Diquark degrees of freedom in the EOS and the compactness of compact stars

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1 Introduction

"Old" questions of relativistic astrophysics, like the internal structure of compact stars, have received a lot of attention recently since they are very important for a solid knowledge of the QCD diagram in the low-T , high μ region as discussed along this Workshop. Among the most simple forms of investigating (indirectly) the nature of high-density matter, more precisely of the equation of state connecting the pressure and the energy density; is by obtaining accurate determinations of masses and radii. A comparison of the static models generated by integration of the Tolman-Oppenheimer-Volkoff equations with observed data should reveal a great deal of information about the equation of state.

While astronomers and physicists alike have longely dreamed of such determinations we have only recently achieved sufficient accuracy to perform some key tests on selected objects, although it should be acknowledged that three decades of compact star astrophysics had produced quite clever arguments to determine masses and radii, even in the cases in which they proved to be wrong.

A paradigmatic example of the above assertions is the celebrated determination of the binary pulsar mass PSR 1913+16, believed to be accurate to the fourth decimal place [7]. Methods based on combinations of spectroscopic and photometric techniques have been recently perfectioned, and have confirmed that at least one X-ray source is significantly above the "canonical" 1.44 $M_{\odot}$; namely Vela X-1 for which a value of $1.87^{+0.23}_{-0.17}\,M_{\odot}$ has been obtained [7].

It is also possible that at least some compact stars are extremely compact, or in other words, that their radii are $\sim 30 - 40\%$ less than the cherished $10\,km$ for $M \sim 1\,M_{\odot}$. Indeed, this is the claim of the analysis of [4] of the binary Her X-1 ($M = 0.98 \pm 0.12\,M_{\odot}$ and $R = 6.7 \pm 1.2\,km$) and [11] in the case of the isolated nearby RX J1856-37 ($M = 0.9 \pm 0.2\ M_{\odot}$ and $R = 6^{-1}_{+2}\, km$). In fact, such compactness is extremely difficult (impossible?) to model using underlying equations of state based on hadrons alone, and a "natural" alternative would be to consider deconfined matter. Naturally, these results will be checked and scrutinized for confirmation.

Could this be related to high-density deconfined matter ? Perhaps, since there is hope to achieve deconfinement densities inside compact stars. Also, there is some evidence that diquarks (e. g. a spin-0, color-antitriplet bound state of two quarks) might occur as a component in the QCD plasma (see [9] and references therein for a review). Such diquarks would be expected to be favoured by Bose statistics and they are quit helpful to model the low-energy hadronic properties. Of course, at very high densities, characterized by interquark distances less than $(10GeV^2)^{-1/2}$, diquarks lose their identity and must eventually dissolve into quarks, even if there is no clear consensus about the onset of the asymptotic regime.

According to this picture we may regard a diquark as any system of two quarks considered collectively. Diquark correlations arise in part from spin-dependent interactions between two quarks. Regardless of the exact mechanism for their origin, it is imperative to have some gain of energy (binding) for the stellar models to work (see below). The bound state is a quite strong assumption, and is not easy to prove (or disprove) in the absence of a reliable way of performing sensible calculations. As a working hypothesis (quite analogous to the well known strange matter), we shall assume simply that this bound state actually exists.

2 Bag-inspired quark-diquark Model

Some time ago it was the interesting controversy about the compactness of Her X-1 that prompted us [3] to consider models in which diquarks are considered fundamental digrees of freedom and treated as effective bosons. The results were derived using an effective $\lambda\phi^4$ model and a derived equation of state given in [10], shown to be valid in this case. We shall not repeat the analysis here, but just point out that (as expected) a self-bound quark-diquark mixture indeed produced very compact models (see Fig.1, curve A). In those models there are two quantities that had to be treated parametrically: the diquark mass m_D and the vacuum energy density B. There is no relation between the two included and therefore other possibilities than the $m_D = 575MeV\ B = 57MeVfm^{-3}$ could be considered. However, it is desirable to reduce the freedom in the parameters to see whether the compactness can be altered.

3 QMDD Model

In a more recent attempt we have modeled the quark - diquark plasma as a relativistic gas of u and d quarks, and (ud) diquarks. We treated non-perturbative effects in the quark mass-density-dependent model of confinement (QMDD model) extended for the diquarks assuming $m_D = m_{D0} + C/n_B^{1/3}$, analogously to quarks. Some information on the parameter C to describe absolutely stable SQM has been found in the range $(155\text{-}171)^2\ MeV^2$ [2] based on known stability arguments. The expression for the thermodynamic potential Ω was derived in the thermodynamic limit for the QMDD model extended for bosons, and the usual thermodynamical quantities derived from Ω with some care(see [2]).

Because Ω depends on the baryon number n_B through the masses m_i; the standard relation $P = -\Omega$ [2, 1] is not valid and we have an extra term generated by this density dependence; namely $P = -\frac{\partial(V\Omega)}{\partial V} = \sum_i \Big(- \Omega_i + n_B \frac{\partial m_i}{\partial n_B}\frac{\partial \Omega_i}{\partial m_i}\Big)$. As stressed in previous works [1], the term $n_B \frac{\partial m_i}{\partial n_B}\frac{\partial \Omega_i}{\partial m_i}$ allows the pressure to be zero at finite baryon number, playing the role of the Bag Constant B in the MIT Bag Model. Needless to say, this is a property related to the parametrization of the mass only, and does not depend on the particle statistics of the particles. However, in the case of bosons this extra term does not contribute at zero temperature. As in the standard case of massive bosons at $T = 0$ the energy density reduces to $E = \sum_i n_i m_i$. Diquarks do not contribute to the pressure as they are all in the ground state, but do contribute to the energy density, the equation of state is expected to be quite soft.

