# Effective Theory for QCD in the LOFF Phase

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## 1 Introduction

Recently, the old idea of color superconductivity [1] has been given a new life [2], after the discovery that QCD at high density and low temperature can undergo a phase transition to a color super-conducting state characterized, for three light quarks, by Color Flavor Locking: CFL (for recent reviews see [3]). In this talk I wish to examine a different phase of QCD also present at low temperature and high baryonic chemical potential [4]; for two flavors this phase is characterized by isospin breaking and the emergence of a crystalline pattern of the diquark condensate. In a recent paper [5] it has been shown that in this phase there exists a quasi particle (phonon) associated to spontaneous breaking of rotational and translational invariance. Its properties can be studied by an effective lagrangian approach [6], [7], [8]; they will be reviewed here together with a short discussion on possible astrophysical implications.

## 2 Crystalline colour superconductive phase

In any possible application of the color superconductivity, for instance in the inner core of neutron stars, flavor symmetry is likely to be broken not only explicitly by quark mass terms, but also by weak interactions. For example in compact stars, considering only two flavors, isospin is broken by $\delta\mu = \mu_u - \mu_d \neq 0$, due to the process:

$$d \to u e \nu \ . \tag{1}$$

When the two quarks in the Cooper pair have different chemical potentials, the vacuum is characterized, for certain values of $\delta\mu$, by a non vanishing expectation value of a quark bilinear breaking translational and rotational invariance. The appearance of this condensate is a consequence of the fact that in a given range of $\delta\mu$ [4], the formation of a Cooper pair with a total momentum

$$\vec{p}_1 + \vec{p}_2 = 2\vec{q} \neq \vec{0} \tag{2}$$

is energetically favored. A similar phenomenon was already observed many years ago in the context of the BCS theory for superconducting materials in presence of magnetic impurities by Larkin, Ovchinnikov, Fulde and Ferrel and the corresponding phase is named LOFF state [9].

The exact form of the order parameter (diquark condensate) breaking space-time symmetries in the crystalline phase is not yet known. In [4] the following ansatz is made:

$$\Delta(\vec{x}) = \Delta e^{i2\vec{q}\cdot\vec{x}} = \Delta e^{i2q\vec{n}\cdot\vec{x}} \ . \tag{3}$$

The value of $|\vec{q}|$ is fixed by the dynamics, while its direction $\vec{n}$ is spontaneously chosen. In the sequel I will assume the ansatz (3) as well. The order parameter (3) induces a lattice structure given by parallel planes perpendicular to $\vec{n}$:

$$\vec{n}\cdot\vec{x} = \frac{\pi k}{q} \qquad (k = 0, \pm 1, \pm 2, ...) \ . \tag{4}$$

We can give the following physical picture of the lattice structure of the LOFF phase: Due to the interaction with the medium, the Majorana masses of the red and green up and down quarks have a periodic modulation in space, reaching on subsequent planes maxima and minima. For the Cooper pair to be formed one needs a color attractive channel, which is the antisymmetric channel; therefore it must be in antisymmetric flavor state if it is in antisymmetric spin state ($S = 0$). The condensate has therefore the form

$$- < 0|\epsilon_{ij}\epsilon_{\alpha\beta 3}\psi^{i\alpha}(\vec{x})C\psi^{j\beta}(\vec{x})|0 >= 2\Gamma_A^L e^{2i\vec{q}\cdot\vec{x}} \ . \tag{5}$$

The analysis of [4] shows that, besides the condensate (5) (scalar condensate), another different condensate is possible, i.e. one characterized by total spin 1 (vector condensate) and by a symmetric flavor state:

$$i < 0|\sigma^1_{ij}\epsilon_{\alpha\beta 3}\psi^{i\alpha}(\vec{r})C\sigma^{03}\psi^{j\beta}(\vec{x})|0 >= 2\Gamma_B^L e^{2i\vec{q}\cdot\vec{x}} \ . \tag{6}$$

Note that in the BCS state the quarks forming the Cooper pair have necessarily $S = 0$.

