Color-flavor locking in strange stars, strangelets, and cosmic rays

Jes Madsen
Institute of Physics and Astronomy
University of Aarhus
DK-8000 Århus C, DENMARK

1 Introduction

Three topics with relation to color superconductivity in strange quark matter are discussed. 1) The r-mode instability in strange stars, which is consistent with the existence of "ordinary" strange quark matter stars but inconsistent with strange stars in a pure color-flavor locked state. 2) Color-flavor locked strangelets, which are more bound than normal strangelets, and have a different charge-mass relation. 3) Estimates of the strangelet flux in cosmic rays, which is relevant for strangelet detections in upcoming cosmic ray space experiments.

2 r-mode instabilities in strange stars

As discussed in several talks during this conference, it is not an easy task to distinguish observationally between neutron stars and strange stars, and it is even more difficult to tell the difference between "ordinary" strange stars and strange stars composed of color-flavor locked (CFL) quark matter and/or quark matter with a two-flavor color superconducting phase (2SC). Precise mass and radius measurements may offer a way because strange stars are generally more compact than neutron stars. Differences in neutrino cooling properties could be another possibility. While some data can be interpreted as being in favor of at least a few objects being strange stars, the situation is still not convincingly settled [1, 2].

Another phenomenon in compact stars has turned out to be a sensitive probe of quark matter properties. This is the r-mode instability, which is a generic instability in all rotating compact stars in the absence of viscous forces [3, 4, 5]. The instability involves horisontal mass-currents in the stellar fluid. These currents couple to gravitational wave emission (like magnetic quadropole radiation). In a rotating star there are r-modes which are counter-rotating as seen from the stellar rest frame, but forward-rotating as seen from infinity. Gravitational wave emission taps positive energy and angular momentum from the mode, which strengthens the mode in the stellar frame, so the mode is inherently unstable for any rate of rotation.

Viscosity may, however, prevent the mode from growing. In ordinary neutron stars, the combination of shear and bulk viscosities, and in particular the effect of "surface rubbing" [6] between the inner fluid and the solid part of the stellar crust dampens the r-mode instability significantly, leaving an interesting regime only for very high temperature and rotation rates close to the mass-shedding limit (the so-called Kepler limit, where the equatorial centripetal and gravitational accelerations are equal).

"Ordinary" strange stars may also have solid crusts of nuclear matter, held afloat by a strong electrostatic potential at the surface of the quark phase (the electrons are not as strongly bound as the quarks, so they form a thin "atmosphere", corresponding to a strong outward directed electrostatic potential), but the maximal crust density is much smaller than for neutron stars, only $\approx 10^{11}$g cm^{-3}, and therefore the surface rubbing is far less important.

In fact, data on pulsar rotation are consistent with the strange star hypothesis if the quark matter is non-superfluid [7, 8], and if the strange stars are either completely bare (making them invisible in x-rays), or have a very thick crust (to assure an internal temperature much higher than the surface temperature; otherwise the most rapid millisecond pulsars would be in a regime with significant spin-down due to the r-mode instability, contrary to observations).

In contrast strange stars purely in a color-flavor locked phase are not permitted by pulsar data [8]. The main contribution to bulk viscosity in strange quark matter is the weak reaction u+d $\leftrightarrow$ s+u, whereas shear viscosity is governed by strong quark-quark scattering. If the quark pairing energy gap is Δ, the characteristic time-scales for viscous damping of r-modes are exponentially increased by factors of $\exp(2\Delta/T)$ and $\exp(\Delta/(3T))$ respectively for bulk and shear viscosity. As the relevant temperature regime in pulsars is $T \ll 1$ MeV (except in the first few seconds after the supernova explosion) this means that viscous damping of r-modes essentially disappears for $\Delta > 1$ MeV, and all color-flavor locked strange matter pulsars would spin down within hours in sharp contrast to observations. Figure 1 illustrates the situation for $\Delta = 1$ MeV. Values of Δ as high as 100 MeV are often assumed; for such values the entire diagram would be r-mode unstable.

