Introduction to Effective Lagrangians for QCD

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A brief introduction to the effective Lagrangian treatment of QCD (in the sense of using fields representing physical particles rather than quarks and gluons) will be given. The historical evolution of the subject will be discussed. Some background material related to a recent model for Gamma Ray Bursters will be given. Finally, some recent work on low energy strong interactions will be mentioned.

1 Need for an effective Lagrangian; historical background

There is little doubt that, at least up to energies probed by present accelerators, QCD is the correct theory of strong interactions. It contains the three light spin 1/2 quark fields u, d and s as well as the three heavy quark fields c, b and t. We will focus attention completely on the light quarks here. The dynamics is governed by an SU(3) Yang Mills [1] theory; roughly this means that the strong force has a general similarity to the ordinary QED force which involves the exchange of a gauge field (photon). However there are eight "photons" rather than a single one in QCD. They may be put into a traceless 3×3 matrix, A_μ and the interaction term in the fundamental Lagrangian for, say, the u quark is $ig\bar{u}\gamma_\mu A_\mu u$, where g is a coupling constant. Notice that u is a column vector with unwritten "color" indices. Unlike the QED case, the QCD "photons" have self- interaction terms with the structures $-igTr(\partial_\mu A_\nu[A_\mu, A_\nu]) + \frac{g^2}{4}Tr([A_\mu, A_\nu][A_\mu, A_\nu])$. These have the consequence at the first loop order of perturbation theory that the effective energy dependent coupling constant behaves as

$$\frac{g^2(E)}{4\pi} \sim 1/ln(\frac{E}{\Lambda}), \tag{1}$$

where E is the energy scale at which the theory is being applied and Λ is a fixed number of order 250 MeV characterizing QCD. At high energies (above several GeV) g is small and perturbation theory is good. However at low energies, where we want to discuss topics like binding of quarks to make mesons, interactions of light mesons, CP violation in K meson decays, Nuclear Physics, etc. etc., g is large so QCD is non perturbative. Then one must adopt some other approach. It seems reasonable to hope that this strong coupling at low energies tightly binds the quarks into mesons which interact weakly enough among themselves to be described by the perturbative treatment of an *effective Lagrangian* constructed from the light meson fields [1] rather than the quarks and gluons. This hope turns out to be realized.

The crucial idea in constructing the effective Lagrangian is to mock up the symmetries observed in nature (and which are displayed by the more fundamental QCD Lagrangian). Apart from the Lorentz and discrete space-time symmetries of the strong interaction, the starting point is the imposition of Wigner's isotopic spin symmetry, now denoted $SU(2)_V$. The proton and neutron belong to a spinor of $SU(2)_V$ while three pion fields representing linear combinations of the three pion charge states belong to a vector. The corresponding transformations are:

$$SU(2)_V: \quad N = \begin{pmatrix} p \\ n \end{pmatrix} \rightarrow U_V N \quad \text{and} \quad \phi = \frac{1}{\sqrt{2}} \boldsymbol{\pi} \cdot \boldsymbol{\tau} \rightarrow U_V \phi {U_V}^\dagger, \tag{2}$$

where U_V is a two dimensional unitary unimodular matrix. The subscript V stands for vector. Yukawa's original theory implies an effective Lagrangian with the $SU(2)_V$ invariant interaction terms:

$$i g_Y \overline{N} \phi \gamma_5 N + \lambda [Tr(\phi^2)]^2. \tag{3}$$

Here g_Y is the Yukawa coupling constant while λ is the coupling constant for a term which gives $\pi\pi$ scattering.

Does the Yukawa Lagrangian actually work? If it is to be a reasonable effective Lagrangian it should, like QED, give reasonable results already at tree level. Now the value of g_Y has been known for about 60 years (determined from the long distance part of the nucleon nucleon force due to pion exchange). Around 50 years ago pions were made in accelerators and pion nucleon scattering measured. At tree level, the amplitude for this scattering process should correspond to nucleon "exchange" and be proportional to g_Y^2. Unfortunately it turns out, near the scattering threshold, to be more than an order of magnitude larger than experiment. This would appear to doom the Yukawa theory as the basis for a perturbative treatment. However, there is a small modification which saves it.

