A Colored Zoo of Quasi-particles and Light Glueballs

Francesco Sannino
NORDITA,
Blegdamsvej 17, DK-2100
Copenaghen O, DENMARK

1 Electroweak for CFL

Here I present a compendium of the effective Lagrangians for three and two flavors QCD describing the relevant color superconductive light degrees of freedom and their electroweak interactions. However I will not consider some more recent developments involving new phases which are presented by other participants in this conference. Possible phenomenological applications include the description of quark stars, neutron star interiors, the physics near the core of collapsing stars and supernova explosions [1, 2, 3].

At high density, in the 3 flavor case, the symmetry group $SU_L(3)\times SU_R(3)\times SU_c(3)$ breaks spontaneously to $SU_{c+L+R}(3)$ leaving behind 16 Goldston bosons. However, being $SU_c(3)$ a gauge group 8 Goldstone bosons are absorbed in the longitudinal components of the massive gluons. So we are left with 8, not colored, physical massless Goldstone bosons. They can be encoded in the unitary matrix [4] transforming linearly under the left-right flavor rotations $U \to g_L U g_R^\dagger$ with $g_{L/R} \in SU_{L/R}(N_f)$. U satisfies the non linear realization constraint $UU^\dagger = 1$. We also require $\det U = 1$. In this way we avoid discussing the axial $U_A(1)$ anomaly at the effective Lagrangian level. (See Ref. [5] for a general discussion of trace and $U_A(1)$ anomaly). We have $U = \exp\left[i\Phi/F\right]$ with $\Phi = \sqrt{2}\Phi^a t^a$ representing the 8 Goldstone bosons and $\mathrm{Tr}\left[t^a t^b\right] = \delta^{ab}/2$. F is the Goldstone bosons decay constant at finite density.

The effective Lagrangian, for massless quarks, globally invariant under chiral rotations (up to two derivatives) and invariant only under the rotational subgroup of the Lorentz group is:

$$L = \frac{F^2}{2}\,\mathrm{Tr}\left[\dot{U}\dot{U}^\dagger - v^2 \vec{\nabla}U \cdot \vec{\nabla}U^\dagger\right] \ . \tag{1}$$

Clearly by rescaling the vector coordinates $\vec{x} \to \vec{x}/v$ we can recast the previous Lagrangian in the form $L = \dfrac{F^2}{2}\,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right]$.

We now augment the theory by the time-honored Wess-Zumino term which is compactly written using the language of differential forms. It is useful to introduce the algebra valued Maurer-Cartan one forms:

$$\alpha = (\partial_\mu U)\, U^{-1}\, dx^\mu \equiv (dU)\, U^{-1} \ , \qquad \beta = U^{-1} dU = U^{-1}\alpha U \ , \tag{2}$$

which transform, respectively, under the left and right $SU(N_f)$ flavor group. The Wess-Zumino effective action is

$$\Gamma_{WZ}[U] = C \int_{M^5} \mathrm{Tr}\left[\alpha^5\right] \ , \qquad C = -i\frac{N_c}{240\pi^2} \ . \tag{3}$$

C assumes the same value then the one used at zero density and N_c is the number of colors (fixed to be 3 in this case). The coefficient is fixed by saturating the global anomalies at the effective Lagrangian level [6]. The 3 flavor case respects the 't Hooft global anomaly matching conditions at non zero density. A relevant feature is that α, being a differential form, is unaffected by coordinate rescaling (actually topological terms being independent on the metric are unaffected by medium effects), and hence the Wess-Zumino term is not modified at finite matter density.

At this point it is important to note[1] that in reference [7] the coefficient of the Wess-Zumino term actually agrees with the one presented here [8, 6] while it is the one presented/computed in [9] which disagrees by a factor three. A relevant consequence of our results is associated to the solitonic (Skyrme) sector of the effective Lagrangian theory. In fact now we have the same winding number as for ordinary QCD and hence we get massive excitations, which after collective quantization, describe spin half particles with the same quantum numbers of ordinary baryons.