These equations have been supplemented with the conditions of equilibrium between D, u and d. By imposing very general stability and equilibrium conditions we found an absolute lower limit on m_{D0} $m_{D0,lim} = \frac{C}{n_b^{1/3}} = 230\Big(\frac{n_0}{n_B}\Big)^{1/3} MeV$. Electrical charge neutrality and baryon number conservation were enforced to calculate the equation of state for each n_B once the parameters (C, m_{D0}) were given. The numerical results were scanned for stability of the mixture . Stability (and hence self-binding [14]) occurs inside a pretty large region in the $(C^{1/2}, M_{D0})$ plane. This region of absolute stability allows the existence of pure quark-diquark stars.It is curious to remark that the *same* parametrization used for simulating the confinement produces a behaviour of the diquark mass that mimics the uprising part of a gap curve (i.e. increasing binding) and thus energetically favoured diquarks as the density increases. This is *not* inconsistent with the idea of dissociation of the diquarks at still higher densities, but in any case it is not expected that a mass density-dependent model can solve the complicated structure of a theory at higher densities (see [8] for a through discussion of this and other details of the model).

4 Discussion

Three examples of the QMDD diquark model are shown in Fig. 1. The resulting equations of state are very soft (models B and C), as expected, but fall short to explain the claimed compactness by at least $\Delta R \sim 1\,km$, even in the most favourable case (model B). When compared with the equation of state for quark-diquark matter presented in [3] (model A) and the softest models of MIT bag models [12] (model D) we may assert that the stellar sequences are in between the former and the latter. Diquarks may be relevant for compact star structure because they allow the existence of stable compact stars with masses of $\sim 1M_{\odot}$ and very small radii. For comparison, the inferred radius and mass of both Her X-1 and RX J1856-37 are shown with their respective error bars. Li *et al.* models SS1 and SS2 [13](not shown in the figure) are examples of other self-bound models of the equation of state which do produce very compact stars, in fact also capable of matching the values of Her X-1 claimed in [4] and of RX J1856-37 [11]. We may state that if evidence for extreme compactness holds, diquarks may be relevant for model building .

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Discussion

Dmitri Diakonov(NORDITA): The spin-0 diquark mass in vacuum has been measured in lattice computer simulations has been measured by the Bielefeld group (M.Hess *et al.*) and claimed to be around $700\,MeV$. It seems that it is unbound.

J.E. Horvath: The m_D appearing in the effective Lagrangian is not necessarily the vacuum mass to which calculations refer, but if the scalar diquark is indeed unbound the possibility of self-bound stars will be gone. Then diquark models may even be useful, although not for explaining small radii.

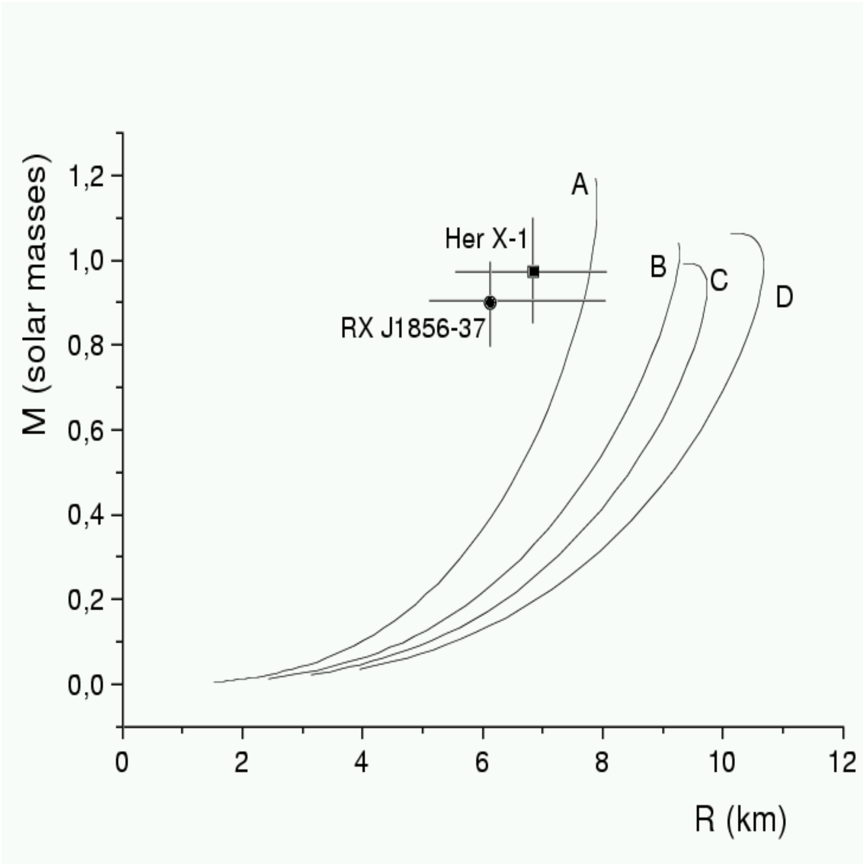

Figure 1: The mass-radius plane, stellar sequences labeled as indicated in the text.