Thus, the r-mode instability seems to firmly rule out that pulsars are color-flavor locked strange stars. (In [8] I showed how electron shear viscosity and/or surface rubbing on a CFL strange star crust might stop the spin-down at rotation periods of order 10 milliseconds at intermediate temperatures. This would still be far too slow to be consistent with observations of pulsars and low-mass x-ray binaries, but even these viscous mechanisms are ruled out by the realisation, that CFL strange quark matter is charge neutral [9], with equal numbers of up, down, and strange quarks, and no electrons. Such a system is without the electron "atmosphere" needed to sustain

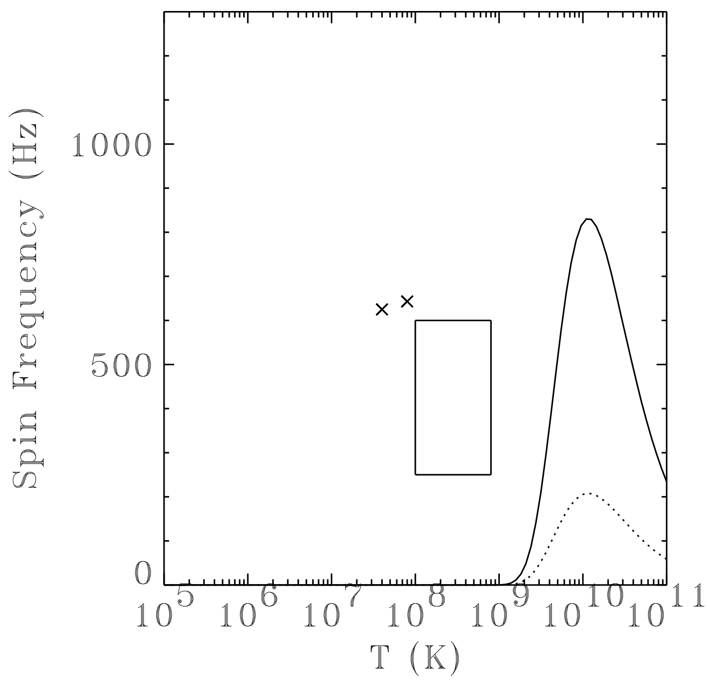

Figure 1: Critical rotation frequencies in Hz as a function of internal stellar temperature for CFL strange stars with an energy gap as small as $\Delta = 1$ MeV. Full curve is for $m_s = 200$ MeV; dotted curve for $m_s = 100$ MeV. The box marks the positions of most low-mass x-ray binaries (LMXB's), and the crosses are the most rapid millisecond pulsars known (the temperatures are upper limits). All strange stars above the curves (i.e. essentially all over the diagram) would spin down on a time scale of hours due to the r-mode instability, in complete contradiction to the observation of millisecond pulsars and LMXB's. Thus CFL strange stars are ruled out.

a crust; thus color-flavor locked strange stars would only exist in a "bare" variety, but these would not be able to rotate at all because of the r-mode instability, and are therefore not the pulsars or low-mass x-ray binaries observed).

For strange quark matter in the 2SC phase the situation is less conclusive. Now only some of the strong and weak reaction channels responsible for shear and bulk viscosities are exponentially suppressed, and the resulting viscous timescales for r-mode damping are increased in a much less dramatic fashion than for the CFL-phase, by factors of 9 and $(9/5)^{1/3}$ respectively for bulk and shear viscosity [8]. As seen in Figure 2 it becomes difficult to reconcile pulsar and LMXB data with 2SC-calculations, but it would be premature to rule it out.