Next we extend the previous effective Lagrangian by incorporating the electroweak intermediate vector mesons as external fields.

$$\begin{aligned} DU \;&=\; \partial U - i\frac{g}{\sqrt{2}}\left[W^+\tau^+ + W^-\tau^-\right]U - i\frac{g}{\cos\theta_W}\, Z^0 \ \left[\tau^3 U - \sin^2\theta_W\, [Q,U]\right] \\ &\quad -i\widetilde{e}\,\widetilde{A}\ [Q,U] - i\widetilde{e}\ \tan\theta\widetilde{G}^8\ [Q,U] \end{aligned} \tag{4}$$

where θ_W is the electroweak angle and

$$\tau^+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \qquad \tau^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \qquad \tau^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \ . \tag{5}$$

The last two terms in Eq. (4) describe the, non anomalous, interaction of the Goldstone bosons with respectively the physical massless photon and the physical massive

[1] I thank D.K. Hong for helping me clarifying this point.

eight gluon. The physical photon and eight gluon states are related to the, in vacuum, ones via:

$$\widetilde{G}^8 = \cos\theta G^8 + \sin\theta A \ , \qquad \widetilde{A} = -\sin\theta G^8 + \cos\theta A \ , \tag{6}$$

with $\cos\theta = \sqrt{3}g_s/\sqrt{3g_s^2 + 4e^2}$. Clearly $\widetilde{e} = e\,\cos\theta$ is the finite density new electric charge.

The $\pi^0 \to \gamma\gamma$ Lagrangian term is obtained by gauging the Wess-Zumino action [6] and is given by:

$$\mathcal{L}_{\pi^0\to\gamma\gamma} = -\frac{30}{4}e^2\, C\, i\mathrm{Tr}\left[t^3 Q^2\right]\pi^0 \frac{\sqrt{2}}{F}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \ , \tag{7}$$

and $\pi^0 = \Phi^3$. After substituting the, in vacuum, photon field with its expression as function of the physical photon and gluon [6] we discover that the process is identical to the in the vacuum one when replacing the, in vacuum, electric charge with $\widetilde{e}$. It is also worth noticing that since the mass of the physical eight gluon is larger than the pion mass the process $\pi^0 \to \widetilde{G}^8\widetilde{G}^8$ is barred kinematically.

2 Electroweak for 2SC

Here I summarize the effective low energy Lagrangian for two flavors (when a 2SC phase sets is) which contains all of the relevant degrees of freedom.

In 2SC the diquark condensate leaves invariant the following symmetry group:

$$[SU_c(2)] \times SU_L(2) \times SU_R(2) \times \widetilde{U}_V(1) \ , \tag{8}$$

where $[SU_c(2)]$ is the unbroken part of the gauge group. The $\widetilde{U}_V(1)$ generator $\widetilde{B}$ is the following linear combination of the previous $U_V(1)$ generator $B = \frac{1}{3}\mathrm{diag}(1,1,1)$ and the broken diagonal generator of the $SU_c(3)$ gauge group $T^8 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}(1,1,-2)$: $\widetilde{B} = B - \frac{2\sqrt{3}}{3}T^8$. The quarks with color 1 and 2 are neutral under $\widetilde{B}$ and consequently the condensate too ($\widetilde{B}$ is $\sqrt{2}\widetilde{S}$ of Ref. [10]). The superconductive phase for $N_f = 2$ possesses the same global symmetry group of the confined Wigner-Weyl phase [8]. In Reference [8], it was shown that the low-energy spectrum, at finite density, displays the correct quantum numbers to saturate the 't Hooft global anomalies. In Reference [11] it was then seen, by using a variety of field theoretical tools, that global anomaly matching conditions hold for any cold but dense gauge theory.

