 7 Form Factors

$A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $V(q^2)$, $T_1(q^2)$, $T_2(q^2)$, $T_3(q^2)$

$$E_V = \frac{m_B}{2}\left(1 - \frac{q^2}{m_B^2} + \frac{m_V^2}{m_B^2}\right)$$

- Large Energy Effective Theory (LEET)
  [Dugan, Grinstein '91; Charles et al. '99]
  For Large $E_V \sim m_B/2$, i.e., $q^2/m_B^2 \ll 1$; Symmetries in the Effective Theory $\Longrightarrow$ Relations among FFs:

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2)$$

- LEET-symmetries broken by perturbation theory

  Factorization Ansatz:
  [Beneke et al.; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

  Perturbative Corrections:

$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

- T_k: Hard Spectator Corrections

$$\Delta\mathcal{M}^{(\mathrm{HSA})} = \frac{4\pi\alpha_s C_F}{N_c} \int_0^1 du \int_0^\infty dl_+ \, M_{jk}^{(B)} M_{li}^{(V)} \mathcal{T}_{ijkl},$$

- $M_{jk}^{(B)}$ and $M_{li}^{(V)}$ B-Meson & V-Meson Projection Operators

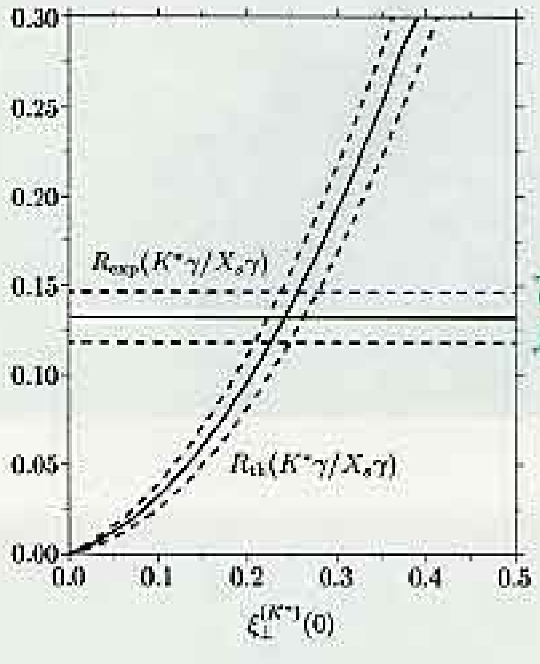
$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3$$
$$\times \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2$$

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) \simeq (7.2 \pm 1.1) \times 10^{-5} \left(\frac{\tau_B}{1.6\ \rm ps}\right) \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$
$$= (7.2 \pm 2.7) \times 10^{-5}$$

[Expt. $(4.22 \pm 0.28) \times 10^{-5}$]

- $$K = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \le K \le 1.7$$

[Beneke, Feldmann, Seidel; Bosch, Buchalla; Parkhomenko, A.A.]



} Expt.

$$R(K^*\gamma/X_s\gamma) \equiv \frac{\mathcal{B}(B \to K^*\gamma)}{\mathcal{B}(B \to X_s\gamma)} = 0.13 \pm 0.02 \Longrightarrow \boxed{\xi_\perp^{(K^*)}(0) = 0.25 \pm 0.04}$$

$$\boxed{T_1^{(K^*)}(0, \bar{m}_b) = 0.27 \pm 0.04}$$

LEET FF

Full QCD FF

$= 0.38 \pm 0.05$ [QCD-SR]

$= 0.32 {}^{+0.04}_{-0.02}$ [Lattice-QCD] Del Debbio et al.

Asymmerties in $B \to \rho\gamma$ Decays

[Parkhomenko,A.A.; Bosch, Buchalla]

- Isospin-Violating Ratios $\Delta^{\pm 0}$

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \qquad \Delta^{\pm 0} = \frac{\Gamma(B^{\pm} \to \rho^{\pm}\gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0\gamma)} - 1$$

$$\Delta_{\rm LO} \simeq 2\epsilon_A \left[F_1 + \frac{\epsilon_A}{2}(F_1^2 + F_2^2)\right]$$

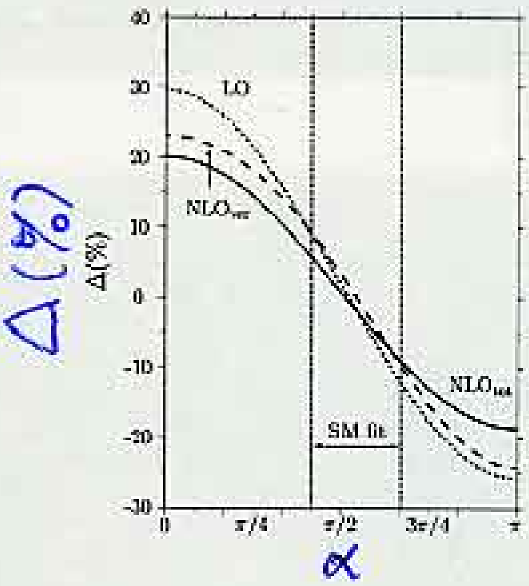
$$\Delta_{\rm NLO} \simeq \Delta_{\rm LO} - \frac{2\epsilon_A}{C_7^{(0)\rm eff}}\left[F_1 A_R^{(1)t}\right.$$

$$\left. +(F_1^2 - F_2^2)A_R^u + \epsilon_A(F_1^2 + F_2^2)(A_R^{(1)t} + F_1 A_R^u)\right]$$

$$\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| e^{i\alpha} = F_1 + iF_2$$

$\epsilon_A = -0.3 \pm 0.03$; Sign of ϵ_A controversial

(Annihilation Cont. in $B^\pm \to \rho^\pm \gamma$)



Braun, AA;
Khodjamirian et al.;
Grinstein, Pirjol;
Stech et al.; ...

Δ measures α

- Direct CP-Asymmetries $\mathcal{A}_{\rm CP}(\rho^\pm\gamma)$ and $\mathcal{A}_{\rm CP}(\rho^0\gamma)$

- Annihilation Contribution important in $\mathcal{A}_{\rm CP}(\rho^\pm\gamma)$

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}$$

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{2F_2(A_I^u - \epsilon_A\, A_I^{(1)t})}{C_7^{(0)\rm eff}\,(1+\Delta_{\rm LO})}$$

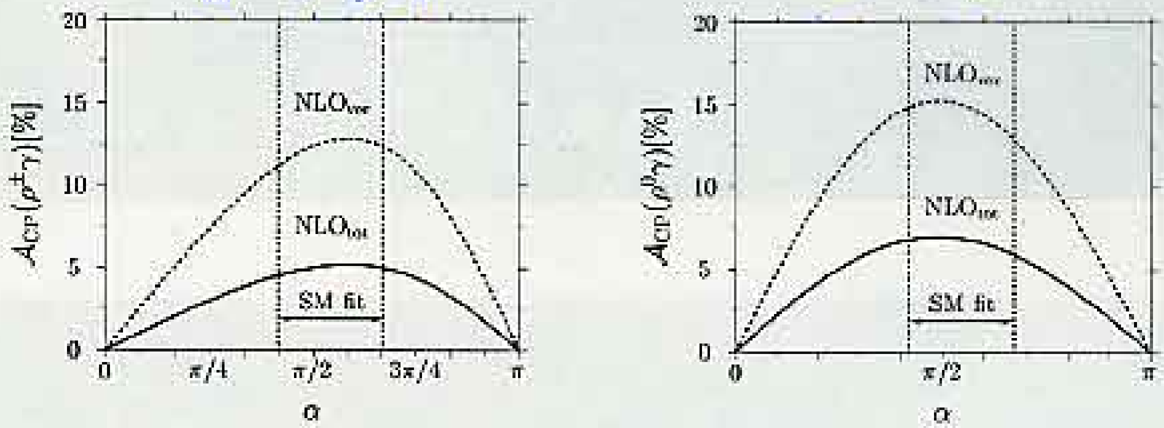
- Annihilation Contribution small in $\mathcal{A}_{\rm CP}(\rho^0\gamma)$

$$\mathcal{A}_{\rm CP}(\rho^0\gamma)(t) = a_{\epsilon'}\cos(\Delta M_d t) + a_{\epsilon+\epsilon'}\sin(\Delta M_d t)$$

$$a_\epsilon(\rho^0\gamma) = \frac{2F_2 A_I^u}{C_7^{(0)\rm eff}\,(1+\Delta_{\rm LO})}$$

[Parkhomenko;A.A.]

$B^\pm \to \rho^\pm\gamma$ $\qquad\qquad$ $\overset{(-)}{B^0} \to \rho^0\gamma$



- Hard Spectator Corrections reduce $A_{CP}(\rho\gamma)$
- $A_{CP}(\rho^\pm\gamma)$ sensitive to μ, m_c/m_b + ϵ_A

$B \to (X_s, X_d)\ell^+\ell^-$ in SM

If the invariant mass $s = (\ell^+ + \ell^-)^2$ of the lepton pair is away from endpoints and resonances, ($s = m_\psi^2, m_{\psi'}^2, \ldots;\ m_\omega^2,\ m_\rho^2$) theor. status for

- dilepton inv. mass spectrum $\frac{d\Gamma}{ds}$
- forward-backward charge asymmetry $\bar{A}_{\mathrm{FB}}(s)$

at similar level as $BR(B \to X_s\gamma)$:

- NLL QCD corrections (Misiak; Buras,Münz).
- NLL matching cond. have a large $\pm 16\%$ matching scale (μ_W) dep.
- Could be removed (Bobeth, Misiak, Urban (1999)), by NNLL matching (two-loop).
- But the prediction for the BR has a $\pm 13\%$ (μ_b) renorm. scale dep. $\to$ two-loop matrix elements needed, e.g.
- $\mathcal{O}(\alpha_s)$ two-loop virtual corrections to $d\Gamma/d\hat{s}$ for $\hat{s} = s/m_b^2 < 0.25$ (Asatryan, Asatrian, Greub, Walker, '01)
  $\Rightarrow$ Reduction in $\delta\Gamma(\mu_b) \approx \pm 6\%$
- Updated analysis in NNLO for $B \to (X_s, K^*, K)\ell^+\ell^-$ + Comparison with Data (AA, L[illegible]hi, Greub, Hiller, '01)

Inclusive $B \to X_s \ell^+ \ell^-$ in NNLO in SM

Dilepton Invariant Mass

$$\frac{d\Gamma(b \to X_s \ell^+ \ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} (1-\hat{s})^2 \times$$
$$\left((1+2\hat{s})\left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\tilde{C}_{10}^{\text{eff}}\right|^2\right) + 4\left(1+2/\hat{s}\right)\left|\tilde{C}_7^{\text{eff}}\right|^2 + 12\text{Re}\left(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}\right)\right)$$

$$\begin{aligned}
\tilde{C}_7^{\text{eff}} &= \left(1+\frac{\alpha_s(\mu)}{\pi}\omega_7(\hat{s})\right) A_7 \\
&\quad -\frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(7)}(\hat{s}) + C_2^{(0)} F_2^{(7)}(\hat{s}) + A_8^{(0)} F_8^{(7)}(\hat{s})\right), \\
\tilde{C}_9^{\text{eff}} &= \left(1+\frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right)\left(A_9 + T_9\, h(\hat{m}_c^2, \hat{s}) + U_9\, h(1,\hat{s}) + W_9\, h(0,\hat{s})\right) \\
&\quad -\frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(9)}(\hat{s}) + C_2^{(0)} F_2^{(9)}(\hat{s}) + A_8^{(0)} F_8^{(9)}(\hat{s})\right), \\
\tilde{C}_{10}^{\text{eff}} &= \left(1+\frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) A_{10},
\end{aligned}$$

- $h(\hat{m}_c^2, \hat{s})$ and $\omega_9(\hat{s})$
  [Bobeth, Misiak; Urban NP B574 (2000) 291]
- $\omega_7(\hat{s})$, and $F_{1,2,8}^{(7,9)}(\hat{s})$
  [Asatrian, Asatrian, Greub, Walker; Phys. Lett. B507 (2001) 162; hep-ph/0109140]

[Asatrian, Asatrian, Greub, Walker; Phys. Lett. B507 (2001) 162; hep-ph/0109140]