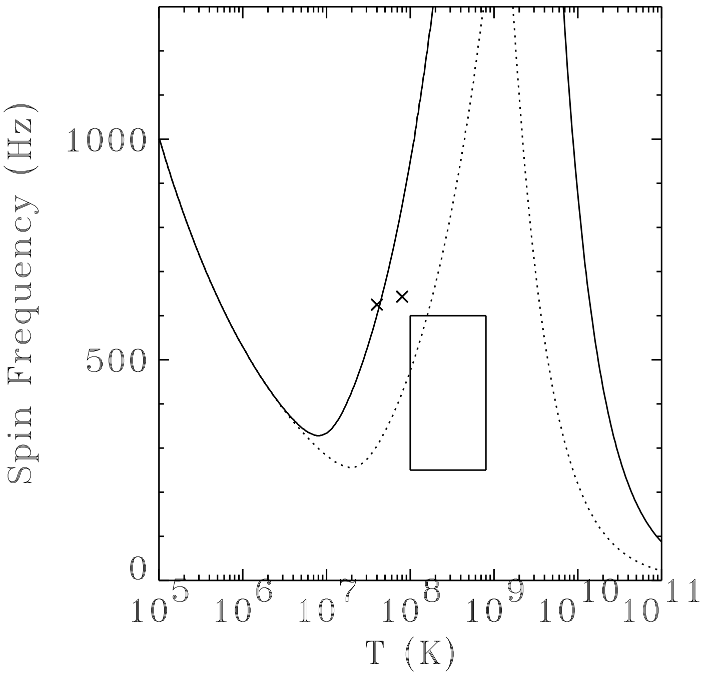

Figure 2: As Figure 1, but for 2SC-stars. Rapid spin-down happens above the parabola-like curves. The rapid millisecond pulsars are uncomfortably close to the instability regime, in particular because the temperatures are upper limits, but it would not seem appropriate to rule out 2SC strange stars on this basis alone.

The discussion above focused on strange stars, i.e. on the assumption of absolutely stable strange quark matter. If quark matter is only metastable CFL and/or 2SC phases could exist in the interior of hybrid stars, with mixed phases and ordinary hadronic matter in the outer parts. While no explicit r-mode studies have been performed for such systems, it is clear that the results will resemble those for ordinary neutron stars. This is because r-modes are located mainly in the outer parts of the star, and they are significantly damped by the boundary condition stemming from the fact that the r-modes cannot propagate in a solid crust.

Even pure quark matter stars could have significant substructure which could make them resemble neutron or hybrid stars in terms of r-mode instabilities. For instance, if the density profile of the star is such that only the central region is in the CFL phase, but the outer part in 2SC, then crystalline like structures like the LOFF-phase could form [10], and a surface rubbing effect might appear, suppressing the r-modes.

Finally magnetic fields and the exact nature of the superfluid vortices could be important. However, the conclusion that pure CFL strange stars are ruled out by the r-mode studies appears to be robust.

3 Color-flavor locked strangelets

If quark matter is in a color-flavor locked phase, it is because this phase has lower energy than other possible phases. In particular, this means that metastability or even absolute stability of strange quark matter becomes more likely than hitherto assumed, based on calculations for "ordinary" strange quark matter [1, 2]. For relevant ranges of strange quark mass, the gain in energy per baryon for bulk strange quark matter is roughly 100 MeV at fixed bag constant for $\Delta = 100$ MeV.

This also makes it relevant to reconsider the properties of finite size quark matter lumps, strangelets, for color superconducting strange quark matter. A first attempt in this direction was a study of color-flavor locked strangelets within the framework of the multiple-reflection expansion of the MIT bag model [11]. In this approach the total energy (mass) of a strangelet can be written as

$$E = \sum_i (\Omega_i + N_i\mu_i) + (\Omega_{\mathrm{pair},V} + B)V, \tag{1}$$

where the sum is over flavors, B is the bag constant, N_i and μ_i are quark number and chemical potential, $\Omega_{\mathrm{pair},V} \approx -3\Delta^2\mu^2/\pi^2$ is the binding energy from pairing (μ is the average quark chemical potential), and the thermodynamic potential of quark flavor i is a sum of volume, surface, and curvature terms derived from a smoothed density of states.

Apart from the pairing energy another crucial difference relative to non-CFL strangelet calculations is the equality of all quark Fermi momenta in CFL strange quark matter. This property, which leads to charge neutrality in bulk without any need for electrons [9], is due to the fact that pairing happens between quarks of different color and flavor, and opposite momenta $\vec{p}$ and $-\vec{p}$, so it is energetically favorable to fill all Fermi seas to the same Fermi momentum, p_F.