Before getting to this modification we note that the discovery of strange particles indicated that the isotopic symmetry group $SU(2)_V$ is too small and that it should be upgraded to $SU(3)_V$ while the meson matrix ϕ in eq.(2) should be upgraded [3] to a 3×3 matrix. The mesons are now pictured as composites, $q\overline{q}$ of a quark and

[1] Baryons can either be included directly or observed to emerge as solitons[2] in the effective meson theory.

anti-quark [4, 5]. Similarly the baryons are regarded as composites qqq. At the fundamental quark level the $SU(3)_V$ symmetry is realized as:

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \to U_V q, \tag{4}$$

where U_V is now a 3×3 unitary matrix. It was understood from the beginning [4, 6] that the symmetry group $SU(3)_V$ has to be an approximate one; it could only be exact in a world in which the nucleons and strange baryons like the Λ are degenerate.

The desired modification of the Yukawa theory which saves the perturbative pion nucleon scattering prediction arises from the extension of the isotopic spin symmetry in another direction. This may be motivated by consideration of the "V-A" theory of beta decay type interactions, discovered [7] a little more than forty years ago. In that well confirmed model, which displays maximal parity violation, the fundamental fermion fields (then nucleons but now quarks) enter into the interaction only through their projection: $q_L = \frac{1+\gamma_5}{2} q$. For a massless fermion, this corresponds to a left handed helicity state (momentum vector opposite to spin vector). The strong interaction, which conserves parity, of course, requires the inclusion also of the right handed projection $q_R = \frac{1-\gamma_5}{2} q$. What this seems to be telling us is that Nature likes to employ the projected (chiral) fields. Around that time, well before QCD was known, it raised the question as to whether Nature also chooses to realize isotopic type symmetries on the chiral components. The chiral (for generality) $U(3)_L \times U(3)_R$ symmetry corresponds to demanding invariance under the transformations:

$$q_{L,R} \to U_{L,R} q_{L,R}, \tag{5}$$

where U_L and U_R are two separate 3×3 unitary matrices. This symmetry is an approximate one since it is manifestly broken by non zero fundamental fermion masses. The vector symmetry corresponds to the special choice $U_L = U_R = U_V$. In order to implement the chiral symmetry in the usual manner it is necessary to further enlarge the meson multiplet. In the simplest version which uses $SU(2)_L \times SU(2)_R$ it means adding to the three pion field components an isosinglet scalar field called the sigma [8], whose coupling to the nucleon is correlated with that of the pion. We will discuss this more soon but for the moment just note that it modifies the pion nucleon scattering diagrams to those shown in Fig 1. The new sigma exchange diagram almost completely cancels the contribution of the nucleon exchange diagrams. The much smaller answer agrees at low energies with experiment to about 15 per cent, if the mass of the sigma is considerably heavier than that of the pions. Other similarly good strong interaction predictions can be made by the same model. Thus it seems that chiral symmetry has "saved" the Yukawa theory as the effective description of strong interactions at low energies.

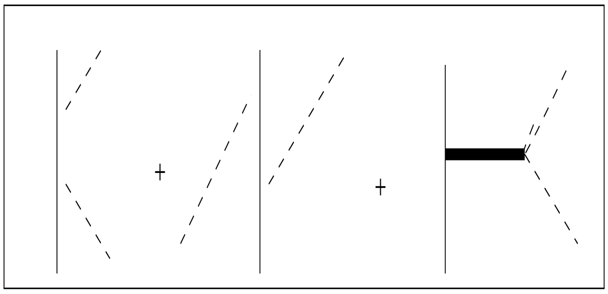

Figure 1: Nucleon=full line,pion=dashed line, sigma=heavy line.

