Tensor Correlations and Pions in Dense Nuclear Matter

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1 Introduction

Despite concerted efforts by many groups over a period of more than three decades, the problem of stellar collapse, supernova production, and neutron star formation is still not understood. The basic physical inputs required for calculations of stellar collapse are, apart from an initial stellar model, the equation of state of hot dense matter, and the rates of neutrino production, absorption, and scattering processes. There are tantalizing hints that, if the microscopic physical input differed from what is presently used in simulations, the "delayed" mechanism [1], in which the outgoing shock wave is revived by deposition of energy by neutrinos streaming out of the stellar core, could account for core collapse supernovae and the formation of neutron stars in the collapse of a massive star.

One such hint comes from a relatively old calculation in which it was found that the allowance for pion-like excitations in the equation of state led to an increase in the explosion energy [2]. This indicates the importance of performing improved calculations of the equation of state in which pionic degrees of freedom are treated realistically. A second hint comes from parametric studies of core-collapse models, which indicate that the outcome of stellar collapse is sensitive to modest changes (a factor of two, say) in the neutrino opacity [3]. The effects of some parts of the nucleon-nucleon interactions on rates of neutrino processes in stellar collapse have been investigated by a number of groups, but so far the tensor part of the interaction has received scant attention, except in the work of Raffelt and colleagues [4]. Given what is known about the role of the tensor force in finite nuclei, there is reason to believe that its effect on neutrino processes in stellar collapse could be significant.

Numerical simulations of stellar collapse provide motivation for taking up the question of how important tensor correlations are in nuclear and neutron matter. In

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addition, theoretical investigations of the equation of state of cold dense matter point to there being strong pionic correlations at densities only slightly above that of saturated nuclear matter [5]. In the latter work it was found that a state resembling a neutral pion condensate was energetically favorable compared with one that did not, contrary to the view accepted for the past quarter of a century that pion condensation is ruled out at such densities by the strong central part of the nucleon-nucleon interaction in the spin-isospin channel. The pionic correlations give rise to maxima in the static longitudinal structure factor for the nuclear spin in neutron matter, and for the nucleon spin-isospin in nuclear matter, as shown in Fig. 1. The definition of the structure factor is given in Eq. (10) below. All these pieces of evidence point to

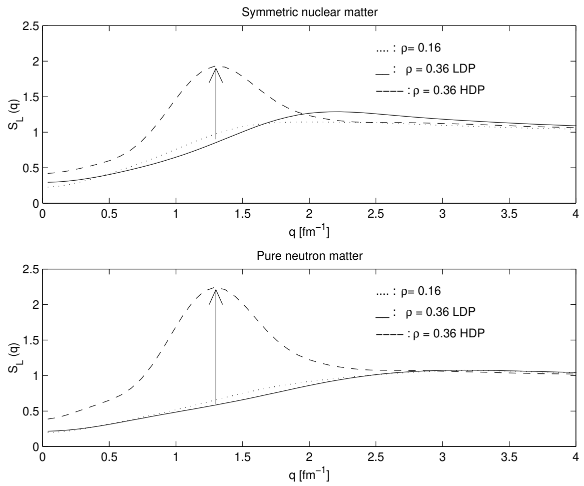

Figure 1: Static structure factor for the longitudinal spin operator in neutron matter and for the longitudinal spin-isospin operator in symmetric nuclear matter as a function of wave number, q. (After Ref. [5]).

the importance of investigating anew tensor correlations in dense matter.

The aspect we shall focus on most here is that of neutrino processes, and how they are affected by non-central forces, of which the pion-exchange interaction between nucleons is an example. Our aim is to provide a general framework within which to describe neutrino processes, including production, scattering and absorption. We shall make connections to Landau's theory of normal Fermi liquids, and indicate how the usual formulation, which assumes that the interaction is invariant under rotation of the spins of the particles, needs to be extended.