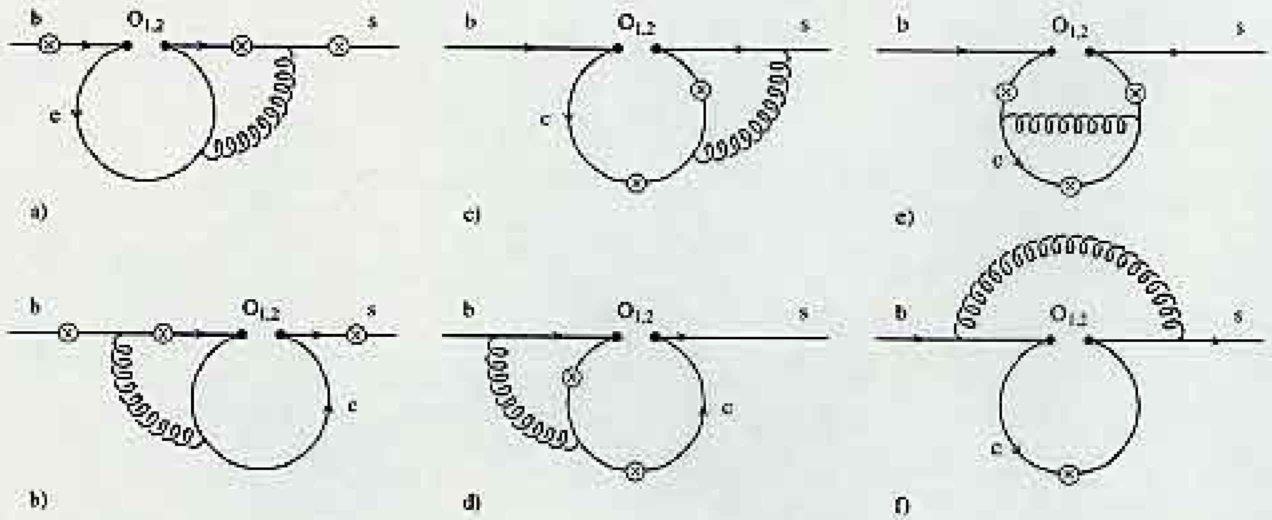


Figure 1: Matrix Elements from the operators $\mathcal{O}_{1,2}$

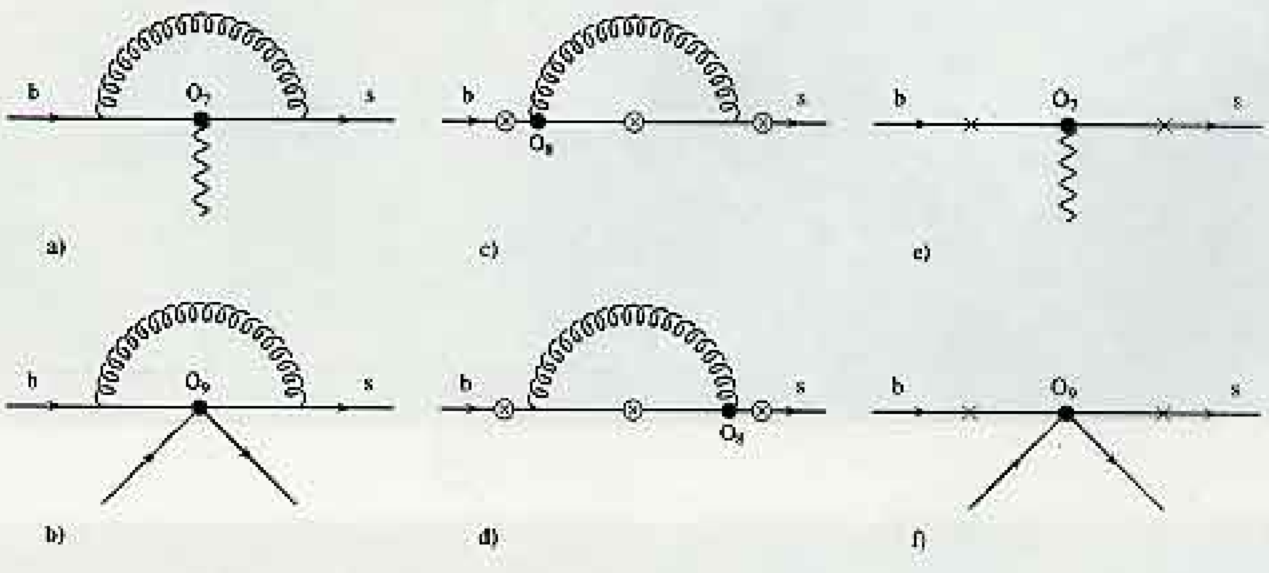


Figure 2: Matrix Elements from the operators $\mathcal{O}_7$, $\mathcal{O}_8$, $\mathcal{O}_9$

- A_7, A_8, A_9, A_{10}, T_9, U_9, W_9 are linear combinations of the Wilson coefficients

$$
\begin{aligned}
A_7 &= \frac{4\pi}{\alpha_s(\mu)} C_7(\mu) - \frac{1}{3} C_3(\mu) - \frac{4}{9} C_4(\mu) - \frac{20}{3} C_5(\mu) - \frac{80}{9} C_6(\mu)\,, \\
A_8 &= \frac{4\pi}{\alpha_s(\mu)} C_8(\mu) + C_3(\mu) - \frac{1}{6} C_4(\mu) + 20\, C_5(\mu) - \frac{10}{3} C_6(\mu)\,, \\
A_9 &= \frac{4\pi}{\alpha_s(\mu)} C_9(\mu) + \sum_{i=1}^{6} C_i(\mu)\, \gamma_{i9}^{(0)} \ln \frac{m_b}{\mu} \\
&\quad + \frac{4}{3} C_3(\mu) + \frac{64}{9} C_5(\mu) + \frac{64}{27} C_6(\mu)\,, \\
A_{10} &= \frac{4\pi}{\alpha_s(\mu)} C_{10}(\mu)\,, \\
T_9 &= +\frac{4}{3} C_1(\mu) + C_2(\mu) + 6\, C_3(\mu) + 60\, C_5(\mu)\,, \\
U_9 &= -\frac{7}{2} C_3(\mu) - \frac{2}{3} C_4(\mu) - 38\, C_5(\mu) - \frac{32}{3} C_6(\mu)\,, \\
W_9 &= -\frac{1}{2} C_3(\mu) - \frac{2}{3} C_4(\mu) - 8\, C_5(\mu) - \frac{32}{3} C_6(\mu)
\end{aligned}
$$

[A.A., Lunghi, Greub, Hiller, DESY 01-217; hep-ph/0112300]

in the NNLO Calculations

	$\mu = 2.5$ GeV	$\mu = 5$ GeV	$\mu = 10$ GeV
α_s	0.267	0.215	0.180
$(C_1^{(0)}, C_1^{(1)})$	$(-0.697, 0.241)$	$(-0.487, 0.207)$	$(-0.326, 0.184)$
$(C_2^{(0)}, C_2^{(1)})$	$(1.046, -0.028)$	$(1.024, -0.017)$	$(1.011, -0.010)$
$(A_7^{(0)}, A_7^{(1)})$	$(-0.353, 0.023)$	$(-0.312, 0.008)$	$(-0.278, -0.002$
$(A_{77}^{(0)}, A_{77}^{(1)})$	$(0.577, -0.0524)$	$(0.672, -0.0391)$	$(0.760, -0.0277$
$(A_{78}^{(0)}, A_{78}^{(1)})$	$(0.109, -0.00520)$	$(0.0914, -0.00193)$	$(0.0707, -0.0002$
$A_8^{(0)}$	-0.164	-0.148	-0.134
$A_{88}^{(0)}$	0.618	0.706	0.786
$(A_9^{(0)}, A_9^{(1)})$	$(4.287, -0.218)$	$(4.174, -0.035)$	$(4.177, 0.107)$
$(T_9^{(0)}, T_9^{(1)})$	$(0.114, 0.280)$	$(0.374, 0.252)$	$(0.575, 0.231)$
$(U_9^{(0)}, U_9^{(1)})$	$(0.045, 0.023)$	$(0.033, 0.015)$	$(0.022, 0.010)$
$(W_9^{(0)}, W_9^{(1)})$	$(0.044, 0.016)$	$(0.032, 0.012)$	$(0.022, 0.008)$
$(A_{10}^{(0)}, A_{10}^{(1)})$	$(-4.592, 0.379)$	$(-4.592, 0.379)$	$(-4.592, 0.379)$

[Asatrian, Asatrian, Greub, Walker; Phys. Lett. B507 (2001) 162; hep-ph/0109140]

$$R \equiv \frac{1}{\Gamma(b \to X_c \ell \nu_\ell)} \frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{d\hat{s}}$$

Scale - dependence of R

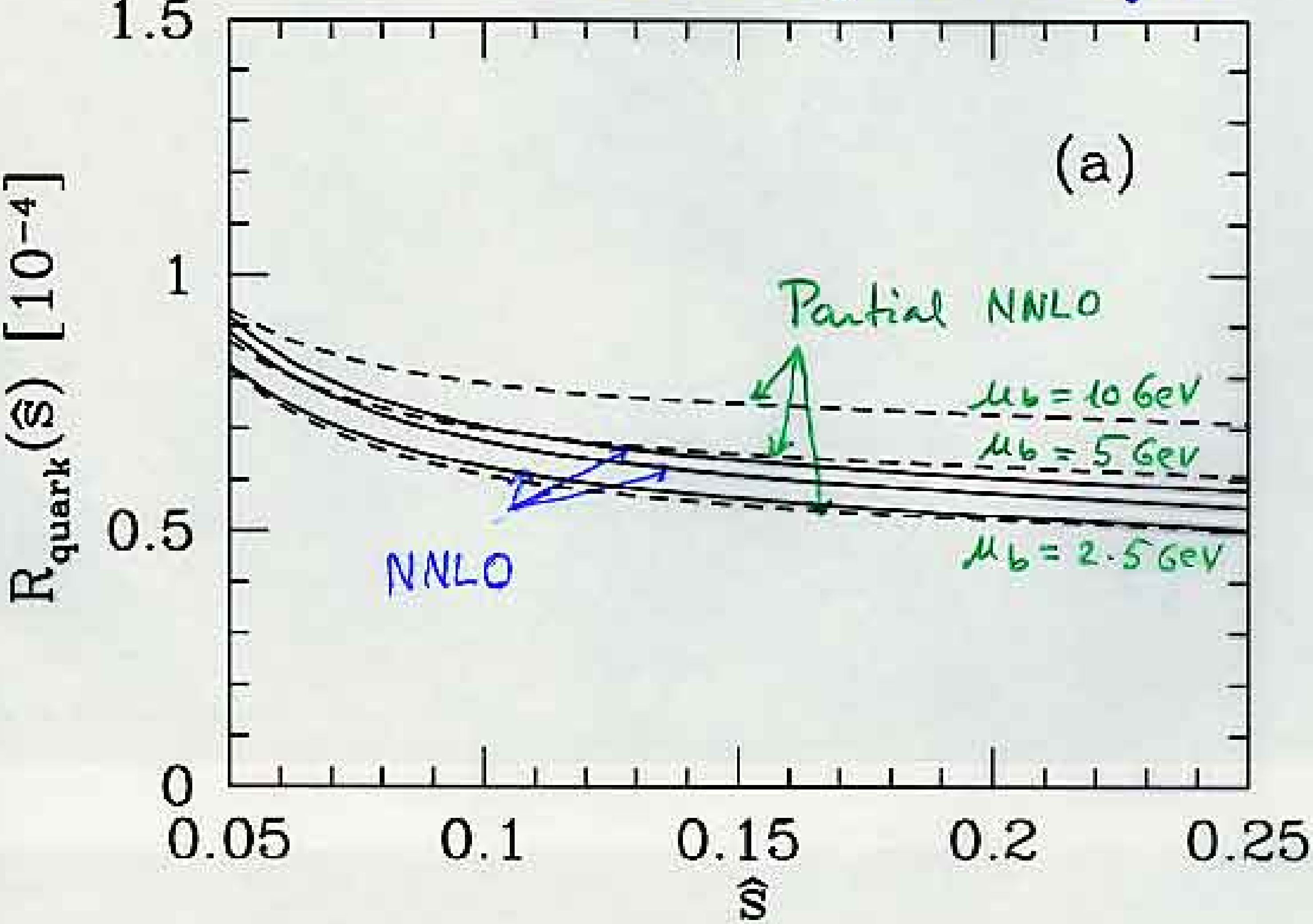


Figure 3: Reduction of scale-dependence in $\mathcal{O}(\alpha_s)$

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Improved Model-Independent Analysis of Semileptonic and Radiative Rare B Decays

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Abstract

We update the branching ratios for the inclusive decays $B \to X_s \ell^+ \ell^-$ and the exclusive decays $B \to (K, K^*)\ell^+\ell^-$, with $\ell = e, \mu$, in the standard model by including the explicit $O(\alpha_s)$ and Λ_{QCD}/m_b corrections. This framework is used in conjunction with the current measurements of the branching ratios for $B \to X_s \gamma$ and $B \to K \ell^+ \ell^-$ decays and upper limits on the branching ratios for the decays $B \to (K^*, X_s)\ell^+\ell^-$ to work out bounds on the Wilson coefficients C_7, C_8, C_9 and C_{10} appearing in the effective Hamiltonian formalism. The resulting bounds are found to be consistent with the predictions of the standard model and some variants of supersymmetric theories. We illustrate the constraints on supersymmetric parameters that the current data on rare B decays implies in the context of minimal flavor violating model and in more general scenarios admitting additional flavor changing mechanisms. Precise measurements of the dilepton invariant mass distributions in the decays $B \to (X_s, K^*, K)\ell^+\ell^-$, in particular in the lower dilepton mass region, and the forward-backward asymmetry in the decays $B \to (X_s, K^*)\ell^+\ell^-$, will greatly help in discriminating among the SM and various supersymmetric theories.