As illustrated in Figure 3, color-flavor locked strangelets have an energy per baryon, E/A, that behaves much like that of ordinary strangelets as a function of A. For high A a bulk value is approached, but for low A the finite-size contributions from surface tension and curvature significantly increases E/A, making the system less stable. The main difference from ordinary strangelet calculations is the overall drop in E/A due to the pairing contribution, which is of order 100 MeV per baryon for $\Delta \approx 100$ MeV for fixed values of m_s and B. Since $\Omega_{\mathrm{pair},V} \propto \Delta^2$, the actual energy gain is of course quite dependent on the choice of Δ.

A significant distinction between the properties of ordinary strangelets and CFL strangelets lies in the charge properties. They have in common a very small charge

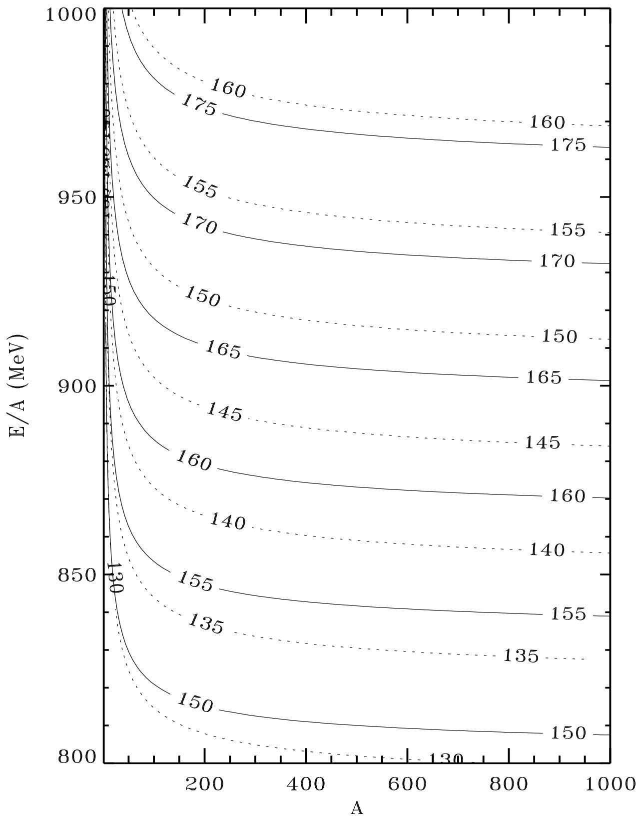

Figure 3: Energy per baryon in MeV as a function of A for ordinary strangelets (dashed curves) and CFL strangelets (solid curves) for $B^{1/4}$ in MeV as indicated, $m_s = 150$ MeV, and $\Delta = 100$ MeV.

per mass unit relative to nuclei, but the exact relation is quite different, and this may provide a way to test color-flavor locking experimentally if strangelets are found in accelerator experiments or (perhaps more likely) in cosmic ray detectors. Ordinary strangelets have (roughly) [12, 13, 14]

$$Z \approx 0.1 \left(\frac{m_s}{150\ \mathrm{MeV}}\right)^2 A; \qquad A \ll 10^3; \tag{2}$$

$$Z \approx 8 \left(\frac{m_s}{150\ \mathrm{MeV}}\right)^2 A^{1/3}; \qquad A \gg 10^3. \tag{3}$$

In contrast, CFL strangelets are described by [11]

$$Z \approx 0.3 \left(\frac{m_s}{150\ \mathrm{MeV}}\right) A^{2/3}. \tag{4}$$

This relation can easily be understood in terms of the charge neutrality of bulk CFL strange quark matter [9] with the added effect of the suppression of s-quarks near the surface, which is responsible for (most of) the surface tension of strangelets. This leads to a reduced number of negatively charged s-quarks in the surface layer; thus a total positive quark charge proportional to the surface area or $A^{2/3}$.