The lowest lying excitations are protected from acquiring a mass by the aforementioned constraints and dominate the low-energy physical processes. Here, see [10], the relevant coset space is G/H with $G = SU_c(3) \times U_V(1)$ and $H = SU_c(2) \times \widetilde{U}_V(1)$ is parameterized by

$$\mathcal{V} = \exp(i\xi^i X^i) \ , \tag{9}$$

where $\{X^i\}$ $i = 1, \cdots, 5$ belong to the coset space G/H and are taken to be $X^i = T^{i+3}$ for $i = 1, \cdots, 4$ while $X^5 = B + \frac{\sqrt{3}}{3}T^8 = \mathrm{diag}(\frac{1}{2}, \frac{1}{2}, 0)$. T^a are the standard generators of $SU(3)$. The coordinates

$$\xi^i = \frac{\Pi^i}{f} \quad i = 1, 2, 3, 4 \ , \qquad \xi^5 = \frac{\Pi^5}{\widetilde{f}} \ , \tag{10}$$

via Π describe the Goldstone bosons.

$\mathcal{V}$ transforms non linearly

$$\mathcal{V}(\xi) \to u_V \, g\mathcal{V}(\xi) h^\dagger(\xi, g, u) h^\dagger_{\widetilde{V}}(\xi, g, u) \ , \tag{11}$$

with $u_V \in U_V(1)$, $g \in SU_c(3)$, $h(\xi, g, u) \in SU_c(2)$ and $h_{\widetilde{V}}(\xi, g, u) \in \widetilde{U}_V(1)$. It is, also, convenient to define:

$$\omega_\mu = i\mathcal{V}^\dagger D_\mu \mathcal{V} \quad \text{with} \quad D_\mu \mathcal{V} = (\partial_\mu - i g_s G_\mu)\mathcal{V} \ , \tag{12}$$

with gluon fields $G_\mu = G^m_\mu T^m$. Following [10] we decompose ω_μ into

$$\omega^{\parallel}_\mu = 2S^a \mathrm{Tr}\left[S^a \omega_\mu\right] \quad \text{and} \quad \omega^{\perp}_\mu = 2X^i \mathrm{Tr}\left[X^i \omega_\mu\right] \ , \tag{13}$$

where S^a are the unbroken generators of H with $S^{1,2,3} = T^{1,2,3}$, $S^4 = \widetilde{B}/\sqrt{2}$. Summation over repeated indices is assumed.

To be able to include the in medium fermions in the picture we define:

$$\widetilde{\psi} = \mathcal{V}^\dagger \psi \ , \tag{14}$$

transforming as $\widetilde{\psi} \to h_{\widetilde{V}}(\xi, g, u) h(\xi, g, u)\widetilde{\psi}$ and ψ possesses an ordinary quark transformations (as Dirac spinor).

The simplest non-linearly realized effective Lagrangian describing in medium fermions, the five gluons and their self interactions, up to two derivatives and quadratic in the fermion fields is:

$$\begin{aligned}
\mathcal{L} = \quad & f^2 a_1 \mathrm{Tr}\left[\omega^\perp_0 \omega^\perp_0 - \alpha_1 \vec{\omega}^\perp \vec{\omega}^\perp\right] + f^2 a_2 \left[\mathrm{Tr}\left[\omega^\perp_0\right] \mathrm{Tr}\left[\omega^\perp_0\right] - \alpha_2 \mathrm{Tr}\left[\vec{\omega}^\perp\right] \mathrm{Tr}\left[\vec{\omega}^\perp\right]\right] \\
+ \quad & b_1 \overline{\widetilde{\psi}} i \left[\gamma^0 (\partial_0 - i\omega^{\parallel}_0) + \beta_1 \vec{\gamma} \cdot \left(\vec{\nabla} - i\vec{\omega}^{\parallel}\right)\right] \widetilde{\psi} + b_2 \overline{\widetilde{\psi}} \left[\gamma^0 \omega^\perp_0 + \beta_2 \vec{\gamma} \cdot \vec{\omega}^\perp\right] \widetilde{\psi} \\
+ \quad & m_M \overline{\widetilde{\psi}^C} \gamma^5 (iT^2) \widetilde{\psi} + \mathrm{h.c.} \ ,
\end{aligned} \tag{15}$$