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Power corrections in $B \to X_s \ell^+ \ell^-$ decays

- $1/m_b$ corrections [A. Falk et al., Phys. Rev. D49 (1994) 4553; AA, Handoko, Morozumi,Hiller, Phys. Rev. D55 (1997) 4105; Buchalla, Isidori, Rey; Nucl. Phys. B511 (1998) 594]

$$\frac{d\Gamma(b \to s\ell^+\ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 |\lambda_{ts}|^2}{48\pi^3}(1-\hat{s})^2 \left[(1+2\hat{s})\left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\right.\right.\right.$$

$$\left.\left. + \left|\tilde{C}_{10}^{eff}\right|^2\right) + \quad 4\left(1+2/\hat{s}\right)\left|\tilde{C}_7^{\text{eff}}\right|^2 G_2(\hat{s}) + 12\text{Re}\left(\tilde{C}_7^{\text{eff}}\tilde{C}_9^{\text{eff}*}\right) G_3(\hat{s}) + G_c(\hat{s})\right]$$

where

$$G_1(\hat{s}) = 1 + \frac{\lambda_1}{2m_b^2} + 3\frac{1-15\hat{s}^2+10\hat{s}^3}{(1-\hat{s})^2(1+2\hat{s})}\frac{\lambda_2}{2m_b^2}$$

$$G_2(\hat{s}) = 1 + \frac{\lambda_1}{2m_b^2} - 3\frac{6+3\hat{s}-5\hat{s}^3}{(1-\hat{s})^2(2+\hat{s})}\frac{\lambda_2}{2m_b^2},$$

$$G_3(\hat{s}) = 1 + \frac{\lambda_1}{2m_b^2} - \frac{5+6\hat{s}-7\hat{s}^2}{(1-\hat{s})^2}\frac{\lambda_2}{2m_b^2}$$

- $1/m_c$ corrections [Buchalla, Isidori, Rey; Nucl. Phys. B511 (1998) 594]

$$G_c(\hat{s}) = -\frac{8}{9}(C_2 - \frac{C_1}{6})\frac{\lambda_2}{m_c^2}\text{Re}\left(F(r)\left[\tilde{C}_9^{\text{eff}*}(2+\hat{s}) + \tilde{C}_7^{\text{eff}*}\frac{1+6\hat{s}-\hat{s}^2}{\hat{s}}\right]\right)$$

where $F(r)$ ($r = \hat{s}/(4\hat{m}_c^2)$) is:

$$F(r) = \frac{3}{2r}\begin{cases} \dfrac{1}{\sqrt{r(1-r)}}\arctan\sqrt{\dfrac{r}{1-r}} - 1 & 0 < r < 1\,, \\ \dfrac{1}{2\sqrt{r(r-1)}}\left(\ln\dfrac{1-\sqrt{1-1/r}}{1+\sqrt{1-1/r}} + i\pi\right) - 1 & r > 1\,. \end{cases}$$

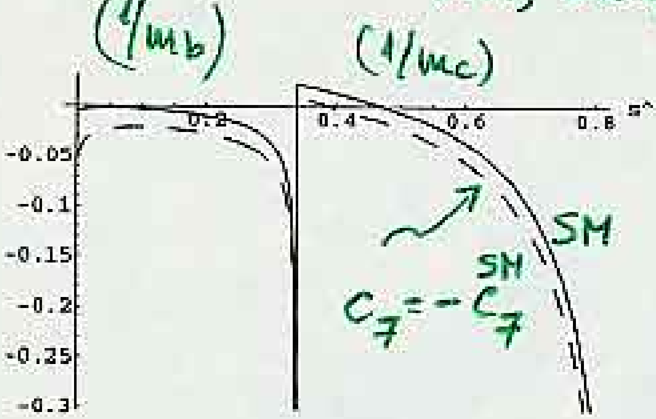


Figure 4: *Relative size $R(\hat{s})$ of power corrections in $B \to X_s \ell^+ \ell^-$ decays: SM (solid), $C_7 = -C_7^{SM}$ (dashed)*

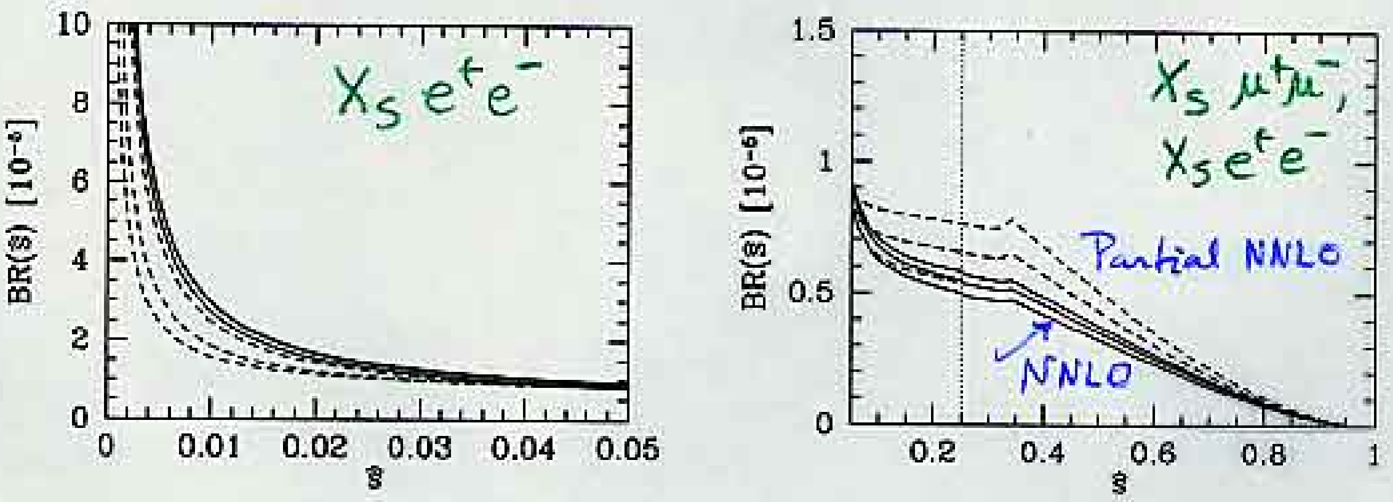


Figure 5: *Partial (dashed lines) vs. full (solid lines) inv. dilepton mass for $B \to X_s e^+ e^-$. Left plot ($\hat{s} \in [0, 0.05]$) the lowest curves are for $\mu = 10$ GeV and the uppermost ones for $\mu = 2.5$ GeV. Right plot: μ dependence is reversed*

- Scale-dependence in NNLO reduced
- $\mathcal{B}(B \to X_s \ell^+ \ell^-)_{\rm NNLO} < \mathcal{B}(B \to X_s \ell^+ \ell^-)_{\rm NLO}$
- The choice $\mu = 2.5$ GeV reduces the NNLO corrections; hence used for $\hat{s} > 0.25$

$B \to (K, K^*) \ell^+ \ell^-$

Hadronic matrix elements and form factors for semileptonic B decays

$B \to X$, $X = V, P$ (vector, pseudoscalar) K, K^*
2 currents, $q = p_B - p_X$

$$\Gamma^1_\mu = \gamma_\mu(1-\gamma_5), \Gamma^2_\mu = \sigma_{\mu\nu} q^\nu (1+\gamma_5)$$

$$\left\langle P \left| \bar{s}\Gamma^1_\mu b \right| B) \right\rangle \supset f_+, f_-$$

$$\left\langle P \left| \bar{s}\Gamma^2_\mu b \right| B \right\rangle \supset f_T$$

$$\left\langle V \left| \bar{s}\Gamma^1_\mu b \right| B) \right\rangle \supset V, A_1, A_2, A_0$$

$$\left\langle V \left| \bar{s}\Gamma^2_\mu b \right| B \right\rangle \supset \underline{\underline{T_1}}, T_2, T_3$$

<u>10</u> non-perturbative q^2-dependent objects (Form Factors)

- quark models Jaus, Wyler, Colangelo et al, Melikhov et al
- QCD sum rules Colangelo, DeFazio, Santorelli, Scrimieri
- HQS and data Isgur, Burdman, Ligeti, Wise
- LCQCD sumrules for heavy-to-light transitions Aliev et al, Ball, Braun '98, Ball, Handoko, Hiller, A.A. '99

<u>check</u> : $\underline{\underline{T_1}}(q^2 = 0) = 0.38$ agrees with $B \to K^* \gamma$ data in LO

error: 15% at $q^2 = 0$ to 20% at $q^2_{max} = (m_B - m_X)^2$.

However NLO Corrections Large in $B \to K^* \gamma$
$\Rightarrow$ K-factor $\simeq 1.6 \Rightarrow T_1(0) \simeq 0.28$

A. A., Ball, Handoko, Hiller

Form Factors in LC-QCD-SR

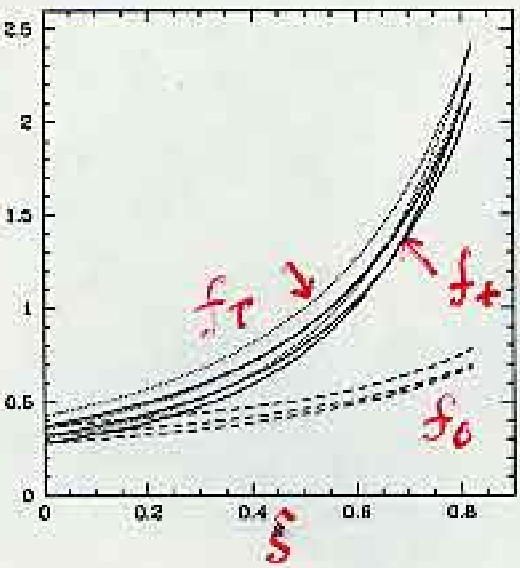


Figure 1: *LCSR form factors with theoretical uncertainties for the $B \to K$ transition as a function of $\hat{s}$. Solid, dotted and dashed curves correspond to f_+, f_T, f_0, respectively. Renormalization scale for f_T is $\mu = m_b$.*

(a) (b)