2 Effective Lagrangian of mesons

One of the many questions raised by the remarks above concerns the consistency of the requirement that the mass of the sigma be appreciably greater than the mass of the pion, with the requirement that they both belong to the same chiral multiplet and hence should be, at least approximately, degenerate in mass. To investigate this further it is easier to focus just on the mesons. The same problem of too large predicted low energy scattering is displayed by pi pi scattering and the solution of including sigma exchanges is also the same.

The meson field multiplet, M_{ab} with the correct chiral properties is schematically constructed from the underlying quark fields as:

$$\overline{q}_{Rb} q_{La} \sim M_{ab} = S_{ab} + i\phi_{ab}. \tag{6}$$

where the decomposition of M into pseudoscalar, $\phi = \phi^\dagger$ and scalar, $S = S^\dagger$ pieces is shown. M has the transformation properties

$$U(3)_L \times U(3)_R : \quad M \rightarrow U_L M U_R{}^\dagger, \quad \text{parity} : \quad M(\mathbf{x}) \rightarrow M^\dagger(-\mathbf{x}). \tag{7}$$

One may check these by noting that the Yukawa like term $\overline{q}_L M q_R + h.c.$ is invariant. The simplest Lagrangian made from M is:

$$\mathcal{L}_{meson} = -\frac{1}{2} Tr(\partial_\mu M \partial_\mu M^\dagger) - V_0(M, M^\dagger) + \sum A_a (M_{aa} + M^\dagger{}_{aa}). \tag{8}$$

The first term is the standard kinetic term while the second term might as well be taken to be the most general non derivative function of the independent chiral $SU(3)_L \times SU(3)_R$ invariants $I_1 = Tr(MM^\dagger)$, $I_2 = Tr[(MM^\dagger)^2]$, $I_3 = Tr[(MM^\dagger)^3]$ and $I_4 = 6(detM + detM^\dagger)$. The last term provides the required chiral as well as flavor (vector) type symmetry breaking, with the real numbers A_a being proportional to the masses m_a of the light quarks. It turns out [9] that many of the most interesting consequences of eq.(8) at tree level are independent of the specific form of V_0; only its symmetry properties are needed. The consequences of chiral symmetry are relations between n and $n-1$ point vertices; for example, trilinear vertices are related to masses while masses are related to "decay constants". Another point of interest is that the presence of the quantity I_4 in V_0 spoils the invariance under "axial baryon number" or $U(1)_A$ transformations of the form $M \rightarrow phase \times M$. If this extra symmetry were allowed to remain it would force [10] one of the eta type mesons to be degenerate with the pi, in clear contradiction to nature.

The names of the 18 particles belonging to M, which comprises a basis for the irreducible representation $(3,3^*)+(3^*,3)$ of $SU(3)_L \times SU(3)_R$ and parity, are listed in Table 1.

It is helpful to first consider the two flavor case wherein M is just a 2×2 matrix. This corresponds to keeping the eight particles in the first two columns of Table 1. However a further simplification is possible since, for only $N=2$, the fundamental representation of $SU(N)$ is equivalent to its complex conjugate. Using the fact that $U\tau_2 = \tau_2 U^*$ for any 2 dimensional unitary, unimodular matrix, U we observe that under an $SU(2)_L \times SU(2)_R$ transformation, $\tau_2 M^* \tau_2 \rightarrow U_L(\tau_2 M^* \tau_2) {U_R}^\dagger$. This is the same transformation as in eq.(7) so we have the following two linear combinations which each transform irreducibly under $SU(2)_L \times SU(2)_R$:

$$\frac{1}{\sqrt{2}}(M + \tau_2 M^* \tau_2) = \sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}, \quad \frac{1}{\sqrt{2}}(M - \tau_2 M^* \tau_2) = i\eta + \mathbf{a}_0\cdot\boldsymbol{\tau}. \tag{9}$$

Thus it is consistent to construct a theory using just the σ and π fields; this is the choice made in the Gell-Mann Levy model [8] discussed above.

