The QCD Phase Diagram and Explosive Astrophysics

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1 Introduction

I was asked by the organizers to give an overview of the QCD phase diagram, geared towards a mixed audience of astrophysicists and particle theorists. I chose to emphasize the phase transition(s) from the normal nuclear phase to the color superconducting and chirally symmetric phases at high density and low temperature[1]. I did so because it is possible that these transitions might occur during violent astrophysical phenomena such as supernovae or neutron star mergers [1], with potentially dramatic consequences. If so, these astrophysical events could provide a window into the physics of QCD.

This contribution is organized as follows. In section 2 I summarize our current understanding of the QCD phase diagram. In section 3 I discuss the phase transition characteristics most relevant to astrophysics, such as critical temperature and latent heat. In section 4 I discuss supernova core collapse and argue that conditions at core bounce might lead to the crossing of a QCD phase boundary. I also briefly discuss neutron star mergers and hypernovae. I conclude with some discussion from the conference.

2 QCD phase diagram

Figure 1 (taken from Rajagopal and Wilczek's excellent review in [1]) shows a possible QCD phase diagram as a function of temperature and chemical potential. The region at low temperature and density (lower left) is the normal nuclear matter phase (the small spur at the bottom is the phase boundary for the formation of nuclear matter). At sufficiently high density and low temperature (lower right) it has recently been

[1]By low temperature here I mean low relative to the QCD scale, or $\sim$ 100 MeV; by astrophysical standards supernova temperatures of tens of MeV are obviously quite high.

established that the ground state of quark matter exhibits Color Superconductivity (CSC), resulting from the Cooper pairing of quark quasiparticles near the Fermi surface [1] – [19]. At high temperature and low density (upper left, extending to the upper right) we expect to find the quark-gluon plasma. I will comment on the more specific features of the diagram below.

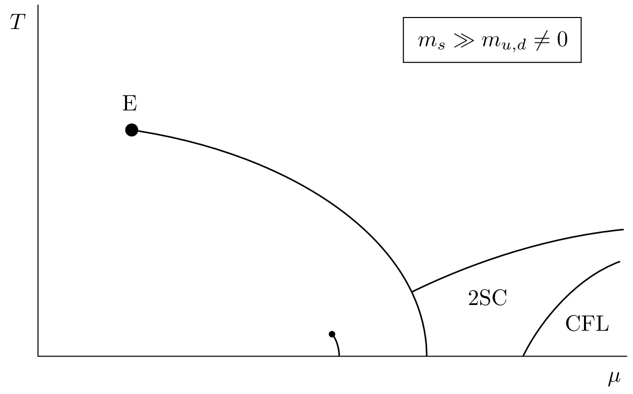

Figure 1: QCD phase diagram (simplified) in the density-temperature plane.

Let me try to give you some flavor of the recent results on color superconductivity. Recall that Cooper pairing (as in ordinary superconductors) involves excitations near the Fermi surface, but with equal and opposite momenta (figure 2). In QCD the excitations have quark quantum numbers, and hence there is a directly attractive channel due to gluon exchange in the anti-triplet ($\overline{3}$) color representation. In the case of electrons in a metal photon exchange is repulsive, and the attractive channel is due to the exchange of phonons (lattice vibrations).

What is common between quark matter and an ordinary metal is the existence of a Fermi surface. The Pauli exclusion principle requires that fermions like quarks or electrons occupy distinct quantum states, hence at zero temperature the ground state consists of filled levels up to some energy (this boundary is the Fermi surface). The lowest energy excitations of this system, called quasiparticles, are states just above the Fermi surface. Now, a Fermi surface is unstable with respect to attractive interactions in the Cooper pairing channel. That is, even an arbitrarily weak interaction in this channel can lead to pairing of quasiparticles, which reduces the overall energy of the system. Heuristically, the spherical symmetry of the Fermi surface (in momentum space, the energy of a quasiparticle only depends on its distance from the surface, and not on its angular position) reduces the pairing dynamics to that of a 1+1 dimensional system, where even weak interactions lead to pairing.