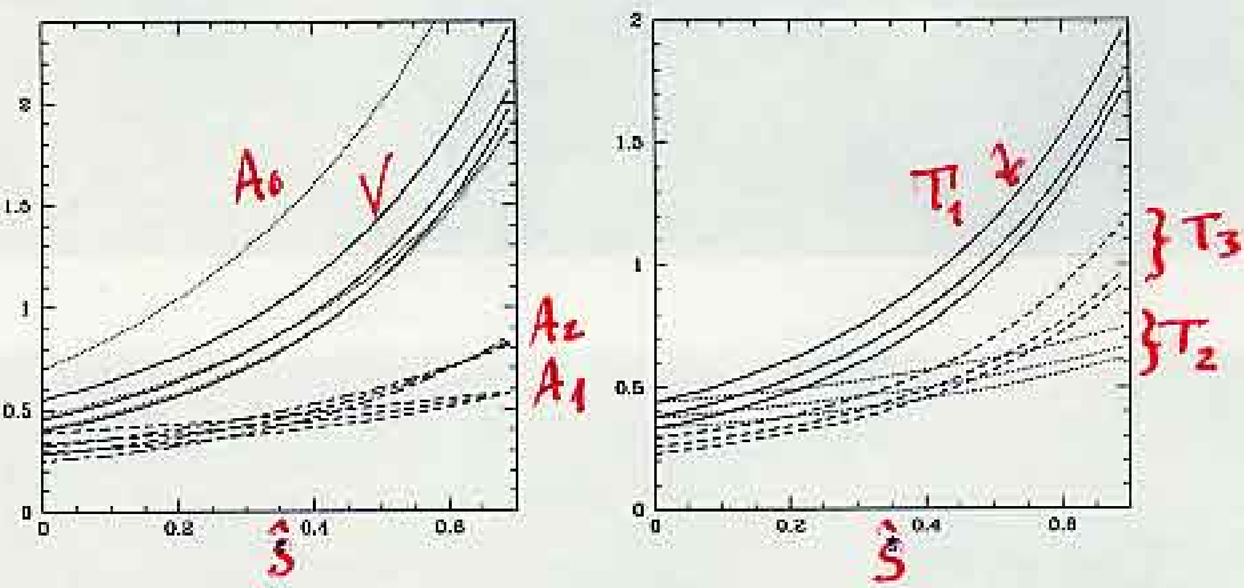


Figure 2: *LCSR form factors with theoretical uncertainties for the $B \to K^*$ transition as a function of $\hat{s}$. In (a), the solid, dotted, dashed and short long dashed curves correspond to V, A_0, A_1, A_2 and in (b), the solid, dotted and dashed curves correspond to T_1, T_2, T_3, respectively. Renormalization scale for T_i is $\mu = m_b$.*

Observables
1. $B \to K$

dilepton invariant mass spectrum, $\hat{s} = q^2/m_B^2$ ($m_\ell = 0$):

$$\frac{d\Gamma}{d\hat{s}} \sim |V_{ts}^* V_{tb}|^2 \left(|\mathcal{C}_9{}^{\text{eff}} f_+ + \frac{2\hat{m}_b}{1+\hat{m}_P} \mathcal{C}_7{}^{\text{eff}} f_T|^2 + |\mathcal{C}_{10} f_+|^2 \right)$$

- no f_- contribution for $m_\ell = 0$
- $|\mathcal{C}_7{}^{\text{eff}}| \ll |\mathcal{C}_9{}^{\text{eff}}|, |\mathcal{C}_{10}|$ and no kinematical enhancement: roughly $\frac{d\Gamma}{d\hat{s}} \sim |f_+|^2$ [-12% effect from $\mathcal{C}_7{}^{\text{eff}} f_T$]
- relate to V-A charged current $B \to \pi \ell \nu_\ell$ decays, determine $|V_{ub}/V_{ts}^* V_{tb}|$ Ligeti,Stewart,Wise '98

- Sensitivity to new physics:
  $B \to X_s \gamma$ data imply $|\mathcal{C}_7{}^{\text{eff}}| \simeq |\mathcal{C}_{7\,SM}^{\text{eff}}|$ two possible branches

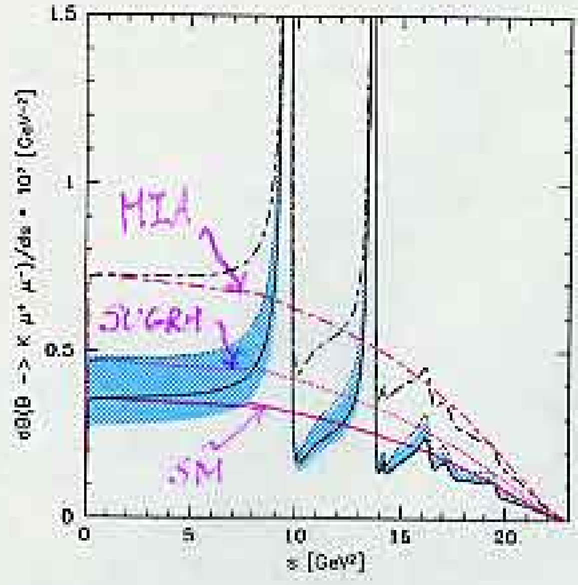
  - $C_7^{eff} \simeq C_{7\,SM}^{eff}$

    or

  - $C_7^{eff} \simeq - C_{7\,SM}^{eff}$

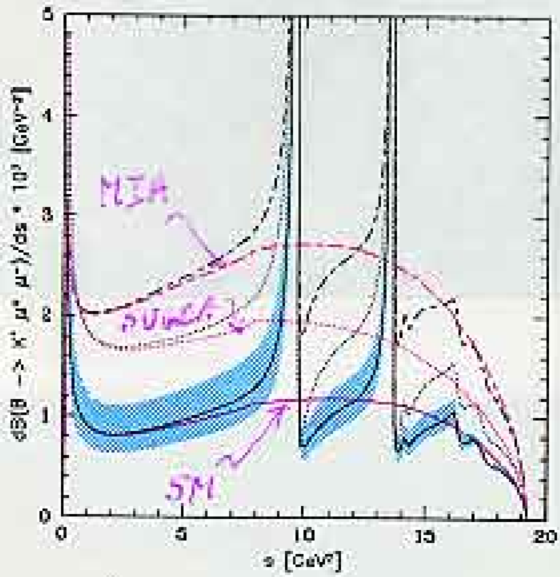
Both realized in Supersymmetry over different regions of SUSY parameters

Ball, Handoko,
Hiller, A-A.



$B \to K\mu^+\mu^-$

Figure 6: *The dilepton invariant mass distribution in $B \to K\mu^+\mu^-$ decays, using the form factors from LCSR as a function of s. All resonant $c\bar{c}$ states are parametrized as in Ref. [29]. The solid line represents the SM and the shaded area depicts the form factor-related uncertainties. The dotted line corresponds to the SUGRA model with $R_7 = -1.2$, $R_9 = 1.03$ and $R_{10} = 1$. The long-short dashed lines correspond to an allowed point in the parameter space of the MIA-SUSY model, given by $R_7 = -0.83$, $R_9 = 0.92$ and $R_{10} = 1.61$. The corresponding pure SD spectra are shown in the lower part of the plot.*

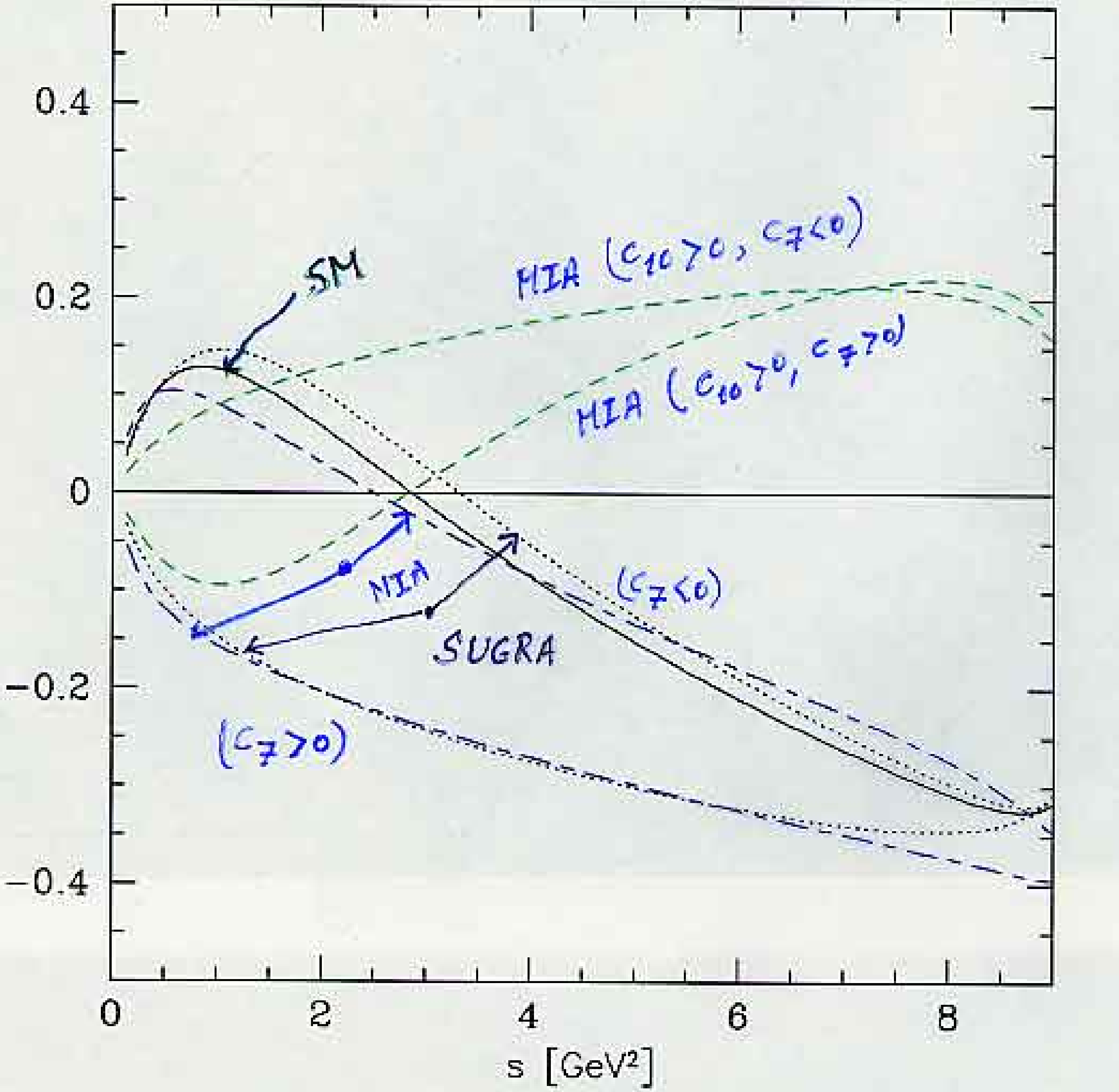


$B \to K^*\mu^+\mu^-$

Figure 7: *The dilepton invariant mass distribution in $B \to K^*\mu^+\mu^-$ decays, using the form factors from LCSR as a function of s. All resonant $c\bar{c}$ states are parametrized as in Ref. [29]. The legends are the same as in Fig. 6.*

Ball, Hiller, Handoko, AA

FB Asymmetry ($B \to K^* \ell^+ \ell^-$)



2. $B \to K^*$

2 distributions, dilepton invariant mass spectrum

- no A_0 contribution for $m_\ell = 0$
- ${C_7}^{\rm eff}/\hat{s}$ enhancement in low $\hat{s}$ region, contributes with $\sim -30\%$, photon pole is dominant for $q^2 < 1\text{GeV}^2$

Like $B \to K$, the following combinations of WC's are involved: $|{C_{10}}^2|$, $|{C_9}^{\rm eff}|^2$, $|{C_7}^{\rm eff}|^2$, $Re({C_7}^{\rm eff}{C_9}^{\rm eff})$

Forward-Backward asymmetry A_{FB} (Marnel, Morozumi; A.A.)