where $\widetilde{\psi}^C = i\gamma^2 \widetilde{\psi}^*$, $i, j = 1, 2$ are flavor indices and

$$T^2 = S^2 = \frac{1}{2} \begin{pmatrix} \sigma^2 & 0 \\ 0 & 0 \end{pmatrix} \ , \tag{16}$$

a_1, a_2, b_1 and b_2 are real coefficients while m_M is complex. The breaking of Lorentz invariance to the $O(3)$ subgroup, following [4], has been taken into account by providing different coefficients to the temporal and spatial indices of the Lagrangian, and it is encoded in the coefficients αs and βs.

To construct the low energy effective theory for the electroweak sector we need to generalize the one form $\omega_\mu = i\mathcal{V}^\dagger D_\mu \mathcal{V}$ by introducing the new covariant derivative:

$$D_\mu \mathcal{V} = (\partial_\mu - ig_s G_\mu - ig' Y\, B^y_\mu)\mathcal{V} = (\partial_\mu - ig_s G_\mu - ig'\frac{B}{2}\, B^y_\mu)\mathcal{V}\ . \tag{17}$$

B^y_μ is the standard hypercharge gauge field and is a linear combination of the electroweak eigenstates associated to the photon field A_μ and the neutral massive vector boson Z^0_μ, i.e.:

$$B^y_\mu = \cos\theta_W A_\mu - \sin\theta_W Z^0_\mu\ , \tag{18}$$

with θ_W the standard electroweak angle.

After diagonalizing the full quadratic terms the new massless eigenstate is interpreted as the, in medium, photon and is a linear combination of the in vacuum photon and eight gluon:

$$\widetilde{A}_\mu = \cos\theta_Q A_\mu - \sin\theta_Q G^8_\mu\ , \tag{19}$$

with $\cos\theta_Q = \sqrt{3}\frac{g_s}{\sqrt{3g_s^2+e^2}}$. The massive state orthogonal to $\widetilde{A}_\mu$ is $\widetilde{G}^8_\mu = \cos\theta_Q G^8_\mu + \sin\theta_Q A_\mu$ and further mixes with Z^0 of, in vacuum, mass m_Z. The new eigenstates are:

$$\hat{G}^8_\mu = \cos\theta_M \widetilde{G}^8_\mu + \sin\theta_M Z^0_\mu\ , \qquad \widetilde{Z}^0_\mu \;=\; \cos\theta_M Z^0_\mu - \sin\theta_M \widetilde{G}^8_\mu, \tag{20}$$

with $\tan 2\theta_M \approx \frac{2f^2\,\tan\theta_W}{9m_Z^2}\,(a_1+2a_2)\,\sqrt{3g_s^2+e^2}$ and we considered the physical limit $m_Z^2 \gg f^2$. In the same limit, as expected, $\widetilde{G}^8$ and Z^0 do not mix much and we can use them as physical eigenstates.

Having identified the correct physical eigenvalues and eigenvectors we now turn to the quark sector. We have $Q = \tau^3 + \frac{B-L}{2}$ with $\tau = \tau_L + \tau_R$ and $\widetilde{e} = e\,\cos\theta_Q$ is the new electric charge while $\widetilde{Q}$ is the associated electric charge operator associated with the field $\widetilde{A}_\mu$:

$$\widetilde{Q} = \tau^3 \times \mathbf{1} + \frac{\tilde{B}-L}{2} = Q\times\mathbf{1} - \frac{1}{\sqrt{3}}\mathbf{1}\times T^8\ . \tag{21}$$