In fact, a similar effect becomes important even in ordinary strangelets, meaning that the standard $A^{1/3}$-result breaks down at very high A [15]. And even more important, this effect is large enough to rule out a potential disaster scenario, where negatively charged strangelets produced in heavy ion colliders could grow by nucleus absorption and swallow the Earth. While ordinary strange quark matter can be negatively charged in bulk if the one-gluon exchange α_S is very prominent [12], the added positive surface charge due to massive s-quark suppresssion is sufficient to make the overall quark charge positive for a large range of A, thus preventing any such disaster [15].

Naturally, only first steps have been made in the effort to describe properties of color-flavor locked strangelets. First of all, the MIT bag model is a phenomenological approximation to strong interaction physics; it is not QCD. Secondly, while finite-size effects were included in the free quark energy calculations, such (unknown) higher order terms were not taken into account in the pairing energy. This approximation seems warranted as long as $\Omega_{\rm pair}$ itself is a perturbation to $\Omega_{\rm free}$. And thirdly, quark level shell effects were only taken into account in an average sense via the smoothed density of states described as a sum of volume, surface and curvature terms. While this is an excellent approximation to the average strangelet properties [16], it misses the interesting stabilizing effects near closed shells [17, 18] that could make certain baryon number states longer lived than one might expect from a glance at Figure 3. A first attempt at approaching finite size effects in 2SC quark matter in a completely different manner is discussed in [19].

4 Strangelet flux in cosmic rays

Two cosmic environments could in principle harbor strangelet formation. The cosmological quark-hadron phase transition 10^{-5} seconds after the Big Bang, and the high density conditions in compact supernova remnants, which may be strange stars composed of quark matter rather than neutron stars.

Cosmologically produced strangelets were for a time believed to be natural dark matter candidates [17]. In that case a significant background of largely neutral strangelets (quark core charge neutralized by electrons) would be moving in our galactic halo at typical speeds of 3–400 km/sec, corresponding to the depth of the galactic gravitational potential. Several experiments have placed limits on the abundance of these nonrelativistic strangelets (sometimes called quark nuggets), but it now seems unlikely that they could form in or survive from the very hot ($T \approx 100$ MeV) environment in the early Universe. A similar problem faces strangelet production in ultrarelativistic heavy ion collisions; it has been compared to making ice cubes in a furnace.

4.1 Strangelet production in collisions of strange stars

A more likely origin of cosmic ray strangelets is from collisions of binary compact star systems containing strange stars. If strange quark matter is the ground state of hadronic matter at zero pressure, it will be energetically favorable to form strange stars rather than neutron stars, and it would be expected that all the objects normally associated with neutron stars (pulsars and low-mass x-ray binaries) would actually be strange stars.

Several pulsars are observed in binary systems containing another compact star; the most famous such system called PSR1913+16 delivered convincing evidence for gravitational wave emission. Such binaries typically move in elliptical orbits, spiraling closer to each other because the system loses energy by gravity wave emission. The remaining lifetime can be estimated, and combined with estimates of the number of these binaries in our galaxy, the expected rate of binary collisions is of order 10^{-4} year^{-1}.

Several numerical studies have been performed in the literature to follow the late stages of inspiral in systems composed of two neutron stars or neutron stars orbiting black holes or white dwarfs, especially to derive the gravitational wave signatures, which are of importance for upcoming gravity wave detection experiments. No detailed calculations have been done for systems containing strange stars, and since there are significant differences in the equation of state it may be dangerous to rely on existing models. Nevertheless, certain features seem robust. While details depend on assumptions about the orbit, most collisions seem to release a fraction of the total mass (of order 10^{-4}–$10^{-1}M_\odot$, where $M_\odot$ is the solar mass) in connection with the actual collision and via tidal disruption in the late stages of inspiral.

No realistic estimates exist at present of the mass spectrum of quark matter lumps released during the actual collision. Lumps of matter released during the tidal disruption phase are expected to be very large. Balancing the tidal force trying to disrupt the star with the surface tension force of strange quark matter leads to a typical fragment baryon number of $A \approx 4 \times 10^{38} \sigma_{20} a_{30}^3$, where $\sigma_{20} \approx 1$ is the surface tension in units of 20 MeV/fm^3 and a_{30} is the distance between the stars in units of 30 km.