$\hat{u} \sim \cos\theta$, $\theta = \sphericalangle(p_B, p_+)$ in dilepton CMS

$$\frac{\mathrm{d}A_{\rm FB}}{\mathrm{d}\hat{s}} = -\int_0^{\hat{u}(\hat{s})} \mathrm{d}\hat{u} \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{u}\mathrm{d}\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} \mathrm{d}\hat{u} \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{u}\mathrm{d}\hat{s}}$$

$$\sim C_{10}\left[\mathrm{Re}({C_9}^{\rm eff}) V A_1 + \frac{\hat{m}_b}{\hat{s}} {C_7}^{\rm eff} (V T_2 (1 - \hat{m}_V) + A_1 T_1 (1 + \hat{m}_V))\right]$$

- probes different combinations of WC's than $d\Gamma/d\hat{s}$
- proportional to C_{10} (sign)
- has characteristic zero $\hat{s}_0$ below $m^2_{J/\Psi}$
- normalized FB-asymmetry $\frac{\mathrm{d}\bar{A}_{\rm FB}}{\mathrm{d}\hat{s}} = \frac{\mathrm{d}A_{\rm FB}}{\mathrm{d}\hat{s}} / \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}$ equivalent to energy asymmetry Cho,Misiak,Wyler, Ali et al '96

position of A_{FB} zero: $(\hat{S}_0)$

$$\mathrm{Re}(C_9{}^{\mathrm{eff}}(\hat{s}_0)) = -\frac{\hat{m}_b}{\hat{s}_0} C_7{}^{\mathrm{eff}} \left(\frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)}(1-\hat{m}_V) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)}(1+\hat{m}_V) \right)$$

- $\hat{s}_0 = 0.10$ (or $q^2 = 2.9\mathrm{GeV}^2$) in the SM
- no A_{FB} zero if $C_7{}^{\mathrm{eff}} > 0$ ($C_{7\,SM}^{\mathrm{eff}} < 0$)

very small FF-related uncertainties Burdman '98; theoretically justified in Large Energy Effective Theory (LEET) Charles et al '98

expansion in $1/E$, E : energy of final hadron

$$E = \frac{m_B^2 + m_X^2 - q^2}{2m_B}$$

~~[illegible] mass of the initial quark~~
valid far from zero-recoil point

theorem: $B \to P, V$ heavy-to-light transitions at lowest order in $1/m_B, 1/E, \mathcal{O}(\alpha_s)$ can be expressed in terms of 3 universal functions $\zeta, \zeta_\perp, \zeta_\parallel$

ratios involved in the zero in LEET:

$$\boxed{\begin{aligned} \frac{T_2}{A_1} &= \frac{1+\hat{m}_V}{1+\hat{m}_V^2-\hat{s}}\left(1-\frac{\hat{s}}{1-\hat{m}_V^2}\right) \\ \frac{T_1}{V} &= \frac{1}{1+\hat{m}_V} \end{aligned}}$$

No hadronic uncertainty, all ζ_i cancel

Ball, Hiller, Harlake, AA

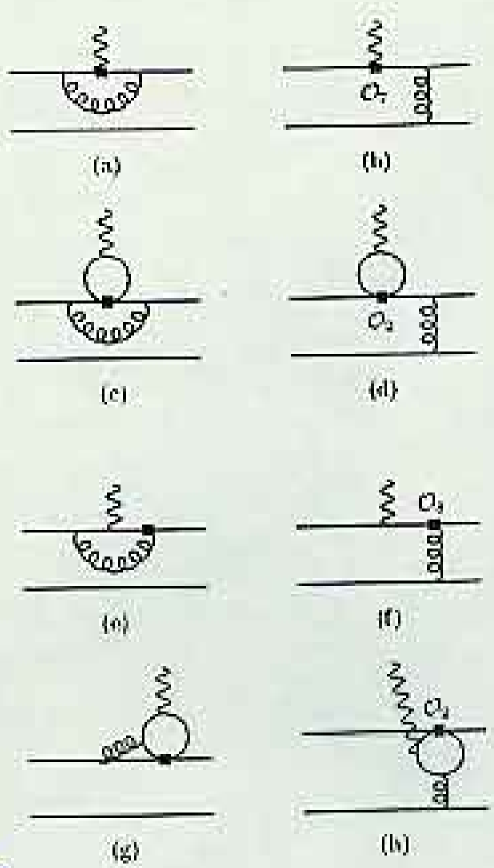
$$\Rightarrow \boxed{C_9^{\mathrm{eff}}(s_0) = \frac{-2 m_b/m_B\, C_7^{\mathrm{eff}}}{s_0}}$$

(In Lowest Order)

$\mathcal{O}(\alpha_s)$ Corrections to FB Asymmetry in $B \to K^* \ell^+ \ell^-$

Beneke, Feldmann;

Beneke, Feldmann, Seidel



+ Vertex Corrections

$\mathcal{A}_{FB}(s_0) = 0$

$$\Longrightarrow \quad C_9 = -\frac{m_b}{s_0} C_7 \left\{ \frac{T_2(s_0)}{A_1(s_0)} (M - m_{K^*}) + \frac{T_1(s_0)}{V(s_0)} (M + m_{K^*}) \right\}$$

Large-E Symmetry

$$\frac{T_2(s_0)}{A_1(s_0)} (M - m_{K^*}) = \frac{T_1(s_0)}{V(s_0)} (M + m_{K^*}),$$

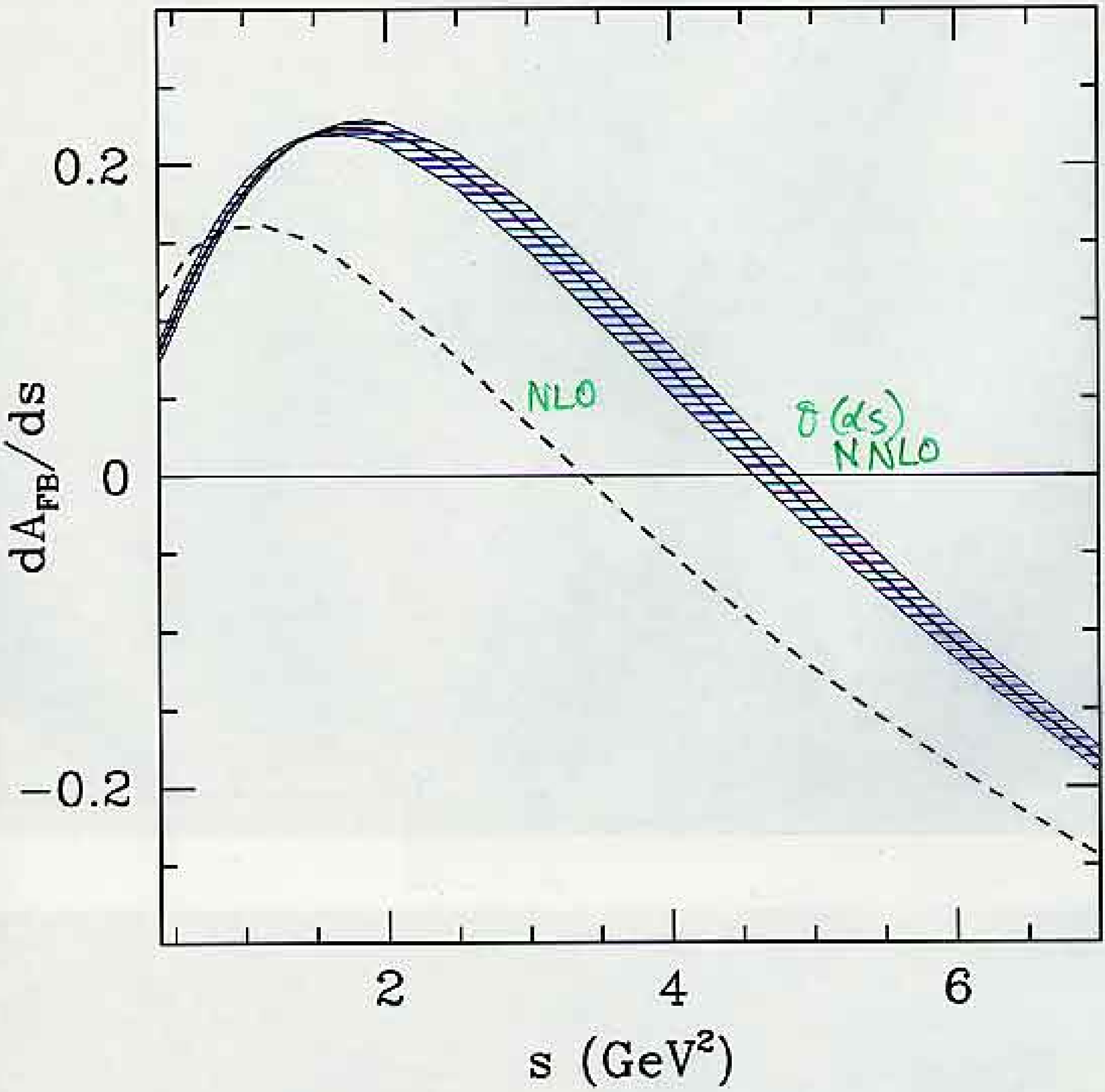
$\mathcal{O}(\alpha_s)$ Corrections

$$C_9 = -\frac{2 M m_b}{s_0} C_7 \left(1 + \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{m_b^2}{\mu^2} - L \right] + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)} \right)$$

$\Longrightarrow$ Significant shift in s_0

Safir, A.A.
[following Beneke et al.]

FB Asymmetry ($B \to K^* \ell^+ \ell^-$)



Perturbative Shift in the Zero of $A_{FB}(s)$

A Model-independent Analysis of $B \to X_s\gamma$

- Assume $\mathcal{H}_{eff}^{\mathrm{SM}}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM physics in $C_{7,8}(\mu_W)$
- Define:

$$R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{\mathrm{tot}}(\mu_W)}{C_{7,8}^{\mathrm{SM}}(\mu_W)}$$

with $C_{7,8}^{\mathrm{tot}}(\mu_W) = C_{7,8}^{\mathrm{SM}}(\mu_W) + C_{7,8}^{\mathrm{NP}}(\mu_W)$

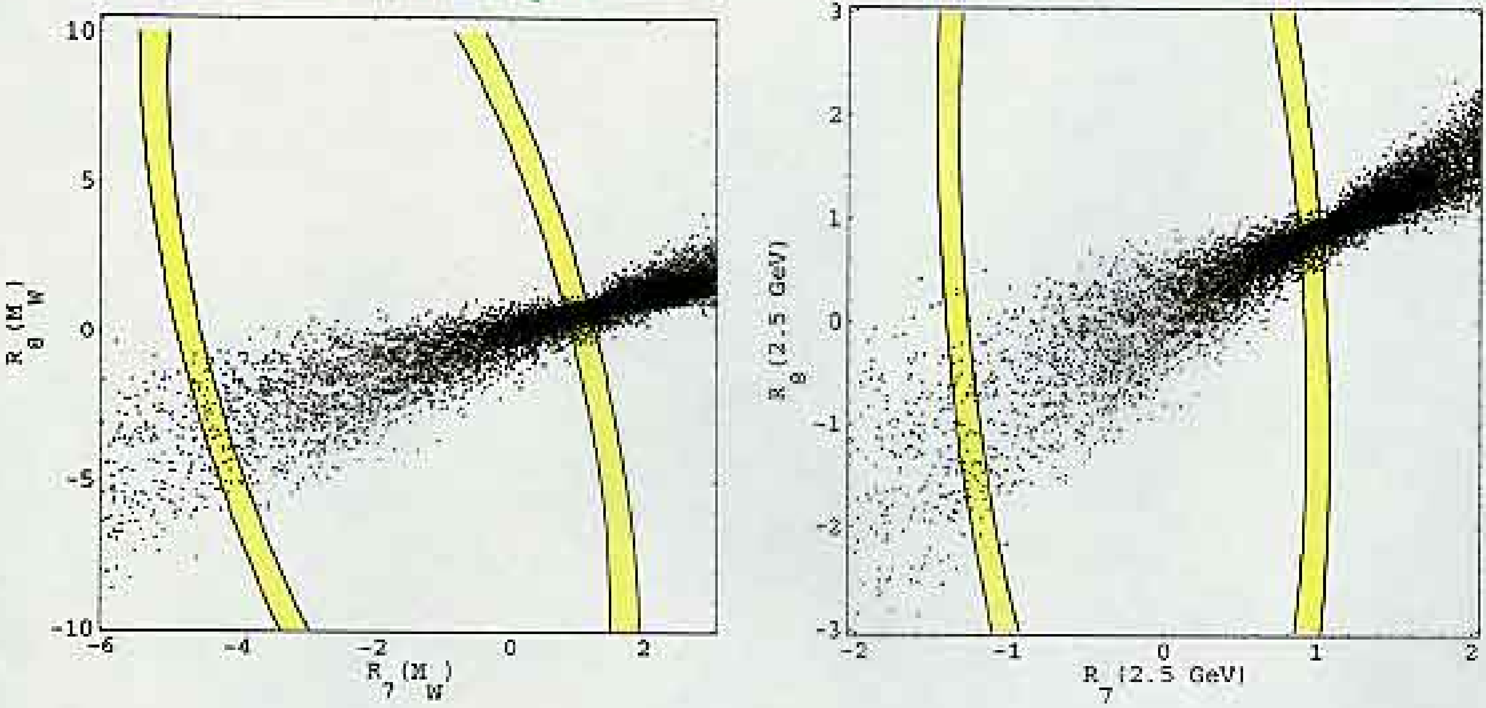
- Set the scale $\mu_W = M_W$, and use RGE to evolve $R_{7,8}(\mu_W) \to R_{7,8}(\mu_b)$
- Current Data $\Longrightarrow$ Tight Constraints on R_7 (at 95% C.L.)