The chiral invariant potential in the Gell-Mann Levy model is taken to be:

$$V_0 = -b(\sigma^2 + \boldsymbol{\pi}^2) + \lambda(\sigma^2 + \boldsymbol{\pi}^2)^2, \tag{10}$$

where b and λ are real, positive constants. Because of the "wrong sign" quadratic term, the parity conserving minimum of the potential will occur for $\boldsymbol{\pi} = 0$ and $(\sigma)_{min}\widetilde{\chi}_0^0$. Once the Lagrangian is rewritten in terms of new "small oscillation" fields defined as deviations from their values at the potential minimum, it is evident that the resulting theory no longer has the full chiral symmetry. This is the familiar phenomenon of spontaneous symmetry breakdown. The remaining symmetry is just isospin invariance ($SU(2)_V$) and, neglecting the quark masses for simplicity, the pion

$spin^{parity}$	$I=1$	$I=0$	$I=1/2$	$I=0$
0^-	$\boldsymbol{\pi}$	η	$K's$	η'
0^+	$\mathbf{a}_0$	σ	$\kappa's$	σ'

Table 1: States represented by M.

is forced to become a Nambu Goldstone boson (i.e. to have zero mass). On the other hand the mass of the sigma is a free parameter. This solves the problem of degenerate pion and sigma, mentioned at the beginning of this section.

At the level of fundamental (idealized to be massless) fermions, it was pointed out [11] that the origin of $(\sigma)_{min}\widetilde{\chi}_0^0$ is, analogously to the theory of superconductivity, a pairing force between (in modern language) q_L and $\overline{q}_R$. This leads to a ground state $|0>$ which is a "condensate" of such pairs and their conjugates, characterized by $<0|\overline{q}_a q_a|0> \widetilde{\chi}_0^0$.

It is interesting to discuss the pi pi scattering amplitude in this model. There is a $\sigma\phi\phi$ interaction as well as the four pion self interaction term shown in eq.(3). The amplitude for any particular choice of pion charges is conventionally specified by a suitable linear combination of the single function $A(s,t,u)$ and its permutations under the interchange of the Mandelstam variables s, t and u. At tree level one has:

$$A(s,t,u) = \frac{2}{F_\pi^2}(m_\sigma^2 - m_\pi^2)(\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s} - 1), \tag{11}$$

where the first term represents the sigma exchange piece and the second term, the contact interaction piece. Furthermore the quantity F_π ($= \sqrt{2}(\sigma)_{min}$ in the model) is identifiable as the hadron factor of the amplitude for $\pi^- \to \mu^-\overline{\nu}_\mu$ and is numerically, 0.131 GeV. At low energies (i.e. just above $s = 4m_\pi^2$) and considering m_σ to be considerably larger than m_π, eq.(11) becomes,

$$A(s,t,u) = \frac{2}{F_\pi^2}(s - m_\pi^2), \tag{12}$$

a formula due to Weinberg [12]. It is in reasonable agreement with experiment and can be noted to emerge from the difference of the two terms in eq.(11) which are each about twenty times larger than the final result.

While this numerical result is very encouraging, from the standpoint of making a perturbation expansion it is not nice that it arises from the near cancellation of two large quantities. Also, historically the sigma meson has been hard to identify from experiment. For the latter reason it was proposed already in [8] to "integrate out" the sigma by imagining its mass to go to infinity. This may be done from the equation of motion $\partial_\mu\partial_\mu\sigma = \frac{\partial V_0}{\partial\sigma}$. For an infinitely heavy particle we neglect the kinetic term so the equation of motion just becomes $0 = \frac{\partial V_0}{\partial\sigma}$, neglecting, at first, the "quark mass terms". Then eq.(10) can be solved for σ:

$$\sigma = \sqrt{F_\pi^2/2 - \boldsymbol{\pi}^2}. \tag{13}$$

Substituting back into the two flavor version of eq.(8) yields the Gell-Mann Levy non linear sigma model:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - \frac{1}{2}(\partial_\mu\sqrt{F_\pi^2/2 - \boldsymbol{\pi}^2})^2. \tag{14}$$

Notice that the substitution of eq.(13) replaces the potential of eq.(10) by a constant. All the interactions in eq.(14) involve only pions and are of derivative type. This reproduces the desired result eq.(12) directly without any need to mention the sigma.

