Neutron Star Masses, Radii and Equation of State

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1 Introduction

We are closing in on neutron stars both observationally and theoretically. Observationally, a number of masses (M) and a few radii (R) have been measured as well as a number of other properties. Theoretically, modern equation of states (EOS) are more reliable due to precision measurements of nucleon-nucleon interactions, detailed calculations of binding energies of light nuclei and nuclear matter which constrain three-body forces, inclusion of relativistic effects, improved many-body and Monte Carlo methods, etc.

Ultimately, we can exploit the one-to-one correspondence between the EOS and the mass-radius relation of cold stellar object:

$$P(\rho) \quad \Leftrightarrow \quad M(R) \tag{1}$$

Observing a range of neutron star M and R thus reveals the EOS (e.g., pressure P versus density ρ) of dense and cold hadronic matter. Possible phase transitions from nuclear matter to quark matter (either can also undergo superfluid transitions at certain densities and temperatures), hyperon matter, kaon or pion condensates, etc., would also be revealed by an anomalous/kinky function $P(\rho)$ and $M(R)$. The higher the order of the transition is, the smoother will $M(R)$ be and very accurate observations will thus be necessary. On the other hand, just one accurately measured neutron star mass and radius would already constrain the EOS significantly. Information on the EOS at high baryon densities and the presence or absence of phase transitions could guide us in solving QCD after decades of unsuccessful attempts.

In the following I shall give a brief account of the present status on neutron star observations and theory referring to [1, 2, 3, 4] for longer reviews. Subsequently, I shall attempt to recount the most likely possible phase transition in dense nuclear matter with emphasis on quark matter and its possible color superconducting states, as this is most relevant at this conference. Finally, I shall point to important developments expected in the near future.

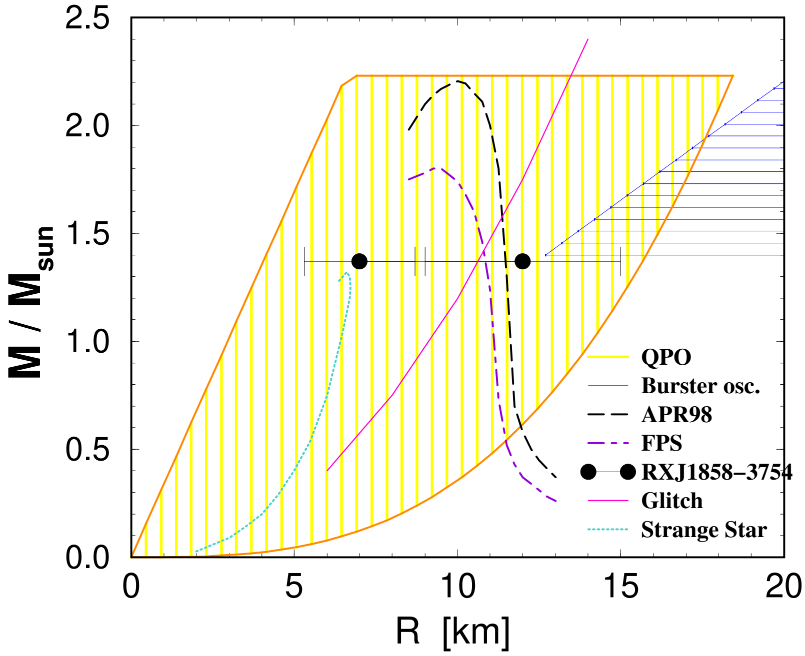

Figure 1: Neutron star masses vs. radius for modern [5] and FPS [6] EOS and strange stars [11]. The hatched areas represent the neutron star radii and masses allowed for orbital QPO frequencies 1060 Hz of 4U 1820-30 (vertical lines, [8, 9]) and for burster oscillations of 4U 1636-53 assuming $M \geq 1.4 M_\odot$ (horizontal lines, [10]) area. Models for glitches in the Vela pulsar constrain masses and radii [11] below the full line. The radii of RX J1856-3754 from Refs. [12, 13] assumes $M = 1.37 M_\odot$.

2 Observed neutron star masses

Only a few masses have been determined from the more than thousand neutron stars, that have been discovered so far:

Binary pulsars: Six double neutron star binaries are known so far, and all of them have masses in the surprisingly narrow range $1.36 \pm 0.08 M_\odot$ [14]. Neutron stars are estimated to have a binding energy of $\sim 10\%$ of their mass. Thus $\sim 1.5\ M_\odot$ of nuclei are needed to obtain a $1.35\ M_\odot$ star. It is suspicious that the Chandrasekhar mass (maximum mass before gravitational collapse sets in) for the iron core inside a large burning star is just around $\sim 1.5\ M_\odot$. It is therefore a tempting conclusion that the iron cores are the progenitors of neutron stars and that all neutron stars are simply produced with $M \simeq 1.35\ M_\odot$. Similarly, white dwarfs are formed in a narrow mass range around $M \simeq 0.6\ M_\odot$ whereas their Chandrasekhar mass is $M \simeq 1.35\ M_\odot$. The latter mass is probably reached by accretion and is responsible for supernova type

SN-1a used as standard candles in cosmology. Calculations of supernova explosions do, however, indicate that neutron stars should be formed with a wide range of masses and so the narrow binary pulsar mass range could be a selection effect in forming a double neutron star system. Whereas selection effects often are important in astrophysics the contrary is the case in particle physics, where one does not believe in accidents but invoke some underlying symmetry.