$$0.78 \leq R_7(2.5 \text{ GeV}) \leq 1.25$$
$$-1.55 \leq R_7(2.5 \text{ GeV}) \leq -1.2$$

- Data allows a larger range for $R_8(2.5 \text{ GeV})$

$B \to X_s \gamma$

$R_7(M_W)$, $R_8(M_W)$ $\quad$ $R_7(2.5\,\text{GeV})$, $R_8(2.5\,\text{GeV})$



[A. A., C. Greub, G. Hiller, E. Lunghi, hep-ph/0201049]

$$
\begin{aligned}
\mu &\in [-1000\ \text{GeV}, 1000\ \text{GeV}] \\
M_2 &\in [100\ \text{GeV}, 1000\ \text{GeV}] \\
M_{\tilde{t}} &\in [100\ \text{GeV}, 1000\ \text{GeV}] \\
M_{H^\pm} &\in [100\ \text{GeV}, 1000\ \text{GeV}] \\
\theta_{\tilde{t}} &\in [-\pi/10, \pi/10] \\
\tan\beta &\in [4, 30]
\end{aligned}
$$

A Model-independent Analysis of $B \to X_s \ell^+ \ell^-$

- Assume $\mathcal{H}_{eff}^{\rm SM}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM physics only in $C_7(\mu_W)$,$C_8(\mu_W)$, $C_9(\mu_W)$, and $C_{10}(\mu_W)$
- : BSM Coefficients: $R_7 - 1$, $R_8 - 1$, $C_9^{\rm NP}$, & $C_{10}^{\rm NP}$
- RGE $\Longrightarrow$ modificications in $\tilde{C}_7^{eff}$, $\tilde{C}_9^{eff}$, $\tilde{C}_{10}^{eff}$
- Impose constraints from $R_7(\mu_b)$ and $R_8(\mu_b)$ from $B \to X_s \gamma$ Data
- Use Data on $B \to (X_s, K^*, K)\ell^+\ell^-$ BRs to constrain $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$
- Two-fold ambiguity due to the sign of C_7^{eff} $\Longrightarrow$ Two-fold ambiguity for $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$

Two Supersymmetric Models & Implications for Rare B-D

- Minimal Supersymmetric Standard Model - Minimal Flavor Violation (MSSM-MFV)

- MSSM Studies of $B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$ in terms of C_i
Bertoloni et al. '91; Cho,Misiak,Wyler; '96; Lunghi et al. '99; Goto et al.; AA, Lunghi, Greub, Hiller, '01

- No New Flavor-Changing Structure

- Gluinos & the First Two Generations of Squarks Assumed Heavy

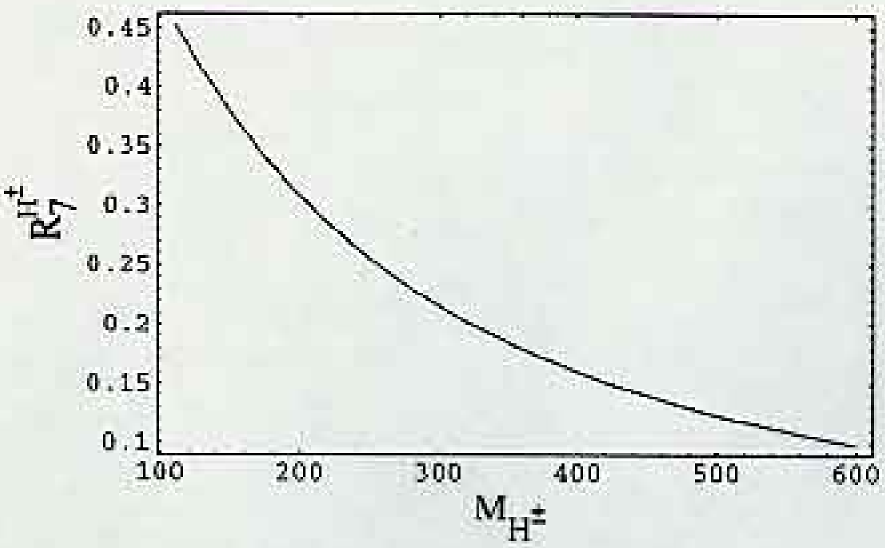
- SUSY Parameters: μ, M_2, $\tan\beta$, $M_{H^\pm}$, $M_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

- MSSM-MFV Effects Small on C_9 and C_{10}
For $C_7^{\rm eff} < 0$
$$C_9^{\rm MFV}(\mu_W) \in [-0.10, +0.11]$$
$$C_{10}^{\rm MFV} \in [0, +1.3]$$

- MSSM-MFV Effects also Small for $C_7^{\rm eff} > 0$

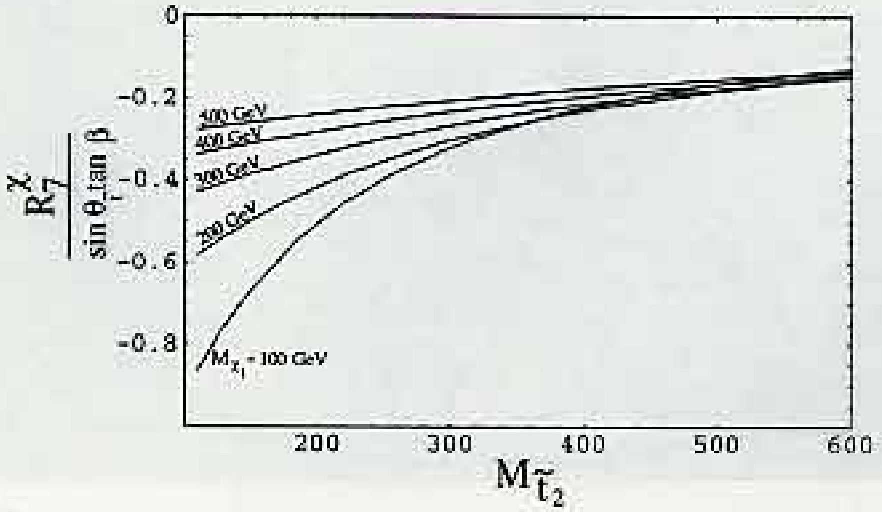
- No Large Deviations from SM Expected in MSSM-MFV

[A.A., Lunghi, Greub, Hiller]



$H^\pm$

Figure 8: $R_7^{H^\pm}(\mathcal{M}_b) \equiv C_7^{H^\pm}(\mathcal{M}_b)/C_7^{\rm SM}(\mathcal{M}_b)$ *vs. mass of the charged Higgs*



$\chi^\pm$

Figure 9: $R_7^\chi(\mathcal{M}_b) \equiv C_7^\chi(\mathcal{M}_b)/C_7^{\rm SM}(\mathcal{M}_b)$ *vs. the mass of the lightest stop in MFV models. The chargino contribution is essentially proportional to* $\sin\theta_{\tilde t}\tan B$ *for not too small* $\sin\theta_{\tilde t}$

Extended MFV models

- Heavy squark–gluino mass spectrum
- The MFV condition is not imposed

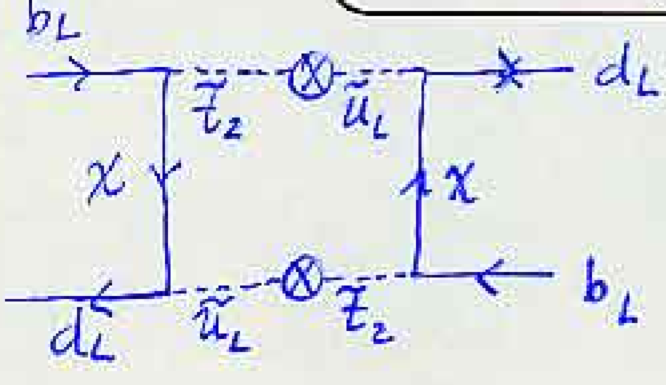
Using the MIA it is possible to show that only 2 insertions can play a role

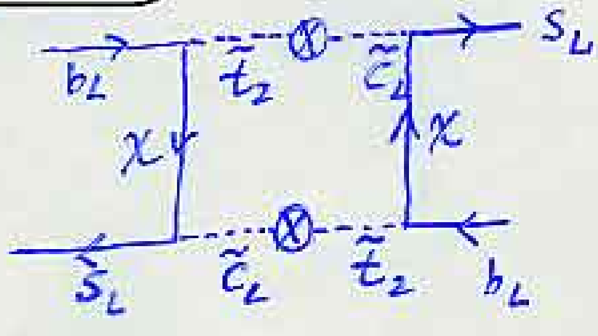
Buras, Romanino, Silvestrini

$\Downarrow \Downarrow \Downarrow$

$$\delta_{\tilde{u}_L\tilde{t}_2} \equiv \frac{M^2_{\tilde{u}_L\tilde{t}_2}}{M_{\tilde{q}}M_{\tilde{t}_2}}\frac{|V_{td}|}{V^*_{td}} \qquad [b \to d]$$

$$\delta_{\tilde{c}_L\tilde{t}_2} \equiv \frac{M^2_{\tilde{c}_L\tilde{t}_2}}{M_{\tilde{q}}M_{\tilde{t}_2}}\frac{|V_{ts}|}{V^*_{ts}} \qquad [b \to s]$$



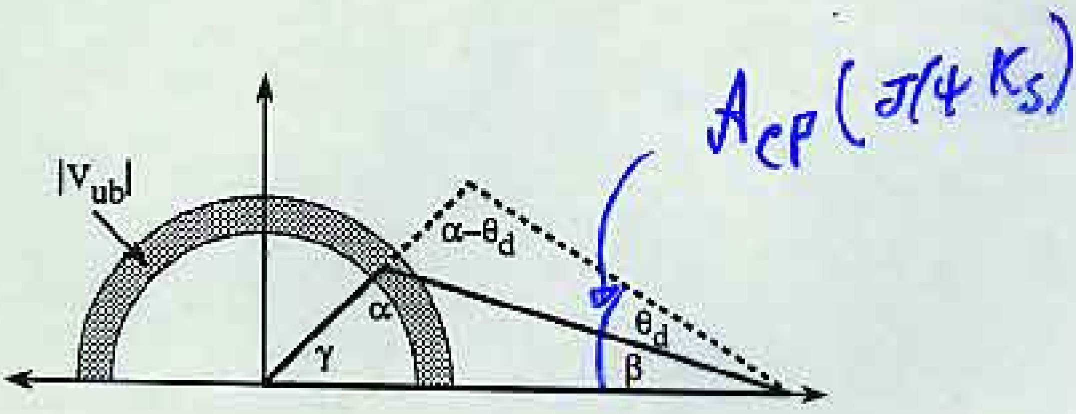


Cohen et al.;
Silva, Wolfenstein

$$\theta_{d,s} \equiv \frac{1}{2} \arg\left(\frac{\langle B_{d,s}|\mathcal{H}_{\text{eff}}^{\text{full}}|\bar{B}_{d,s}\rangle}{\langle B_{d,s}|\mathcal{H}_{\text{eff}}^{\text{SM}}|\bar{B}_{d,s}\rangle}\right)$$

Decay	Quark Process	A_{CP}
$B_d^0 \to \pi^+\pi^-$	$\bar{b} \to \bar{u}u\bar{d}$	$\sin 2(\alpha - \theta_d)$
$B_d^0 \to D^+D^-$	$\bar{b} \to \bar{c}c\bar{d}$	$-\sin 2(\beta + \theta_d)$
$\Rightarrow$ $B_d^0 \to \psi K_s$	$\bar{b} \to \bar{c}c\bar{s}$	$-\sin 2(\beta + \theta_d + \omega)$

$\omega \sim O(\lambda^2) \ll 1$



Solid triangle corresponds to the CKM unitarity condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. *The angles* $(\alpha - \theta_d)$ *and* $(\beta + \theta_d)$ *are measured;* α, β *and* θ_d *may then be reconstructed from knowledge of* $|V_{ub}|$.