Now, recall that QCD exhibits a property called asymptotic freedom, which means that short distance (large momentum transfer) interactions are weak, whereas long distance (small momentum transfer) interactions are strong. At very high quark

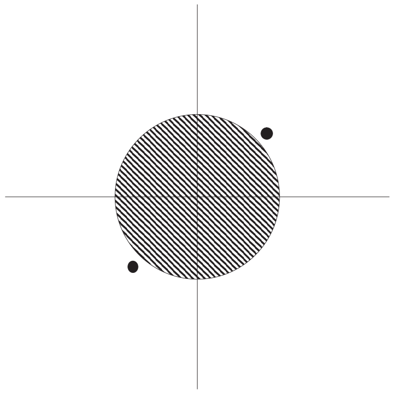

Figure 2: Excitations on opposite sides of the Fermi surface.

densities, most interactions occur over short distances, and we therefore have very good control over calculations. However, at quark densities likely to be found in a neutron star, roughly a few per cubic Fermi, the typical interactions are quite strong, precluding reliable quantitative results. Nevertheless, the indications of at least an *instability* to the formation of Cooper pairs is still evident, even if we can't say more about the details.

In figure 1 there are two distinct CSC phases displayed, the 2SC and CFL phases. In the former, the up and down quarks pair in the isosinglet channel, and the strange quark is left to pair with itself in an exotic (possibly spin 1) channel. In the latter, the flavor and color orientations of all three quarks are correlated in a non-trivial way (see figure 3): pairing in a particular flavor channel corresponds to a particular orientation in color space. In this talk I will assume that figure 1 is correct in that the 2SC phase has the lowest energy at intermediate density. The transition from nuclear matter to the 2SC phase is likely to be first order [18], as we will discuss further below.

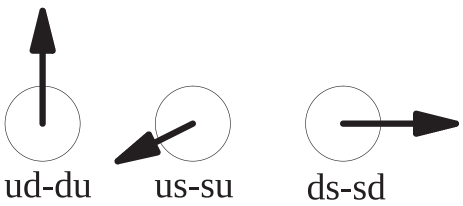

Figure 3: Di-quark orientation in the CFL phase. The arrow represents a $\overline{3}$ orientation in color space.

Before leaving the subject of the QCD phase diagram, let me discuss another transition – the chiral phase transition – that is likely to occur as we increase the baryon number density. Chiral symmetry has to do with unitary rotations among different flavors of quarks:

$$q_i^{L,R} \rightarrow U_{ij} q_j^{L,R} \quad , \tag{1}$$

where i labels the flavor (i = up, down, strange) and U is a unitary matrix. L and R label whether the quark is left handed or right handed. Chiral symmetry reflects the fact that, in the limit of small quark masses, the strong interactions (gluons) can't tell the difference between different flavors of left and right handed quarks.

These symmetries are broken by quark condensation in the QCD ground state:

$$\langle \overline{q}_L q_R \rangle \neq 0 \quad ,$$

but are restored in the 2SC phase and in the quark-gluon plasma. (However, in the CFL state chiral symmetries do remain broken at high density.) Their restoration involves a phase transition, again at quark densities of roughly a few per cubic Fermi. What can we say about this transition?

Because it occurs at a density where the effective coupling is strong, we can't make quantitative predictions. However, we can learn something about the order of the transition from something called "universality". The logic is as follows: if the transition is second order (or higher), then at the precise point of transition there must be very long wavelength excitations in the system. A heuristic way to understand this is to visualize the effective potential near the transition. In a second order transition the potential becomes very flat as the second derivative changes sign from positive to negative. (In contrast, in a first order transition the system tunnels or fluctuates out of the disfavored vacuum while the second derivative is still positive.)

Technically, a second order transition implies fluctuations of infinite wavelength (or equivalently, zero mass excitations). There are not very many models which describe such low-energy dynamics. To be self-consistent they need to exhibit something called an infrared fixed point in the renormalization group evolution of the coupling constant. Roughly speaking, the low-energy dynamics must become almost scale invariant as the correlation length diverges. The candidate models describing the low-energy dynamics must possess the same symmetry properties as the underlying system, but otherwise can be quite simple. In some cases, there are no candidate models with the right symmetries and low-energy particle content that exhibit an infrared fixed point. In this case the transition is predicted to be first order. Based on this kind of analysis, both the 2SC and chiral phase transition at finite density are likely to be first order [17, 18].

So, we have two candidate phase transitions which, as we discuss below, might have astrophysical implications. Obviously, a number of issues are still unresolved. For example, are there two separate phase transitions, or does chiral symmetry restoration coincide with color superconductivity? The renormalization group analysis of color superconductivity [8, 11] suggests that once the effective excitations start to look like quarks, there is a pairing instability and the system is likely to be superconducting. However, one could also imagine a chirally restored phase with nucleonic excitations, so chiral symmetry does not necessarily imply CSC. In any case, both transitions are likely to occur at temperatures and densities that might be achieved by astrophysical phenomena.