It is possible to give a slightly more convenient form for this model, which was obtained independently [13] of ref. [8]. In terms of the unitary matrix "chiral field" $U = exp(2i\phi/F_\pi)$ just write:

$$\mathcal{L} = -\frac{F_\pi^2}{8} Tr(\partial_\mu U \partial_\mu U^\dagger). \tag{15}$$

Of course, the matrix, U is defined by its power series expansion. In order to verify the equivalence of eqs.(14) and (15) one may make use of Chisholm's theorem [14]. This states that the transformation (analog of a point transformation in classical mechanics):

$$\chi_a = \phi_a + \gamma_{abc}\phi_b\phi_c + \gamma_{abcd}\phi_b\phi_c\phi_d + \cdots \tag{16}$$

between the sets of scalar fields ϕ_a and χ_a leads to equivalent results at tree level. An advantage of this formulation is that it can straightforwardly be extended to the three flavor case just by considering ϕ in eq.(15) to be the 3×3 matrix of pseudoscalar fields. Notice that U transforms in the same (linear) way M does in eq.(7); this forces ϕ to transform non-linearly, which give the model its name.

A quick mnemonic for going from the three flavor linear model in eq.(8) to the three flavor non-linear model is to first make the "polar decomposition", $M = BU$ with B hermitian and U unitary, and then replace B by the the assumed $SU(3)$ symmetric "vacuum" value $< B >= \frac{F_\pi}{2} diag(1,1,1)$. Substituting this form into eq.(8) yields finally:

$$\mathcal{L} = -\frac{F_\pi^2}{8} Tr(\partial_\mu U \partial_\mu U^\dagger) + function(detU) + \frac{F_\pi}{2}\sum A_a(U_{aa} + U^\dagger_{aa}). \tag{17}$$

The second term will be non-trivial only if there are nine rather than eight pseudoscalars (i.e. the η' is included) so that $detU\widetilde{\chi}_1^0$. The quantities A_a, proportional to the quark masses, can be related, by using eq.(17) or eq.(8), to the meson masses. In this way one can recapture the initially surprising result [15] for the strange to non-strange quark mass ratio:

$$\frac{A_3}{(A_1 + A_2)/2} = \mathcal{O}(25). \tag{18}$$

This may be contrasted with the value of about 1.4 expected in the qualitatively successful non-relativistic quark model. Similarly one gets [16] for the corresponding iso-spin violating quark mass ratio:

$$\frac{A_2 - A_1}{(A_1 + A_2)/2} = \mathcal{O}(\frac{1}{2}), \tag{19}$$

which may be compared with a value about 0.01 expected in the non-relativistic quark model. These results suggest that the first three (and especially the first two) quark masses are very small. Since these quark masses are the source of the intrinsic chiral $SU(3)_L \times SU(3)_R$ symmetry breaking in the Lagrangian, the goodness of the low energy chiral Lagrangian predictions becomes understandable. In the present picture the non-relativistic quark model masses are identified as arising from the spontaneous breakdown of chiral symmetry; they would be non-zero even if the true quark masses referred to above were zero.

3 QCD ingredients and Gamma Ray Bursters

So far, the arguments leading to eq.(17) were the ones used before QCD was discovered; they are general, based only on flavor symmetries of the strong interaction, and do not need to be changed. Of course QCD has strengthened their acceptance and led to many new insights. In the present context, for example, the violation of the $U(1)_A$ symmetry noted after eq.(8) has been clarified. Even though the QCD Lagrangian written at the classical level obeys this symmetry, it is illusory since quantum corrections (called the Adler Bell Jackiw [17] anomaly) spoil the conservation of the would be symmetry current in the full color gauge theory.