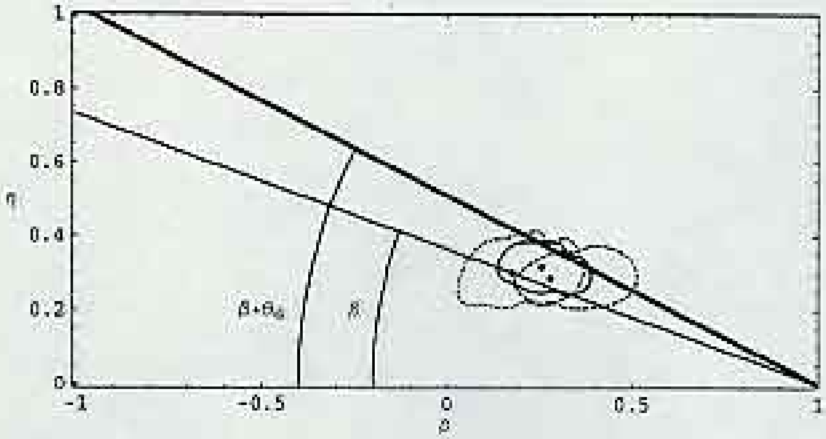
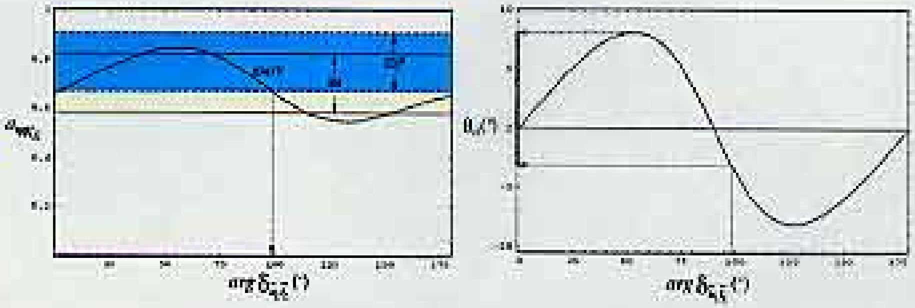


Figure 1: *Allowed 95 % C.L. contours in the (ρ, η) plane. The solid contour corresponds to the SM case, the dashed contour to the Minimal Flavour Violation case with ($f = 0.4$, $g = 0$) and the dashed-dotted contour to the Extended-MFV model discussed in the text ($f = 0$, $g_R = -0.2$, $g_I = 0.2$). (From Ref. 25.)*

$a_{J/\psi K_s}$



$-3^\circ \leq \theta_d \leq 8^\circ$

Figure 2: *The CP asymmetry $a_{\psi K_S}$ as a function of $\arg\delta_{\tilde{u}_L\tilde{t}_2}$ expressed in degrees. The solid curve corresponds to the Extended-MFV model ($f = 0$, $|g| = 0.28$). The light and dark shaded bands correspond, respectively, to the allowed 1 σ region in the SM ($0.58 \leq a_{\psi K_S} \leq 0.82$) and the current 1 σ experimental band ($0.67 \leq a_{\psi K_S} \leq 0.91$). The plot on the right shows the correlation between $\arg\delta_{\tilde{u}_L\tilde{t}_2}$ and the angle θ_d: $\theta_d = \frac{1}{2}\arg(1 + f + |g| e^{2i\arg\delta_{\tilde{u}_L\tilde{t}_2}})$, (mod π). The experimentally allowed region flavours $0^\circ < \arg\delta_{\tilde{u}_L\tilde{t}_2} < 100^\circ$ that translates into $-3^\circ < \theta_d < 8^\circ$. (From Ref. 25.)*

$$\boxed{b \to d\gamma}$$

$$R = \frac{2\mathcal{B}(B^0 \to \rho^0\gamma)}{\mathcal{B}(B \to K^*\gamma)} = \underbrace{\left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3}\xi}_{R_{SM}} \left|\frac{C_7^d(M_b)}{C_7^s(M_b)}\right|^2$$

$$\begin{cases} C_7^s = C_7^W + C_7^{H^\pm} + C_7^\chi \\ C_7^d = C_7^W + C_7^{H^\pm} + C_7^\chi + C_7^{MI} \end{cases}$$

$$C_7^{MI} \simeq \left|\frac{V_{ud}}{V_{td}}\right| \sum_{i=1}^{2} \frac{M_{\tilde{q}} M_{\tilde{t}_2} M_W}{6M_{\chi_i}^3} \frac{\sqrt{2}\tilde{U}_{i2}\tilde{V}_{i1}\sin\theta_{\tilde{t}}}{\cos\beta_S} f_2^{MI} \delta_{\tilde{u}_L\tilde{t}_2}$$

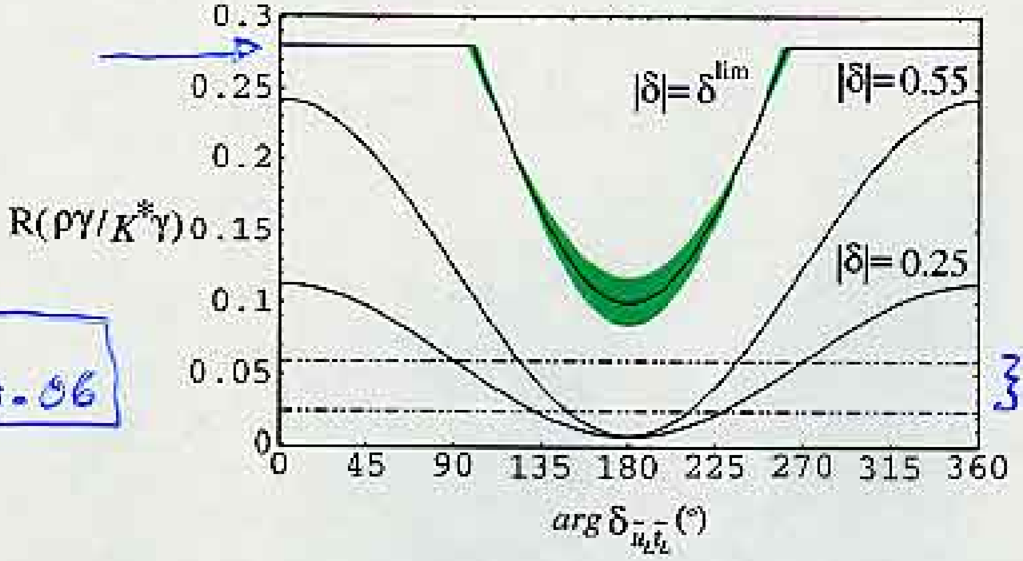
$$\equiv \overline{C}_7^{MI} \delta_{\tilde{u}_L\tilde{t}_2}$$

$$\boxed{R = R_{SM} \left|1 + \delta_{\tilde{u}_L\tilde{t}_2} \frac{\eta^{\frac{16}{23}}\overline{C}_7^{MI}}{C_7^s(M_b)}\right|^2}$$

Lunghi, AA

R^{Expt}_{VL}

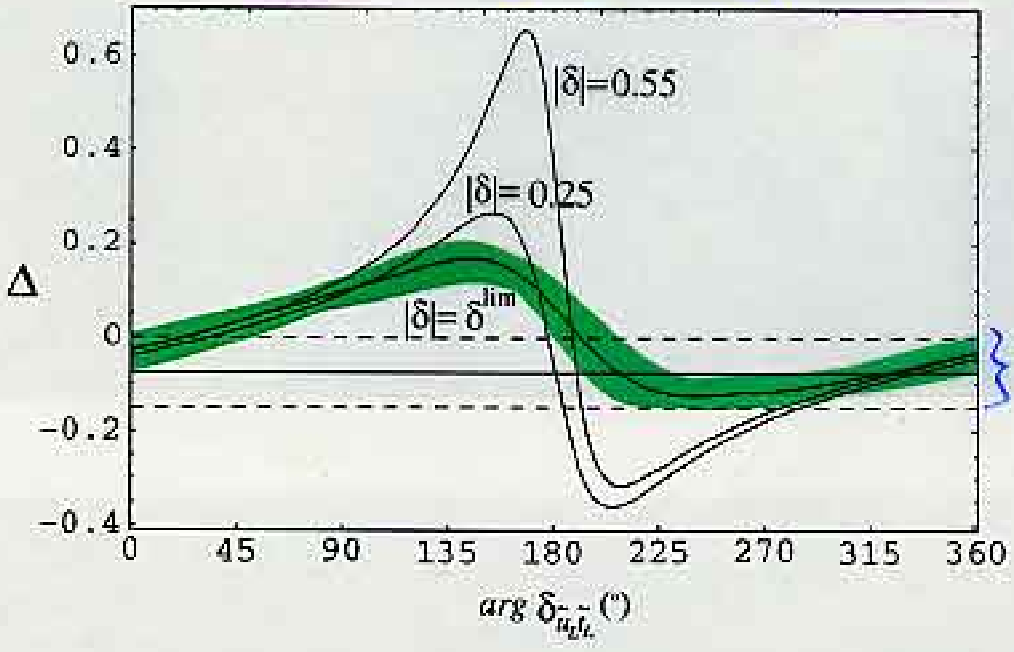
$0.025 \leq R^{SM} \leq 0.06$



} SM

Figure 10: *The ratio $R(\rho\gamma/K^*\gamma) = \mathcal{B}(B \to \rho\gamma)/\mathcal{B}(B \to K^*\gamma)$ as a function of* $\arg \delta_{\tilde{t}_2\tilde{u}_L}$ *(in degrees) in the Extended-MFV model, satisfying the present experimental upper bound $R(\rho\gamma/K^*\gamma) < 0.28$ (at 90% C.L.). The solid lines are obtained for $\bar{\rho}$ and $\bar{\eta}$ set to their central values and for $|\delta_{\tilde{u}_L\tilde{t}_2}| = \delta^{\lim}$, 0.55 and 0.25. The shaded region in the top curve represents the 1 σ uncertainty due to the fit of the unitarity triangle. The dashed lines indicate the 1 σ SM prediction.*

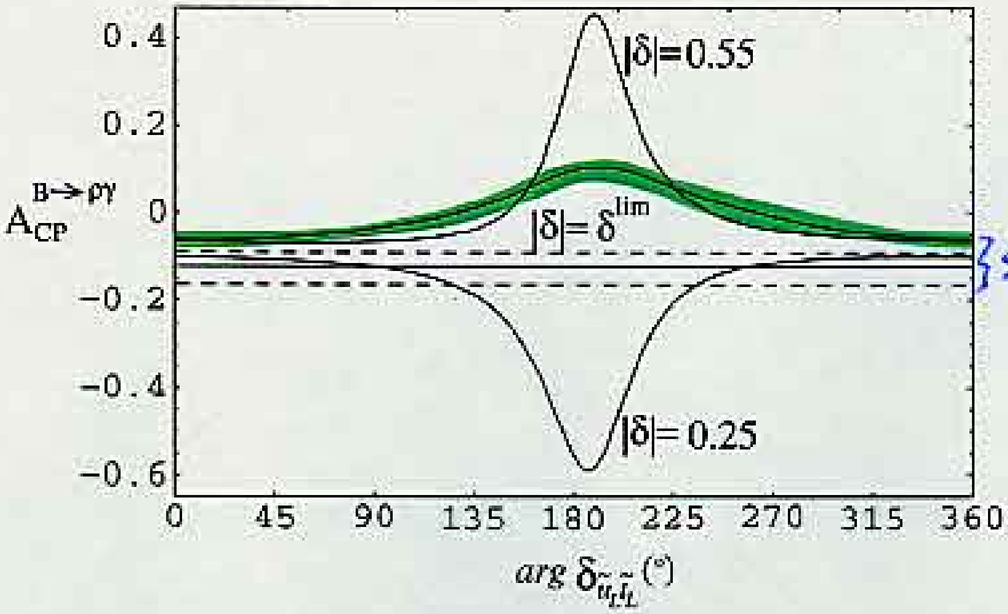
$\Delta^{SM} \in [-0.25, 0.06]$



} SM

Figure 11: *The isospin breaking ratio Δ as a function of* $\arg \delta_{\tilde{t}_2\tilde{u}_L}$ *(in degrees). See the caption in Fig. 10 for further explanations.*