Millisecond binary pulsars with white dwarf companions have less precise mass determination. Promising progress is reported, e.g., for PSR J0437-4715 [15] where a pulsar mass of $M = (1.58 \pm 0.18)M_\odot$ (error bars are 2σ) is found.

Vela X-1 and **Cygnus X-2** are X-ray binary pulsar/burster with high/low mass companions respectively. From X-ray pulse delays, optical radial velocities and constraints from X-ray eclipse, their masses have been determined. For Vela X-1: $M = 1.87^{+0.23}_{-0.17}\, M_\odot$ [16], and for Cygnus X-2: $M = 1.8 \pm 0.4 M_\odot$ [17].

QPO's are neutron stars emitting X-ray's at frequencies of the orbiting accreting matter. Such *quasi-periodic oscillations* (QPO) have been found in 12 binaries of neutron stars with low mass companions. If the QPO originate from the innermost stable orbit [8, 9] of the accreting matter, their observed values imply that the accreting neutron star has a mass $\simeq 2.3M_\odot$ in the case of 4U 1820-30. If not, the QPO's constrain the EOS as shown in Fig. 1.

3 Observed neutron star radii

The small size of neutron stars makes it very difficult to observe them directly and measure their radius. Estimates have been obtained using quite different methods, which has the benefit that the systematic errors are also different.

RX J1856.5-3754 is our nearest known neutron star. It is non-pulsating and almost thermally radiating. It has been studied recently with the Hubble space telescope by Walter et al. [18]. Its surface temperature is $T \simeq 57$ eV and its distance is, from parallax measurements and circumstantial evidence, about $d \sim 61$ pc. From the measured flux

$$F = \sigma_{SB} T^4 R^2 / d^2 \tag{2}$$

one obtains a radius of $R_\infty = R/\sqrt{1 - 2GM/R} \simeq 7$ km which is incompatible with almost any EOS. Kaplan et al. [19] have reanalysed the HST data and find only half the parallax and thus twice the distance and radius $R_\infty \simeq 15$ km corresponding to $R \simeq 12$ km for $M = 1.4M_\odot$. Its age would be almost a million years which is compatible with standard modified URCA cooling. The spectrum is, however, suppressed in the optical part as compared to the X-rays. Recent more detailed analyses of the spectrum [12, 20] attempting to model the neutron star atmosphere and its absorption, as well as including magnetic fields does not improve the spectral fit. A much better description is obtained from a two temperature model, i.e., a small hot spot and a colder area with a larger radius $\sim 9(d/61pc)$ km.

Gravitational lensing by our nearest neutron star may be observed in june 2003, when J1856.5-3754 will pass within ~ 0.3 arcseconds of a background star as it flies through space with proper motion of 0.332 arcseconds per year. According to [21] (see, however, [19]) it should just be possible to measure that the 26.5 magnitude background star moves by 0.6 milliarcseconds due to the gravitational field of the neutron star. This accuracy requires that the Hubble space telescope is extended with the Advanced Camera for Surveys. If possible, its mass could be derived and our nearest known neutron star would then also be the first one with both mass and radius determined.

X-ray bursts are thermonuclear explosions of accreted matter on the surface of neutron stars. After accumulating hydrogen on the surface for hours, pressure and temperature become sufficient to trigger a runaway thermonuclear explosion seen as an X-ray burst that lasts a few seconds [22]. Assuming that the burst spectra are black-body one can from the resulting temperature and measured flux estimate the neutron star radius if its distance is known. Often the radius is underestimated because only a hot spot emits or the spectrum contains a hard tail. Some bursts do, however, give radii of order ~ 12 km with a period of almost constant (Eddington) luminosities.

Quescent neutron stars are non-accreting X-ray binaries of which some are emitting thermally. Recently, a few quescent neutron stars have been discovered in galactic globular clusters where the distance is known relatively accurately. CXOU 132619.7-472910.8 in NGC 5139 has $T = 66 \pm 5$ eV and $R_\infty = 14.3 \pm 2.1$ km (90% confidence limit) and similar radii although with larger uncertainties are obtained from half a dozen other quescent neutron stars [23].