3 Phase transition parameters

At asymptotic density, the nature of the CSC order parameter, the binding energy density and the critical temperature are all precisely calculable. Less is known about intermediate densities of several times nuclear density. However, there are strong indications of a Cooper pairing instability, and estimates of the resulting gap are of order $\Delta \sim$ 40-120 MeV at a quark chemical potential μ of ≈ 400 MeV. The nature of the chiral phase transition is similarly poorly determined, although as we discussed in the previous section it is likely to be first order. Rather than discuss the two transitions separately, we will focus on the CSC transition, keeping in mind that if a first order chiral boundary is crossed, the consequences should be similar.

Consider the possible implications of CSC for the collapse and explosion of massive stars ($M > 8M_\odot$). We will argue that it is quite likely that at the moment of maximum compression of the collapsing Fe core, the densest part of the star crosses the critical density into the phase where CSC is energetically favored. The release of latent energy has the potential to generate an explosive shockwave which powers the resulting supernova (SN). Current simulations of supernovae are generally unable to reproduce the explosive behavior observed in nature: the shockwave generated by the mechanical bounce of the nuclear core stalls before reaching the surface, unless an appeal is made to neutrino reheating combined with non-spherical phenomena such as rotation or convection [20, 21]. We also note that the energy liberated in a CSC phase transition is potentially sufficient to power hypernovae (HN) [22], which have been linked to gamma ray burst events [23].

First, let us summarize some results on CSC from the recent literature. Precise results are only valid at asymptotic densities where the effective QCD coupling is small, however they should still be useful guide when dealing with intermediate densities. In any case, our qualitative results will be insensitive to factors of 2 in these formulas:

• Gap size: $\Delta \sim$ 40 - 120 MeV

• Critical temperature: $T_c \simeq .57\Delta$

• At asymptotic density the binding energy density is $E_{CSC} = \frac{5.8}{4\pi^2}\Delta^2\mu^2$. Simple dimensional analysis (given the absence of any small parameter) also suggests a value in this range. Note that we are interested here in the latent *energy* associated with the first order transition to the CSC phase (or any other transition that occurs as the baryon density is increased beyond several times nuclear density). The baryon density changes on astrophysical timescales, or very slowly on the timescale of QCD dynamics, and the vacuum state at zero temperature (or at $T << T_c$) is found by minimizing the energy in the sector of the Hilbert space with fixed baryon number. Studies such as [19], which involve the free energy Ω at finite chemical potential (but not fixed density) are appropriate for determining pressure equilibrium between nuclear and CSC matter in circumstances in which baryon number can flow across a boundary (e.g. in a neutron star), but do not address a possible SN transition. (See discussion for further remarks.)

• Phase diagram: the normal nuclear phase is separated from the CSC phase (most likely the 2SC two flavor condensate phase, although possibly the CFL phase [17, 18, 19]) by a first order boundary.

4 Astrophysics of Core Collapse

Now let us review the standard scenario of Fe core collapse which is believed to lead to type II supernovae [20]. Nuclear burning during the 10^7 year lifetime of the star leads to a shell structure, with the inner core eventually consisting of Fe ash. Because iron cannot participate in further exothermic nuclear reactions, there is an eventual cooling and collapse of the Fe core, whose mass is likely to be $(1-2)\ M_\odot$ (or, roughly the Chandresekhar mass). The collapse of this core is only halted by neutron degeneracy, which leads to a stiffening of the equation of state. The resulting bounce produces a shockwave, whose energy of $\sim 10^{51}$ ergs is a small fraction of the total available gravitational binding energy released by the collapse:

$$E_b \sim 3 \cdot 10^{53} \text{ergs} \left(\frac{M_{core}}{M_\odot} \right)^2 \left(\frac{R}{10\text{km}} \right)^{-1} . \tag{2}$$

Most of this energy escapes in the form of neutrinos during the supernova, as was observed in the case of SN1987a.

The pressure in the collapsed core at the instant of the bounce is most likely *greater* than the corresponding pressure in any remnant neutron star. In order to cause a bounce, the kinetic energy E_b of the infalling material (which is a sizeable fraction of a solar mass!) must be momentarily stored as compressional potential energy in the (sub)nuclear matter. This additional mechanical squeezing at the bounce suggests that if the critical density for CSC is ever reached in a neutron star, it will be reached at this instant.