2 Neutrino processes

Rates of neutrino processes in dense matter are affected significantly by nucleon-nucleon interactions. Among early works on this subject we mention Refs. [6] and [7] on neutrino scattering processes in degenerate matter. Rates of neutrino processes in dense matter were calculated with allowance for the effect of nucleon-nucleon interactions on the energies of the quasiparticles in Ref. [8]. In addition, the influence of the finite lifetime of excitations in the nuclear medium on rates of neutrino processes has been considered in Refs. [4] and [9]. In the latter calculations, the non-central character of the nucleon-nucleon interaction plays an important role. Finally, in Ref. [10], the calculations of Refs. [6] and [7] have been extended to allow for partial degeneracy of the nucleons, and for non-zero momentum transfers. In the latter work the screening of the weak interaction matrix elements by the central part of the nucleon-nucleon interaction was taken into account. Tensor correlations play an important role in determining the rates of neutrino processes in dense matter, both for the neutrino processes of importance in stellar collapse, as we shall explain in greater detail below, as well as for the modified Urca process which is important for the cooling of neutron stars.

Since the weak coupling constant is small, it is generally an excellent approximation to calculate the rate of neutrino processes in dense matter using the standard golden-rule result for the transition rate. Furthermore, the effects of other interactions, such as electromagnetic ones, between leptons and hadrons are generally small, and the expression for the rate Q of transitions may be written in the form [9]

$$Q \sim {G_F}^2 \; n \int \frac{d^3\mathbf{q}d\omega}{(2\pi)^4} S_{\mu\nu}(\mathbf{q},\omega) N^{\mu\nu}(\mathbf{q},\omega) \tag{1}$$

where G_F is the weak coupling constant, n is the baryon density, $S_{\mu\nu}(\mathbf{q},\omega)$ is a dynamical structure factor for the hadron system, and $N^{\mu\nu}(\mathbf{q},\omega)$ is a function that depends only on the leptons. The quantity $\mathbf{q}$ denotes the momentum transfer to the nucleon system, and ω the energy transfer. The weak interaction has the form of a product of current operators, either the vector current or the axial vector one, for the leptonic and hadronic systems. For definiteness, let us assume that the hadron system consists of non-relativistic nucleons. In this case, the leading contribution to a current is the time component which, for the vector current, is proportional to the nucleon density and, for the axial vector current, to the spin density. More generally one must include relativistic effects, such as weak magnetism (see e.g. Ref. [11]), and other hadronic degrees of freedom.

2.1 Neutral-current processes

To illustrate the qualitative effects of a non-central interaction, such as the tensor interaction, on neutrino processes, we consider as an example scattering of neutrinos by the weak neutral-current interaction. The structure factor of interest in calculating the rate of the process is then the one for the spin density. The tensor interaction has three qualitatively different effects. These are conveniently examined for processes in which the momentum and energy transfers are small compared with the typical energies and momenta of a nucleon in the medium. One effect is to change the expectation value of the spin of an excitation in the nuclear medium to a value different from that for an isolated nucleon. This effect is well-known in the context of nuclear magnetic moments, where the spin-orbit and tensor components of the nucleon-nucleon interaction modify the nuclear moment [12]. A second effect is to introduce into the effective interaction between nucleon-like excitations a non-central component – this modifies screening phenomena. A third effect is that the spin operator, which, if one neglects contributions due to exchange currents, is a one-body operator when expressed in terms of nucleon degrees of freedom, acquires two- and higher-body contributions for the excitations in the nuclear medium.

2.2 Landau theory

To illustrate the effects of non-central interactions described above, we consider the special case of long wavelengths, and temperatures low compared with the Fermi temperature. In this regime the concepts and machinery of Landau's theory of normal Fermi liquids may be applied [13]. We shall examine the form of the spin response for a one-component system. As we mentioned above, this is relevant for calculating the rates of neutral-current processes that proceed via the axial vector contribution of the interaction. If interactions are central, the static magnetic susceptibility of an interacting Fermi liquid with a single fermion component may be expressed in the form

$$\frac{\chi}{\chi_0} = \frac{m^*/m}{1+G_0} \tag{2}$$

where χ_0 is the susceptibility of a non-interacting Fermi gas of the same density, m^* is the effective mass of a quasiparticle, and G_0 is the Landau parameter describing the isotropic part of the spin-dependent component of the interaction. When there are non-central forces, this expression must be modified, and we shall consider the three effects mentioned above in turn.