Burst oscillations in the X-ray flux during the first seconds of the bursts have recently been exploited to determine the compactness or redshift M/R. Whereas the average amplitude increases with the growing size of the hot spot, the amplitude is strongly modulated by the rotational period of the neutron star. The flux does not completely disappear when the hot spot is on the back side due to bending of the light in the gravitational field. This way $M/R < 0.16$ can be extracted [10] for 4U 1636-53 and a radius $R > 12 - 13$ km is obtained for a $M = 1.4M_\odot$ neutron star (see Fig. 1). Corrections from aberration, doppler shifts, etc. are being investigated. Yet, the oscillation analyses is another new promising method by which we can obtain neutron star masses and radii.

Absorption lines in the neutron star photospheres should be detectable with the spectrographs on board Chandra and XMM. The gravitational redshift and pressure broadening of absorption lines determine M/R and M/R^2 respectively and would thus complement other mass and radius information. First results were, however, disappointing. Only in RX J1856.5-3754 were there indications for one or two lines

[13] but higher resolution is required and will come with future observation time.

4 Modern equation of states of dense matter

The relation between $M(R)$ and $P(\rho)$ is often presented (in particular by observers) by a wide variety of curves calculated from "old" EOS thus implying that one can get anything from theory. This is very unfair as many old EOS's are inconsistent with known nucleon-nucleon interactions and/or saturation density and binding energy of nuclei/nuclear matter and should therefore not be taken seriously.

Modern microscopic EOS's are actually converging [4, 3]. The NN interaction is now well determined and constrain potential models leaving only minor differences. Relativistic effects have been included and current scattering experiments at intermediate energies will determine the NN interactions at higher momenta relevant for higher densities. Three-body interactions can be constrained in order to fit nuclear binding and saturation density as well as binding energies of light nuclei up to $A \leq 8$. Many-body and Monte Carlo techniques are now much more accurate. The resulting "modern" EOS [5] are quite reliable up to a few times nuclear saturation densities. Above they are expected to break down but can be constrained by causality conditions. It is boldly predicted [5] that neutron stars in the mass range $M \sim 0.8-1.8M_\odot$ all have radius just around $R \simeq 11.5$ km (see Fig. 1).

The uncertainties in the EOS at densities $\rho \gtrsim 3\rho_0$ affect $M(R)$ for the heavy neutron stars only. By making the EOS stiffer at high densities in a smooth way, the maximum mass can increase up to $M \lesssim 2.0-2.3M_\odot$ [4] but not much higher due to the causality ($c_s < c$) condition. Rotation can increase it by $\sim 10\%$ for the millisecond pulsars.

Phase transitions generally soften the EOS and lower the maximum mass. If the $M = 2.25M_\odot$ mass in 4U 1820-30 stands this would rule out any major phase transition and allow only the stiffest EOS.

Many other phase transitions in neutron stars have been considered. It is expected that superfluid phases of neutrons and protons exist at least in certain density regions at low temperatures although T_c have not been calculated reliably for strongly interacting and correlated nuclear matter. At typical neutron star densities neutrons and protons are superfluid as well due to $^1S_0$ and, in the case of protons, also $^3P_2$ pairing [4]. These superfluid and superconducting components will have drastically different transport properties than normal Fermi liquids. Generally the resistance, specific heat, viscosities, cooling, etc. are suppressed by factors of order $\sim \exp(-\Delta_i/T)$, where Δ_i is the gap of quarks, nucleons or electrons. A superfluid neutron gas in the inner crust is assumed in the description of glitches [11] and provides constraints on the EOS (see Fig. 1).

Eventually at very high densities nucleon degrees of freedom must be replaced by quark ones but it is not known whether core densities of neutron stars are sufficient.

Such quark stars will be discussed in the following section.

More speculative phases are π^0, π^- and K^- condensates as well as hyperons (Σ^-, Λ,...). In [5] a condensate of virtual π^0 is found in a narrow density interval due to strong tensor correlations. The K^- energy can be calculated at low densities and a naive extrapolation would lead to condensate at high densities $\rho \gtrsim 4\rho_0$. However, correlations in nuclear matter invalidates such an extrapolation and makes a K^- condensate unlikely [3]. Hyperons are found to appear at rather low densities $\rho \gtrsim 2-3\rho_0$ in a number of models [25]. Due to limited information on hyperon-nucleon two- and three-body interactions one cannot exclude the presence of hyperons in cores of neutron stars but their effect on the binding energy and thus the EOS is minor whereas their effect on μ_e could be substantial [3].