It goes without saying that the hypotheses of [4] are rather restrictive, as these authors assume only two flavors and make the ansatz of a plane wave behavior, Eq. (3). In any event their results are as follows. Assuming a pointlike interaction as the origin of the Fermi surface instability, the LOFF state is energetically favored in a small range of values of $\delta\mu$ around $\delta\mu \sim 0.7\Delta$. The actual value of the window range compatible with the presence of the LOFF state depends on the calculation by which the crystalline color state is computed. While the small interval is based on a local interaction, assuming gluon exchange, as in [10], the window opens up considerably.

The order parameters (5) and (6) spontaneously break rotational and translational symmetries. Associated with this breaking there will be Nambu Goldstone Bosons (NGB) as in a crystal; these quasi-particles are known as phonons. Let us discuss them in some detail.

# 3 Phonons in the LOFF phase

For a generic lattice structure it is known that there are three phonons associated to the breaking of space symmetries. However one can show [5] that in order to describe the spontaneous breaking of space symmetries induced by the condensates (5) and (6) one NGB is sufficient. The argument (see Fig. 1) is as follows: Rotations and translations are not independent transformations, because the result of a translation plus a rotation, at least locally, can be made equivalent to a pure translation. The NGB

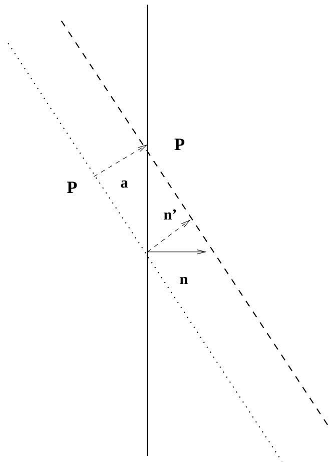


Figure 1: In the point P the effect of the rotation $\vec{n} \to \vec{n}'$ and the effect of the translation $\vec{x} \to \vec{x} + \vec{a}$ tend to compensate each other.

is a long wavelength small amplitude variation of the condensate $\Delta(\vec{x}) \to \Delta(\vec{x})e^{i\phi/f}$, with

$$\phi/f \;=\; 2q(\vec{n}+\delta\vec{n})\cdot(\vec{x}+\delta\vec{x}) - 2q\vec{n}\cdot\vec{x} \;=\; 2q\vec{R}\cdot\vec{x} + T - 2q\vec{n}\cdot\vec{x} \;, \tag{7}$$

where we have introduced the auxiliary function $T$, given by $T \;=\; 2\,q\,\vec{R}\cdot\delta\vec{x}$.

Now the lattice fluctuations $\phi/f$ must be small; from this it follows that $T$ and $\vec{R}$ are not independent fields and therefore $T$ must depend functionally on $\vec{R}$, i.e. $T = F[\vec{R}]$, which, using again (7), means that

$$\Phi \equiv 2q\vec{n}\cdot\vec{x} + \phi/f = 2q\vec{R}\cdot\vec{x} + F[\vec{R}] \equiv G[\vec{R},\,\vec{x}] \;. \tag{8}$$

The solution of this functional relation has the form

$$\vec{R} = \vec{h}[\Phi] \tag{9}$$

where $\vec{h}$ is a vector built out of the scalar function $\Phi$. By this function one can only[1]

[1]In principle there is a second vector, $\vec{x}$, on which $\vec{R}$ could depend linearly, but this possibility is excluded because $\vec{R}$ is a vector field transforming under translations as $\vec{R}(\vec{x}) \to \vec{R}\,'(\vec{x}\,') = \vec{R}(\vec{x})$.

form the vector $\vec{\nabla}\Phi$ ; therefore we get

$$\vec{R} = \frac{\vec{\nabla}\Phi}{|\vec{\nabla}\Phi|} \,, \tag{10}$$

which satisfies $|\vec{R}| = 1$ and $< \vec{R} >_0 = \vec{n}$. In terms of the phonon field $\phi$ the field $\vec{R}$ is given at the first order in $\phi$ by the expression

$$\vec{R} = \vec{n} + \frac{1}{2fq}\left[\vec{\nabla}\phi - \vec{n}(\vec{n}\cdot\vec{\nabla}\phi)\right] \,. \tag{11}$$

The interaction term between the NGB and the quarks is contained in

$$\mathcal{L}_{int} = -\frac{1}{2}\, e^{i\Phi} \sum_{\vec{v}} \left[\Delta^{(s)}\epsilon_{ij} + \Delta^{(v)}(\vec{v}\cdot\vec{R})\sigma^1_{ij}\right] \epsilon^{\alpha\beta 3}\psi_{i,\alpha,\vec{v}}\, C\, \psi_{j,\beta,-\vec{v}} + h.c. - (L \to R) \;; \tag{12}$$

from this lagrangian, following the same bosonization procedure of [7], one can obtain the effective lagrangian for the massless scalar field $\phi$ [5] and derive its dispersion law. It is worth observing that the fields $\psi_{j,\beta,\pm\vec{v}}$ in (12) are velocity dependent fields; they are computed at opposite velocities, as off-diagonal terms in the average over velocities cancel out.