When the interaction has tensor components, which are non-central, application of a magnetic field will tend to polarize quasiparticles in a direction different from that of the applied field. The magnetic moment of a quasiparticle is then described by a tensor which, from the requirement of rotational invariance, must have the form

$$\mu_{ij}(\mathbf{p}) = \mu_0\delta_{ij} + (3/2)\mu_2(\hat{p}_i\hat{p}_j - \delta_{ij}/3), \tag{3}$$

where $\hat{\mathbf{p}}$ is a unit vector in the direction of the momentum, and μ_0 and μ_2 are coefficients. The interaction between two quasiparticles $f_{\mathbf{p}\boldsymbol{\sigma},\mathbf{p}'\boldsymbol{\sigma}'}$ may be written in the general form

$$f_{\mathbf{p}\boldsymbol{\sigma},\mathbf{p}'\boldsymbol{\sigma}'} = f_{\mathbf{p}\mathbf{p}'} + g_{\mathbf{p}\mathbf{p}'}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}' + t_{\mathbf{p}\mathbf{p}'}(3\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\;\boldsymbol{\sigma}'\cdot\hat{\mathbf{k}} - \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'), \tag{4}$$

where there is tensor contribution in addition to the usual expression for a central interaction. Here $\hat{\mathbf{k}}$ is a unit vector in the direction of $\mathbf{p} - \mathbf{p}'$. If we assume that we may neglect all but the isotropic parts of the quasiparticle interaction, one finds for the magnetic susceptibility the result [14]

$$\chi = \frac{{\mu_0}^2 N(0)}{1 + G_0 - {K_0}^2/8} - \frac{\mu_0\mu_2 N(0) K_0}{1 + G_0} + \frac{1}{2}N(0){\mu_2}^2 + \chi_{\mathrm{M}}, \tag{5}$$

where $K_0 = N(0)t$ is the tensor Landau parameter, $N(0)$ being the density of quasiparticle states at the Fermi surface, $G_0 = N(0)g$ and χ_M is the multipair contribution to the susceptibility. In this expression, the effects of tensor interactions have been included only to second order in all but the first term. The first term in Eq. (5) corresponds to the result derived in Ref. [15] if renormalization of the magnetic moment is neglected.

One reason for the great success of Landau Fermi-liquid theory for liquid ^{3}He and for electrons in metals is that the forces between particles are central to a good first approximation. In addition, the quantities of greatest physical interest are the particle number density and the spin density. What the result for the static susceptibility demonstrates is that many more parameters are needed to characterize the long-wavelength properties of a normal Fermi liquid with non-central forces, compared with the two needed for a liquid with only central forces.

2.3 Kinematics of neutrino processes

Now let us consider scattering of neutrinos by dense matter. For the simultaneous conservation of momentum and energy in the process, the magnitude of the energy transfer ω to the neutrino must be less than cq, where $\mathbf{q}$ is the momentum transfer. In other words, the four-momentum transfer from the nuclear medium to the neutrino must be space-like. For production of neutrino-antineutrino pairs via neutral-current processes, the requirements are the opposite, since the total energy of the pair must be greater than cq, $\mathbf{q}$ in this case being the total momentum of the pair, or the four-momentum transfer must be time-like. It is therefore of interest to explore which sorts of transitions in the nuclear medium can contribute to the various classes of neutrino process. For simplicity, let us again consider momentum transfers small compared with the Fermi momentum of the nucleons, and temperatures low compared with the Fermi temperature. The energy of a single quasiparticle-quasihole pair is given by $\omega = \epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}-\mathbf{q}} \approx \mathbf{v}_{\mathbf{p}} \cdot \mathbf{q}$, where $\mathbf{v}_{\mathbf{p}} = \boldsymbol{\nabla}_{\mathbf{p}} \epsilon_{\mathbf{p}}$ always has a magnitude less than $v_{\mathrm{F}} q$, where v_{F} is the velocity of a quasiparticle at the Fermi surface. Thus single pair excitations in the nuclear medium always have space-like four momentum. Consequently annihilation of a single quasiparticle-quasihole pair cannot create a neutrino-antineutrino pair, while it can contribute to scattering of a neutrino. This clearly indicates the important role that excitations containing two or more quasiparticle-quasihole states play in neutrino pair emission.