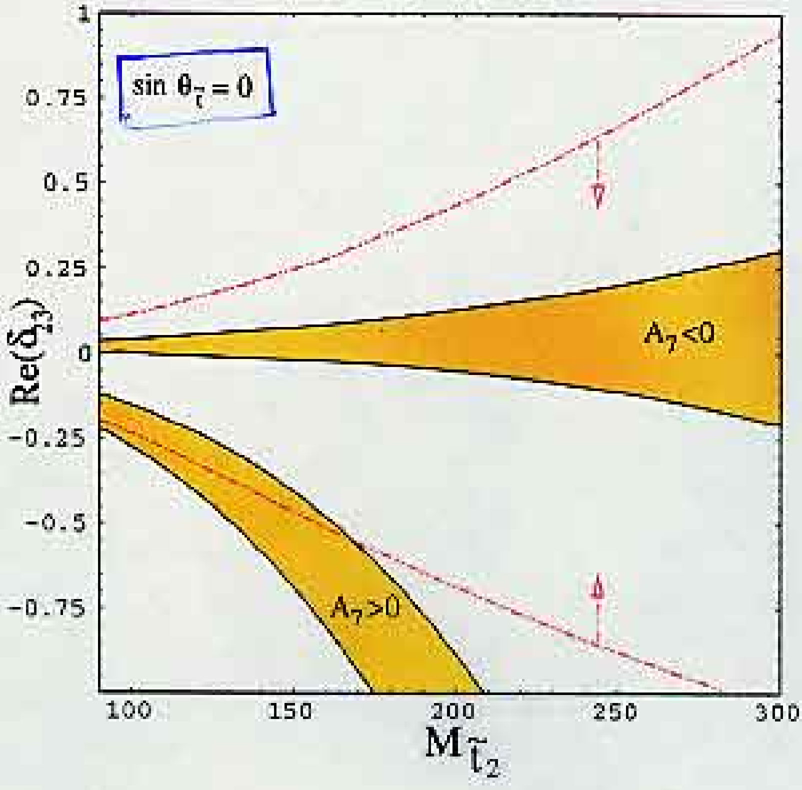
$A_{CP}^{SM} \in [-0.20, -0.06]$



SM

Figure 12: *The CP asymmetry in* $B^\pm \to \rho^\pm\gamma$ *as a function of* $\arg(\delta_{\tilde{t}_2\tilde{u}_L})$ *(in degrees). See the caption in Fig. 10 for further explanations.*

Constraints in Extend-MFV Model



$B \to (X_s, K, K^*) l^+ l^-$

$B \to X_s \gamma$

Figure 10: Bounds on $\delta_{\tilde{t}_2\tilde{c}_L}$ as a function of $M_{\tilde{t}_2}$. $\theta_{\tilde{t}} = 0$ is set to 0 and the mass of the lightest chargino is set to $M_{\chi 1} = 100\text{GeV}$

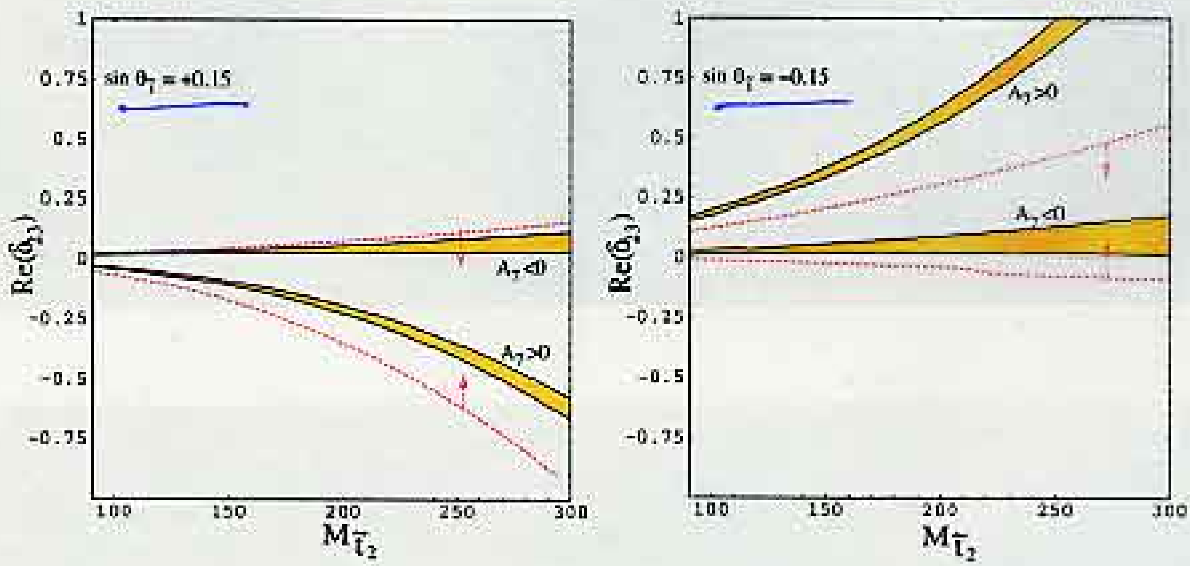


Figure 11: Bounds on $\delta_{\tilde{t}_2\tilde{c}_L}$ as a function of $M_{\tilde{t}_2}$. $\theta_{\tilde{t}} = \pm 0.15$, $\tan\beta = 4$ and the mass of the lightest chargino is set to $M_{\chi 1} = 100\text{GeV}$

Summary

- SM is in comfortable agreement with data on $B \to X_s\gamma$; a non-trivial CKM unitarity test

- Supersymmetric theories also in agreement with data!
Two-fold ambiguity on the sign: $C_7^{\rm tot} > 0$ and $C_7^{\rm tot} < 0$ solutions allowed; will be resolved in FCNC semileptonic decays

- SM is in agreement with the present limits (and one measurement) in semileptonic rare B-decays $B \to (X_s, K^*, K)\ell^+\ell^-$

- Theoretical precision in exclusive decay compromised by the imprecise knowledge of form factors; Inclusive decays $B \to X_s\ell^+\ell^-$ under theoretical control

- Despite theoretical uncertainties, the experimental sensitivity on rare semileptonic B decays is already strong enough to provide non trivial bounds on the SUSY parameter space

- Dilepton invariant mass distribution and Forward-Backward asymmetry crucial measurements in rare B-decays
$\Longrightarrow$ precise determination of Wilson coefficients
$\Longrightarrow$ Precision tests of SM in flavour physics, or discovery of BSM-Physics; Supersymmetry is a case in point

Table 3: Rare B decay branching ratios in the SM and experiments

Decay Modes	$\mathcal{B}$(SM)	Measurements and 90% C.L. Upper Limits
• $(B^{\pm}, B^0) \to X_s\gamma$	$(3.35 \pm 0.30) \times 10^{-4}$	$(3.22 \pm 0.40) \times 10^{-4}$ [CL,AL,BE]
• $B^0 \to K^{*0}\gamma$	$(7.0 \pm 2.7) \times 10^{-5}$	$(4.44 \pm 0.35) \times 10^{-5}$ [CL,BA,BE]
• $B^{\pm} \to K^{\pm}\gamma$	$(7.4 \pm 2.7) \times 10^{-5}$	$(3.82 \pm 0.47) \times 10^{-5}$ [CL,BE]
$(B^{\pm}, B^0) \to X_d\gamma$	$(1.6 \pm 1.2) \times 10^{-5}$	–
$B^{\pm} \to \rho^{\pm} + \gamma$	$(0.5 - 1.2) \times 10^{-6}$	$< 1.3 \times 10^{-5}$ [CLEO]
$B^0 \to \rho^0 + \gamma$	$(0.3 - 0.6) \times 10^{-6}$	$< 5.6 \times 10^{-6}$ [BELLE]
$B^0 \to \omega + \gamma$	$(0.3 - 0.6) \times 10^{-6}$	$< 0.92 \times 10^{-5}$ [CLEO]
$B_s^0 \to \phi + \gamma$	$(5.4 \pm 1.5) \times 10^{-5}$	$< 2.9 \times 10^{-4}$ [ALEPH]
$B_s^0 \to K^* + \gamma$	$(1.0 \pm 0.6) \times 10^{-6}$	–
$(B^{\pm}, B^0) \to X_s e^+e^-$	$(6.9 \pm 1.0) \times 10^{-6}$	$< 10.1 \times 10^{-6}$ [BELLE]
$(B^{\pm}, B^0) \to X_d e^+e^-$	$(3.4 \pm 2.5) \times 10^{-7}$	–
$(B^{\pm}, B^0) \to X_s \mu^+\mu^-$	$(4.2 \pm 0.7) \times 10^{-6}$	$< 19.1 \times 10^{-6}$ [BELLE]
$(B^{\pm}, B^0) \to X_d \mu^+\mu^-$	$(2.1 \pm 1.5) \times 10^{-7}$	–
• $(B^{\pm}, B^0) \to K\ell^+\ell^-$	$(0.3 \pm 0.12) \times 10^{-6}$	$0.75^{+0.25}_{-0.21} \pm 0.09 \times 10^{-6}$ [BELLE]
$(B^{\pm}, B^0) \to K^* e^+e^-$	$(1.6 \pm 0.5) \times 10^{-6}$	$< 5.1 \times 10^{-6}$ [BELLE]
$(B^{\pm}, B^0) \to K^* \mu^+\mu^-$	$(1.2 \pm 0.4) \times 10^{-6}$	$< 3.0 \times 10^{-6}$ [BELLE]
$(B^{\pm}, B^0) \to X_s\, \nu\bar{\nu}$	$(4.0 \pm 1.0) \times 10^{-5}$	$< 7.7 \times 10^{-4}$ [ALEPH]
$(B^{\pm}, B^0) \to X_d\, \nu\bar{\nu}$	$(2.3 \pm 1.5) \times 10^{-6}$	–
$(B^{\pm}, B^0) \to K\, \nu\bar{\nu}$	$(3.2 \pm 1.6) \times 10^{-6}$	–
$(B^{\pm}, B^0) \to K^*\, \nu\bar{\nu}$	$(1.1 \pm 0.55) \times 10^{-5}$	–
$B_s^0 \to \gamma\gamma$	$(9.0 \pm 4.0) \times 10^{-7}$	$< 1.1 \times 10^{-4}$ [L3]
$B^0 \to \gamma\gamma$	$(4.0 \pm 2.0) \times 10^{-8}$	$< 3.8 \times 10^{-5}$ [L3]
$B_s^0 \to \tau^+\tau^-$	$(7.4 \pm 1.9) \times 10^{-7}$	–
$B^0 \to \tau^+\tau^-$	$(3.1 \pm 1.9) \times 10^{-8}$	–
$B_s^0 \to \mu^+\mu^-$	$(3.5 \pm 1.0) \times 10^{-9}$	$< 7.7 \times 10^{-7}$ [CDF]
$B^0 \to \mu^+\mu^-$	$(1.5 \pm 0.9) \times 10^{-10}$	$< 2.6 \times 10^{-7}$ [CDF]
$B_s^0 \to e^+e^-$	$(8.0 \pm 3.5) \times 10^{-14}$	–
$B^0 \to e^+e^-$	$(3.4 \pm 2.3) \times 10^{-15}$	–