A significant fraction of the tidally released material is originally trapped in orbits around the binary stars. The typical orbital speeds of the lumps are here $0.1c$, and collisions among lumps are abundant. Assuming the kinetic energy in these collisions mainly goes to fragmentation of the lumps into smaller strangelets (i.e. that the kinetic energy is used to the extra surface and curvature energies necessary for forming N lumps of baryon number A/N from the original baryon number A), it can be shown for typical bag model parameters that the resulting strangelet distribution peaks at mass numbers from a few hundred to about 10^3. This is well within the interesting regime for the upcoming cosmic ray experiment Alpha Magnetic Spectrometer AMS-02 on the International Space Station [21] (a prototype AMS-01 was flown on the Space Shuttle mission STS-91 in 1998). AMS-02 is a roughly 1 m^2 sterad detector which will analyze the flux of cosmic ray nuclei and particles in unprecedented detail for three years or more following deployment in 2005. It will be sensitive to strangelets in a wide range of mass, charge and energy [22].

4.2 Strangelet flux at AMS-02

Strangelet propagation in the Milky Way Galaxy is in many ways expected to be similar to that of ordinary cosmic ray nuclei. Except for a possible background of slow-moving electrically neutral quark nuggets confined solely by the gravitational potential of the Galaxy, strangelets are charged and are therefore bound to the galactic magnetic field. They lose kinetic energy by electrostatic interactions with the interstellar medium, and they gain energy by Fermi acceleration in shock waves, for example from supernovae. Even if accelerated to relativistic speeds, scatterings on impurities in the magnetic field makes the motion resemble a diffusion process. The solar wind as well as the Earth's magnetic field become important for understanding the final approach to the detector. Also, strangelets may undergo spallation in collisions with cosmic ray nuclei, nuclei in the interstellar medium, or other strangelets.

Detailed studies of all of these phenomena have recently been started in order to understand strangelet propagation in more detail. Much depends on the charge-to-mass relation, but the details of propagation are not even well understood for ordinary nuclei, so clearly some uncertainty in the expectations for the strangelet flux at AMS is inevitable.

In the following it will be assumed that strangelets share two of the features found experimentally for nuclei, namely a powerlaw energy distribution: $N(E)dE \propto E^{-2.5}$, and an average confinement time in the galaxy of 10^7 years. Assuming strangelets to move close to the speed of light, and ascribing one baryon number, A, to them all, the strangelet flux at AMS-02 would be

$$F = 3 \times 10^{12} A^{-1} (\mathrm{m}^2 \text{ y sterad})^{-1} \times R_{-4} \times M_{-2} \times V_{100}^{-1} \times t_7 \times \mathrm{GCfraction} \quad (5)$$

where R_{-4} is the number of strange star collisions in our Galaxy per 10^4 years, M_{-2} is the mass of strangelets ejected per collision in units of $10^{-2} M_\odot$, V_{100} is the effective galactic volume in units of 100 kpc^3 over which strangelets are distributed, and t_7 is the average confinement time in units of 10^7 years. All these factors are of order unity if strange matter is absolutely stable, though each with significant uncertainties.

Finally, GCfraction is the fraction of strangelets surviving the geomagnetic cutoff. Taking this cutoff to be at rigidity 6 GeV/c, assuming the standard $E^{-2.5}$ powerlaw for the energy distribution with a cutoff at $\beta \equiv v/c = 0.01$, and assuming a charge-mass relation $Z = 0.3A^{2/3}$ as derived for color-flavor locked strangelets, the resulting strangelet flux at AMS-02 becomes

$$F = 5 \times 10^5 (\mathrm{m}^2 \text{ y sterad})^{-1} \times R_{-4} \times M_{-2} \times V_{100}^{-1} \times t_7. \tag{6}$$

By coincidence, this result (valid for $A < 6 \times 10^6$; for larger A GCfraction equals 1 and the previous expression applies) is independent of strangelet mass.