# 4 Glitches and the LOFF phase

The pulsars are rapidly rotating stellar objects, characterized by the presence of strong magnetic fieds and by an almost continuous conversion of rotational energy into electromagnetic radiation. The rotaton periods can vary in the range $10^{-3}$ sec up to a few seconds; these periods increase slowly and never decrease except for occasional glitches, when the variation of frequency can be $\delta\Omega/\Omega \approx 10^{-6}$ or smaller.

Glitches are a typical phenomenon of the pulsars, in the sense that probably all the pulsar have glitches. The ordinary explanation of the glitches is based on the idea that these sudden jumps of the rotational frequency are due to the movements outwards of rotational vortices in the neutron superfluid and their interaction with the crust. This is one of the main reasons that allow the identification of pulsars with neutron stars, as only neutron stars are supposed to have a metallic crust. A schematic diagram of glitches is shown in Fig. 2

The interesting aspect of the LOFF phase discussed in this talk is that even in quark stars, provided one is in a color superconductivity phase, one can have a crystal structure given by a lattice characterized by a geometric array where the gap parameter varies periodically. Therefore also these stars might have glitches and the possibility arises that some of the pulsars are indeed quark stars [11]. In a more conservative vein one can also imagine that the LOFF phase be realized in the inner

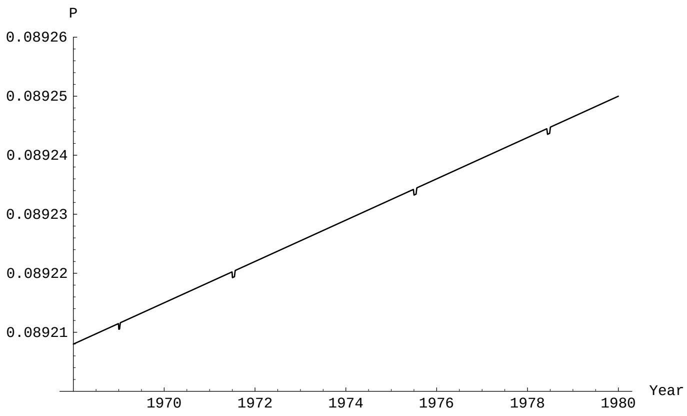


Figure 2: Schematic view of the period $P$ of the PSR 0833-45 (Vela) showing four glitches in the years 1969-1980.

core of neutron star; in this case the crystalline color superconductivity could be partly responsible for the glitches of the pulsar. A detailed analysis of this scenario is however premature as one should first complete the study of the LOFF phase in two directions, first by including the strange quark and, second, by sorting out the exact form of the color lattice.

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# Discussion

**J. Madsen (University of Aarhus):** How would you visualize the LOFF phase? I.e.: What are the "ions" in the crystal and how are they organized?

**Nardulli:** In the LOFF phase the condensate varies periodically in space with minima and maxima. Maxima are places where the condensate is bigger and therefore the fermions have the largest Majorana mass. The lattice of "ions" could be visualized as parallel planes where the Cooper pairs are more tightly bounded.

**D. K. Hong (Pusan National University):** When you derived the effective theory of the LOFF phase, you used the same Fermi velocity for up and down quarks. But it would be much easier to analyze the effective theory using different Fermi velocities that satisfy the constraint $\mu_u\vec{v}_u + \mu_d\vec{v}_d = 2\vec{q}$.

**Nardulli:** The use of one velocity has the advantage of a Fermi velocity superselection rule, which reduces the complexity of the loop integrals. It arises because in the $\mu \to \infty$ limit the rapid oscillations of the exponential factor in the lagrangian cancel all the contributions except those satisfying $\vec{v}_u + \vec{v}_d = 0$.