Simulations of the core bounce result in densities of at least several times nuclear density $(5 - 10 \cdot 10^{14} \text{g/cm}^3)$, and temperatures of roughly 10-20 MeV [20]. This temperature is likely less than T_c for CSC, possibly much less[2], and hence the core of the star traverses the phase diagram horizontally in the density-temperature plane, crossing the critical density boundary into the CSC phase. It is important to note that the core region at bounce is probably *cooler* than post-bounce, since degenerate neutrinos tend to heat the proto-neutron star as they diffuse out [24]. Studies quoting larger SN temperatures such as $T \sim 30$ MeV generally refer to this later stage [25].

Once the core crosses into the CSC part of the phase diagram, the transition proceeds rapidly, on hadronic timescales. Because the transition is first order, it proceeds by nucleation of bubbles of the CSC phase in the normal nuclear background. The rate of bubble nucleation will be of order $(\mathrm{fm})^4$ ($\mathrm{fm} = 10^{-13}$cm.), due to strong coupling. (In a system governed by a dimensional scale Λ, the nucleation rate is given by $\Gamma \sim \Lambda^4 e^{-S}$, where S is the action of the Euclidean bounce solution interpolating between the false (normal nuclear) vacuum and a bubble of critical size. At strong coupling, S is of order one, so there is no large exponential suppression of the nucleation rate. The scale Λ is of the order of Δ or $\mu = 400$ MeV.)

Causality requires that the mechanical bounce of the core happen over timescales larger than the light crossing time of the core, or at least $10^{-4}s$. Hence, the phase transition occurs instantaneously on astrophysical timescales. A nucleated bubble of CSC phase expands relativistically – liberated latent heat is converted into its kinetic energy – until it collides with other bubbles. Because the system is strongly coupled, these collisions lead to the rapid production of all of the low energy excitations in the CSC phase, including (pseudo)Goldstone bosons and other hadrons. The resulting release of energy resembles an explosion of hadronic matter.

To estimate the total CSC energy released in the bounce, we use the result that the ratio of CSC binding energy density to quark energy density is of order $\left(\frac{\Delta}{\mu}\right)^2$. For $\mu \sim 400$ MeV, and $\Delta \sim$ 40-120 MeV, this ratio is between .01 and .08, or probably a few percent.

$$E \sim \left(\frac{\Delta}{\mu}\right)^2 M_{core} \ . \tag{3}$$

In other words, the total energy release could be a few percent of a solar mass, or 10^{52} ergs! This is significantly larger than the energy usually attributed to the core shockwave, and possibly of the order of the gravitational collapse energy E_b. The implications for SN simulations are obviously quite intriguing.

In [26] it was suggested that strange matter formation might overcome the energetic difficulties in producing type-II supernova explosions. While there are strong arguments that the CSC transition should be first order, and reasonable order of mag-

[2]It is conceptually easier to think about the case where T is much less than T_c, since in this case the Free energy ($F = U - TS$) liberated by the transition is predominantly energy, with only a small component related to entropy. The relevant dynamics is governed by energetics rather than Free energetics.

nitude estimates of the latent heat [1, 19], it is not clear to us why there would be supercooling in a strange matter transition. The conversion of up quarks to strange quarks must proceed by the weak interactions, but the rate is still much faster than any astrophysical timescale. Thus, the population of strange quarks is likely to track its chemical equilibrium value as the pressure of the core increases. There may be an important effect on the nuclear equation of state from strangeness (e.g. a softening of the pressure-density relationship), but we do not see why there should be explosive behavior.

Our results are also relevant to hypernovae [22], which are observed to have ejecta kinetic energies 10-100 times larger (of order 10^{52-53} ergs) than those of ordinary type II supernovae. Accounting for this extra kinetic is extremely challenging in standard scenarios. However, for exceptionally massive stars with $M < 35 M_\odot$ (for $M > 35 M_\odot$ the hydrogen envelope is lost during H-shell burning, and the core size actually decreases [27]) there is a large core mass which leads to a larger release of CSC binding energy. In fact, the released energy might depend nonlinearly and sensitively on the star's mass at the upper range. For example, the fraction of M_{core} which achieves critical density might be a sensitive function of the mass of the star.