As should be evident from the discussion above, there are many uncertainties involved in the calculation of the strangelet flux at AMS-02. A systematic study of these issues has been initiated and should significantly improve our understanding of the strangelet production and propagation. But ultimately we must rely on experiment. So far it is reassuring, that the simple flux estimates above lead us to expect a very significant strangelet flux in the AMS-02 experiment.

A discovery of strangelets would of course be a very significant achievement in itself. Getting data on the charge-to-mass relation may even allow an experimental test of color-flavor locking in quark matter.

This work was supported in part by The Danish Natural Science Research Council, and The Theoretical Astrophysics Center under the Danish National Research Foundation. I thank Jack Sandweiss and Dick Majka for discussions and collaboration on strangelet detection with AMS-02.

References

[1] For a review and references on the physics and astrophysics of “ordinary” strange quark matter, see J. Madsen, Lecture Notes in Physics **516**, 162 (1999) [arXiv:astro-ph/9809032].

[2] For reviews and references on color superconducting quark matter, see M. Alford, Annu. Rev. Nucl. Part. Sci. **51**, 131 (2001) [arXiv:hep-ph/0102047]; K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333.

[3] N. Andersson, Astrophys. J. **502**, 708 (1998).

[4] J. L. Friedman and S. M. Morsink, Astrophys. J. **502**, 714 (1998).

[5] For a review on r-mode instabilities, see N. Andersson and K. D. Kokkotas, Int. J. Mod. Phys. **D10**, 381 (2001).

[6] L. Bildsten and G. Ushomirsky, Astrophys. J. **529**, L33 (2000).

[7] J. Madsen, Phys. Rev. Lett. **81**, 3311 (1998).

[8] J. Madsen, Phys. Rev. Lett. **85**, 10 (2000).

[9] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. **86**, 3492 (2001).

[10] M. Alford, J. Bowers and K. Rajagopal, Phys. Rev. D **63**, 074016 (2001).

[11] J. Madsen, Phys. Rev. Lett. **87**, 172003 (2001).

[12] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).

[13] M. S. Berger and R. L. Jaffe, Phys. Rev. C **35**, 213 (1987); **44**, 566(E) (1991).

[14] H. Heiselberg, Phys. Rev. D **48**, 1418 (1993).

[15] J. Madsen, Phys. Rev. Lett. **85**, 4687 (2000).

[16] J. Madsen, Phys. Rev. D **50**, 3328 (1994).

[17] E. P. Gilson and R. L. Jaffe, Phys. Rev. Lett. **71**, 332 (1993).

[18] J. Schaffner-Bielich, C. Greiner, A. Diener and H. Stöcker, Phys. Rev. C **55**, 3038 (1997).

[19] P. Amore, M. C. Birse, J. A. McGovern and N. R. Walet, arXiv:hep-ph/0110267.

[20] E. Witten, Phys. Rev. D **30**, 272 (1984).

[21] The AMS homepage is http://ams.cern.ch.

[22] A. Chikanian, J. Madsen, R. Majka and J. Sandweiss, work in progress.

Discussion

F. Weber (Notre Dame): In your flux determination, did you assume that all neutron stars are in fact strange stars or that there exist two separate families of compact stars?

Madsen: All compact stars were assumed to be strange stars. Coexistence of two separate families is very unlikely if strange matter is the ground state at zero external pressure, since the Galaxy would be "polluted" by quark lumps from binary pulsar collisions. These lumps would trigger transition to strange stars in supernova cores.

F. Sannino (Nordita): How do you disentangle (CFL) or strange droplets experimentally? What is the signature?

Madsen: Strangelets have very low charge for a given baryon number compared to nuclei. CFL-strangelets have a charge-mass relation that differs from "ordinary" strangelets.

J.E. Horvath (Sao Paulo): A low charge-to-mass ratio for strangelets would not allow substantial acceleration in supernova shocks, thus the spectrum (and also the confinement time) is not necessarily the one measured in cosmic rays. How does this change the estimates of the rates in the expected range of center-of-mass energies?

Madsen: Clearly the propagation in the Galaxy requires further study, and we are trying to address the relevant mechanisms in detail. One should keep in mind, though, that these issues are not fully understood even for ordinary cosmic ray nuclei.