A related phenomenon may be used to throw some light on the ground state of the strong gauge theory and provide the basis for certain semi-quantitative estimates. Neglecting the light quark masses, the QCD Lagrangian written at the classical level is invariant under scale transformations, $x_\mu \to \lambda x_\mu$. Classsically, this implies the existence of a corresponding Noether current D_μ satisfying the conservation law, $\partial_\mu D_\mu = 0$. However this conservation law is also illusory due to quantum corrections. It must be replaced by [18],

$$\partial_\mu D_\mu = -\frac{\beta(g)}{g} Tr(F_{\mu\nu} F_{\mu\nu}) = H, \tag{20}$$

where $\beta(g)$ is the QCD beta function and $F_{\mu\nu}$ is the field strength tensor of QCD. H is a convenient abbreviation for the right hand side. Several authors [19, 1] have considered the possibility of restricting the effective Lagrangian to mock up this equation. This requires the potential term, V_0 of eq.(8) to contain the scalar glueball field, H and to satisfy,

$$H = dTr(M\frac{\partial V_o}{\partial M} + M^\dagger \frac{\partial V_0}{\partial M^\dagger}) + 4H\frac{\partial V_0}{\partial H}) - 4V_0, \tag{21}$$

where d is the scale dimension of M. In order to see how this works let us focus on the simple case in which the quark fields are absent (pure color gauge theory). We assume that the whole theory is approximated by the self interactions of the single scalar glueball H. Then the appropriate effective Lagrangian is,

$$\mathcal{L}_H = -\frac{a}{2}H^{-3/2}(\partial_\mu H)^2 - \frac{1}{4}H ln(\frac{H}{\Lambda^4}), \tag{22}$$

where a is a dimensionless constant and Λ is a QCD energy scale. The first term is a scale invariant kinetic type term while the second term provides the solution of the anomaly condition eq.(21). It is easily seen that the minimum of the potential in eq.(22) occurs for $< H >= \Lambda^4/e$, at which point the vacuum energy density takes the negative value $< V >= -\Lambda^4/(4e)$. In fact, a negative value is required in the bag model picture [21] of quark confinement. In that model, if a "bubble" of non-perturbative, zero energy vacuum is made it would tend to collapse to lower the total energy. However, if quarks or gluons are put inside the bubble, their kinetic energy opposes this effect and results in a stable state. To study the glueball field, h in the simple model of eq.(22) we set $H = \Lambda^4/e + Zh$ and expand. For a given scale Λ, both Z and a may be gotten by specifying the mass m_h.

It may be amusing to note that an effect of electromagnetism can be added to the model through its contribution to the scale anomaly. One should replace the second term in eq.(22) by $-\frac{1}{4}(H + H_{EM})ln(\frac{H}{\Lambda^4})$, where $H_{EM} = -\frac{\alpha}{2\pi}\sum Q_a^2 \mathcal{A}_{\mu\nu}\mathcal{A}_{\mu\nu}$. Here $\mathcal{A}_{\mu\nu}$ is the electromagnetic field strength tensor and the Q_a are the quark charges. This would give an estimate of the glueball decay into two photons in the approximation that it dominates the energy momentum tensor. Of course this model is oversimplified for ordinary QCD since, unlike the case of the pseudoscalar mesons, the glueball will not have low mass.