Another alternative is that hypernovae are the result of neutron star mergers rather than the explosion of an individual star. This possibility has been examined in relation to hypernovae as the engines of gamma ray bursts (GRBs) [23]. It seems quite plausible that in the merger of two cold neutron stars a significant fraction of the stars' mass undergoes the CSC transition (i.e. crosses the critical pressure boundary for the first time; in this case the temperature is probably negligible relative to the CSC scale Δ). This provides a substantial new source of energy beyond gravitational binding, and may solve the "energy crisis" problem for this model of GRBs [23].

Finally, we note that the trajectory of the SN core in the temperature-density phase diagram might be rather complicated. The parameters suggest a density transition (at $T < T_c$), but subsequent reheating of the core due to the explosion, or to neutrino diffusion [24] might raise the temperature above T_c, and lead to additional transitions across the temperature boundary [25]. When $T \sim T_c$, the Free energies ($F = U - TS$) of the normal and CSC phases are comparable, due to the larger entropy of the normal phase. The evolution of bubbles in this regime is governed by relative Free energies rather than energetics alone, and the transition is presumably less explosive than the pressure transition at $T << T_c$.

5 Summary

Our understanding of QCD at high density has evolved dramatically over the past two years, leading to remarkable progress in understanding the QCD phase diagram and the color superconducting state of quark matter. We have argued here that phase transitions at high density and low temperature (relative to the QCD scale) might play a role in violent astrophysical phenomena. In particular, the densities and temperatures associated with core collapse in massive stellar evolution suggest that CSC could play an important role in type II supernovae, or possibly hypernovae (GRBs).

Our assumptions concerning key parameters are conservative, and taken from distinct (and heretofore independent) regimes of inquiry: stellar astrophysics and dense quark matter. Yet, they point to the interesting possibility that supernova explosions are powered by CSC binding energy. It is well established that the shock-wave energy from core collapse is insufficient to produce an explosion, and recent results incorporating Boltzman transport of neutrinos show that neutrino reheating is also insufficient unless non-spherical phenomena such as rotation or convection are taken into account [21]. We are optimistic that future progress in simulations will tell us much about whether and how latent energy from QCD plays a role in stellar explosions.

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Discussion

Student: How does one identify the order of a phase transition? What is the difference between first and second order?

Hsu: Formally speaking, the order refers to the level of discontinuity exhibited by thermodynamic quantities (in the infinite volume limit) at the transition. In an Nth order transition, the Nth derivative of the free energy (with respect to, e.g., temperature or density) is discontinuous, while lower order derivatives are continuous. So, in a first order transition the derivative of the free energy itself is discontinuous, whereas in a second order transition the first derivative of the free energy is continuous and the second derivative is not. One can imagine even smoother transitions where only some high derivative of the free energy is discontinuous. Alternatively, one can distinguish by noting that the order parameter exhibits a jump in the first order case, but evolves smoothly in the 2nd and higher order cases. Of course, it is only first order transitions (with nonzero latent heat) that can lead to the explosive phenomena considered here.

Mark Alford (University of Glasgow): (paraphrasing) In our recent investigations of a model of the nuclear matter–quark matter interface in neutron stars, we find that while the free energy of the quark phase is lower, the energy density of the quark core is actually higher than in the nuclear phase. If this is the case, how does the phase transition proceed?

Hsu: In this discussion I assumed that there is a high density state (either the CSC or chirally restored phase) with lower energy than the normal nuclear phase. Note that I really mean energy here, not free energy: $F = H - \mu N = -P$. If we consider the phase diagram as a function of baryon density (not baryon chemical potential), the existence of a new phase implies a state with lower energy than the normal nuclear phase at the same density. If the transition is first order, there is the possibility of latent energy release. In a core bounce, the baryon density is increased on a timescale too short for bulk baryon number flow. So, the relevant question is what happens when you cross a boundary in the baryon density direction. The system will try to tunnel into the lower energy phase, and the release of latent energy powers the explosion.

On the other hand, if the compression is slow enough for significant rearrangement of baryon number, then it is governed by free energetics, or F. In the model you considered, the quark matter phase has lower free energy, but higher energy and baryon density than the nuclear phase. The system can only form the quark phase by aggregating baryon number, and there is probably not enough time for this to happen during a supernova core bounce. The conversion from nuclear to quark phases also requires energy input, which presumably has to come from gravitational collapse.