2.4 Relative importance of single-pair and multipair states

The discussion above indicates the different roles that single-pair and multipair states play, and here we investigate their relative strength. One simple result may be obtained for operators $\mathcal{O}$ that satisfy a local conservation law of the usual form

$$\frac{\partial \mathcal{O}(\mathbf{r})}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j}(\mathbf{r}) = 0, \tag{6}$$

where $\mathbf{j}(\mathbf{r})$ is the associated current density. On taking the matrix element of this equation between an excited state j and the ground state and Fourier transforming in space, one finds

$$\omega_{j0} (\mathcal{O}_{\mathbf{q}})_{j0} = \mathbf{q} \cdot (\mathbf{j}_{\mathbf{q}})_{j0}. \tag{7}$$

This shows that the matrix element of the density satisfies the condition

$$(\mathcal{O}_{\mathbf{q}})_{j0} = \frac{\mathbf{q} \cdot (\mathbf{j}_{\mathbf{q}})_{j0}}{\omega_{j0}}. \tag{8}$$

Thus for an excited state with non-zero energy, the matrix element of the operator tends to zero as q tends to zero, provided only that the matrix element of the current remains finite. Therefore, since the vector current is conserved, there will be no multipair contributions to the time component of the current. However, the axial current is not conserved, and consequently multipair contributions will be present in general.

To determine how large these multipair contributions are, a number of approaches are possible. One is to calculate them directly from a nucleon-nucleon interaction and microscopic many-body theory. Another is to make use of sum rules to put bounds on multipair contributions. To illustrate the latter method, let us look at the structure function, defined by

$$S(q, \omega) = \sum_j | < j | \mathcal{O}_{\mathbf{q}} | 0 > |^2 \delta(\omega - \omega_{j0}) = -\mathrm{Im}\chi(q, \omega)/\pi. \tag{9}$$

In the limit $q \to 0$, only multipair states contribute to the static structure factor

$$S(q) = \frac{1}{n} \sum_j | < j|\mathcal{O}_{\mathbf{q}}|0 > |^2, \tag{10}$$

and to the energy-weighted sum

$$W(q) = \sum_j | < j|\mathcal{O}_{\mathbf{q}}|0 > |^2 \omega_{j0}. \tag{11}$$

Another moment of interest is the static response function, $\chi(q, 0)$, which is given by

$$\chi(q, 0) = \sum_j 2| < j|\mathcal{O}_{\mathbf{q}}|0 > |^2 / \omega_{j0}. \tag{12}$$

Let us denote the multipair contribution to the structure function by $S_{\mathrm{M}}(q, \omega)$. From the fact that the integrand is positive for positive ω, it follows that

$$\int_0^\infty \frac{S_{\mathrm{M}}(q, \omega)}{\omega} (\omega - \overline{\omega})^2 \geq 0, \tag{13}$$

where the mean excitation energy $\overline{\omega} = W(q)/S(q)$. This leads to the following condition on the multipair contribution to the static response function χ_{M}:

$$\chi_{\mathrm{M}}(0) \geq 2 \, \frac{nS(q)}{\overline{\omega}}. \tag{14}$$

If one were to take the $q \to 0$ limit of the calculations of the static spin structure factor and the energy-weighted sum calculated in Ref. [5], one would be led to the conclusion that multipair excitations make up more than ∼60% of the total static response function. This estimate is surprisingly large, but we should stress that the calculations in Ref. [5] were designed to shed light on correlations at non-zero wave numbers, where pionic correlations were found to enhance the static structure factor, and not to give accurate results for the long-wavelength response. It is possible that the calculations of Ref. [5] overestimate the structure factor at small wave numbers. Consequently, improved calculations of the static structure function are needed before it will be possible to place better bounds on the magnitude of multipair contributions to the response.

3 Concluding remarks

From the discussion above it is apparent that tensor correlations potentially play an important role in the microscopic physics needed as input to simulations of stellar collapse and supernovae. Similar conclusions apply to other processes that we do not have space to consider here, among them charged current processes, such as the modified Urca process [16], which is important for the cooling of neutron stars.

Many open problems remain. With respect to neutrino processes, improved expressions are needed for the modification of axial vector matrix elements in a nuclear medium, and for the Landau parameters [17]. To obtain an improved equation of state, the effects of pionic correlations need to be included in a realistic manner.

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