At this conference many interesting talks were presented on the treatment of QCD at high density (large chemical potential μ). This is a fascinating field [22] since it gives rise to exotic new phases of matter whose properties may be computed perturbatively at large μ. These new phases may be neatly described by effective Lagrangians similar to the one describing ordinary QCD. Possible applications may include matter in compact stars. As an example, Ouyed and Sannino have recently proposed a very imaginative model [23] to understand the puzzling behaviors of Gamma Ray Bursters. They argue it is possible that under the crust of a "quark star" there is a surface layer which could be in the so-called 2SC (two flavor superconducting) phase. In this phase the $SU(3)_C$ color gauge symmetry is spontaneously broken to $SU(2)_C$ and a two flavor chiral symmetry $SU(2)_L \times SU(2)_R$ is preserved in the massless quark limit. They go on to show that the lightest physical hadronic states should actually be glueballs and that the effective Lagrangian is given by an appropriate modification of eq.(22). The gamma ray emissions are born in the $H \to 2\gamma$ process mentioned above. Finally an ingenious mechanism in which the state of the surface layer shuttles between the 2SC and QGP (quark gluon plasma) phases is proposed to explain the observed episodic character of the gamma ray emissions.

4 Going away from threshold.

Gasser and Leutwyler [24] have worked out a systematic procedure for treating eq.(17) beyond tree level (neglecting the second term). Since it is a non-renormalizable theory new counter terms have to be added at every order to cancel divergences. There are ten such terms at one loop order (Two examples : $c_3 Tr(\partial_\mu U \partial_\mu U^\dagger \partial_\nu U \partial_\nu U^\dagger)$ and $c_5 Tr(\partial_\mu U \partial_\mu U^\dagger A(U + U^\dagger))$ where $A = diag(A_1, A_2, A_3)$). If one counts $\mathcal{O}(A/F_\pi) = \mathcal{O}(E^2)$, then the tree diagrams are $\mathcal{O}(E^2/F_\pi^2)$, the one loop diagrams are $\mathcal{O}(E^4/F_\pi^4)$ and so forth. Thus if E is sufficiently small the higher terms in the expansion should be suppressed. This procedure has worked very well to correlate a large number of experimental results near threshold. It should be remarked that the counter terms also contain arbitrary finite pieces, which are adjusted to fit the data.

How far up in energy can this chiral perturbation approach practically take us? To get an idea consider the experimental points shown in Fig. 2 for the real part, $R_0^0(s)$ of the $I = J = 0$ pi pi partial wave amplitude. The chiral perturbation series should essentially give a polynomial fit to this shape, which up to 1.0 GeV is crudely reminiscent of one cycle of the sine curve. Of course,

$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \cdots. \tag{23}$$

By using MAPLE or a similar program it will only take a minute to convine yourself that the number of terms needed to get a decent approximation to one cycle of the sine curve is that just shown. This suggests that something like 7 loop chiral perturbation theory would be required to explain pi pi scattering up to about 1 GeV. An alternative approach, as the data itself suggests, is to explicitly include resonances. For example the fit shown in Fig. 2 was computed [25] using a unitarized tree amplitude from a chiral Lagrangian in which scalar mesons and vector mesons have been consistently added. The sigma, which was earlier "sent to infinity", now turns out to be a broad resonance of mass about 560 MeV. There has been a lot of work [26] in this area recently and it seems exciting, though still in progress. Generalization to a full nonet of scalars suggests [27] the light scalars actually are more likely to be of $qq\overline{q}\overline{q}$ type [28] than of $q\overline{q}$ type. Probably an even better approximation is to include [29] mixing between $qq\overline{q}\overline{q}$ and $q\overline{q}$ states.

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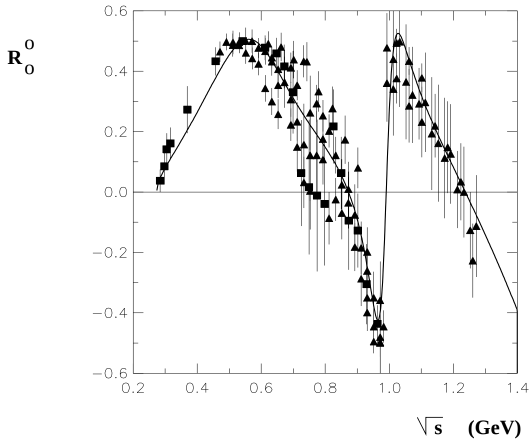

Figure 2: Pi Pi scattering amplitude.

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