By expanding around $\mathcal{V} = 1$ the relevant interaction terms we derive [6] the following full modified quark coupling to the neutral weak current:

$$b_1\frac{g}{\cos\theta_W}\, Z^0_\mu \overline{\psi}\gamma^\mu\left[T^3_L - \sin\theta_W^2 Q - \frac{b_2-b_1}{3\,b_1}\sin\theta_W^2(B+Q-\widetilde{Q})\right]\psi\ , \tag{22}$$

where we used Eq. (21) and $X^5 = B + Q - \widetilde{Q}$. Hence we discover that the neutral weak current is directly affected by finite density effects even when neglecting the small physical mixing between the eighth gluon $\widetilde{G}^8$ and Z^0. We also observe that the modified electroweak coupling only emerges for quarks with color indices 1 and 2, since $X^5 = \frac{1}{2}\mathrm{diag}(1,1,0)$. We expect relevant phenomenological consequences of our result for the cooling history of compact objects [12] (see Ref. [6] for more details).

3 The $SU_c(2)$ Glueball Effective Lagrangian

The $SU_c(2)$ gauge symmetry does not break spontaneously and confines. If the new confining scale [13] is lighter than the superconductive quark-quark gap the associated confined degrees of freedom (light glueballs) [14] can play, together with the massless quarks a relevant role for the physics of Quark Stars featuring a 2SC superconductive surface layer [3].

Defining by H the mass-scale dimension 4 composite field describing, upon quantization, the scalar glueball [15] we can construct the simplest light glueball action, in medium, for the SU(2) color theory:

$$S_{G-ball} = \int d^4x \left\{ \frac{c}{2}\sqrt{b}\,H^{-\frac{3}{2}} \left[\partial^0 H \partial^0 H - v^2 \partial^i H \partial^i H\right] - \frac{b}{2} H \log\left[\frac{H}{\hat{\Lambda}^4}\right] \right\} \ . \tag{23}$$

This Lagrangian correctly encodes the underlying $SU_c(2)$ trace anomaly [14]. The glueballs move with the same velocity v as the underlying gluons in the 2SC color superconductor. $\hat{\Lambda}$ is the intrinsic scale associated with the theory and can be less than or of the order of few MeVs [13, 14] while c is a constant of order unity. The glueballs are light (with respect to the gap) and barely interact with the ungapped fermions. They are stable with respect to the strong interactions unlike ordinary glueballs. We define the mass-dimension one glueball field h via $H = \langle H \rangle \exp\left[h/F_h\right]$. By requiring a canonically normalized kinetic term for h one finds $F_h^2 = c\sqrt{b\langle H\rangle}$, while the glueball mass term is $M_h^2 = \sqrt{b}\sqrt{\langle H\rangle/2c} = \sqrt{b}\hat{\Lambda}^2/2c\sqrt{e}$, which is clearly of the order of $\hat{\Lambda}$.

Once created, the light $SU_c(2)$ glueballs are stable against strong interactions but not with respect to electromagnetic processes. The following decay width, at non zero baryon density, was obtained by saturating the electromagnetic trace anomaly in [14]:

$$\Gamma\left[h \to \gamma\gamma\right] \approx 1.2 \times 10^{-2} \cos\theta_Q^4 \left[\frac{M_h}{1\ \mathrm{MeV}}\right]^5 \ \mathrm{eV}\ , \tag{24}$$

where $\alpha = e^2/4\pi \simeq 1/137$. For illustration purposes we consider a glueball mass of the order of 1 MeV which leads to a decay time $\tau \sim\ 5.5 \times\ 10^{-14}s$. We used $\cos\theta_Q \sim\ 1$ since $\theta_Q \sim 2.5^\circ$ [14].

The present light glueball analysis is limited to the zero temperature and high matter density case. However it is interesting to investigate the effects of a non zero temperature.

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