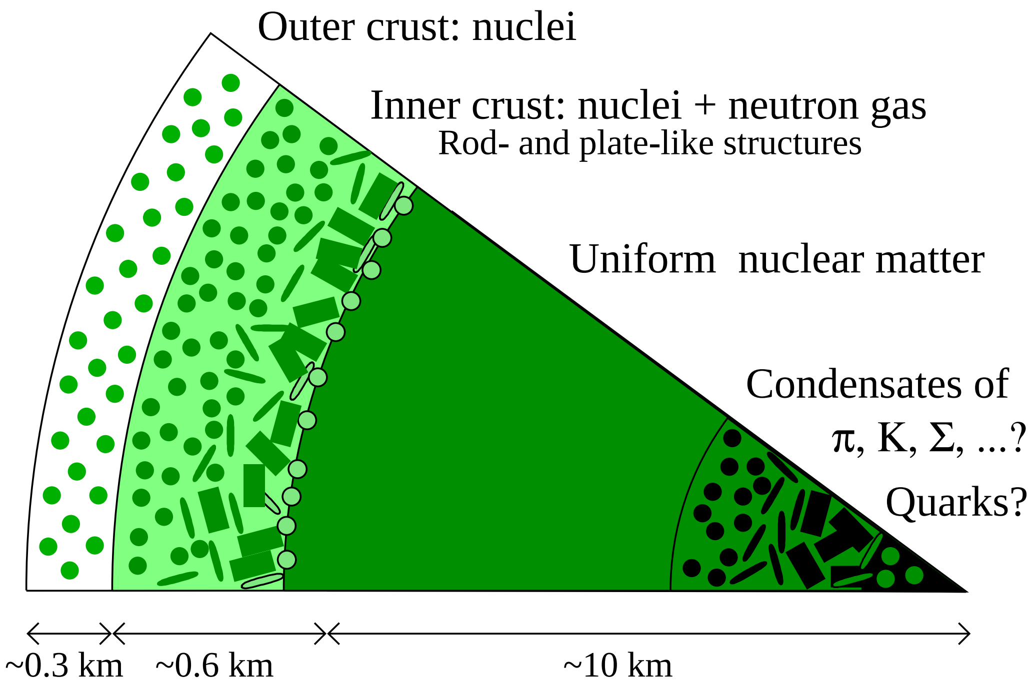

Figure 2: Cross section of a $\sim 1.4 M_\odot$ neutron star. The ~ 1 km thick crust consist of neutron rich nuclei in a lattice and a uniform background of electrons and, in the inner crust, also a neutron gas. The interior of the neutron star contains a nuclear liquid of mainly neutrons and $\sim 10\%$ protons at densities above nuclear matter density n_0 increasing towards the center. Here pressures and densities may be sufficiently high that the dense cold strongly interacting matter undergoes phase transitions to, e.g., quark or hyperon matter or pion or kaon condensates appear. Typical sizes of the nuclear and quark matter structures are $\sim 10^{-14}$ m but have been scaled up to be seen.

5 Quark stars

Natures marvelous variety of EOS's results in the existence of several different stable stellar objects all around one solar mass. These can be ordinary stars, white dwarfs, neutron stars, black holes - and possibly also quark stars. Quark stars come in several categories depending on the details of the nuclear to quark matter phase transition:

- The pure quark stars also called "strange" stars consist of up, down and strange quarks with electrons to fulfil charge neutrality. Possibly a crust of nuclei is suspended above the surface of the quark star. In simple bag model EOS a rather low bag constant and strange quark mass is required to make strange stars and strangelets. If the SAX J1808.4-3658 really has $R \simeq 6$ km based on accretion in magnetic fields [11] or RX J1856-1754 has $R \simeq 7$ km from the one-temperature fit in Ref. [12] that would indicate strange stars rather than normal neutron stars.

- Hybrid stars have a core of quark matter and a mantle of nuclear matter. The quark core size depends on the EOS and vanishes for large bag constants leading to a normal (nucleons only) neutron star.

- Mixed stars have a mixed phase of nuclear and quark matter over a range of density or radius. The mixed phase appear in two-component systems, where the two components: neutrons and protons or up and down quarks [26]. It is, however, required that the interface tension is sufficiently small so that the surface and Coulomb energies of the associated structures are small [27]. If not, a hybrid star results.

A mixed phase of quark and nuclear matter has lower energy per baryon at a wide range of densities [26] if the Coulomb and surface energies associated with the structures are sufficiently small [27, 4]. The mixed phase will then consist of two coexisting phases of nuclear and quark matter in droplet, rod- or plate-like structures (see Fig. 2) in a continuous background of electrons much like the mixed phase of nuclear matter and a neutron gas in the inner crust of neutron stars [28]. Another requirement for a mixed phase is that the length scales of such structures must be shorter than typical screening lengths of electrons, protons and quarks.

In the mixed phase the nuclear and quark matter will be positively and negatively charged respectively. β-equilibrium determines the chemical potentials and densities in the coexisting phases. Total charge neutrality

$$n_e = (1-f)n_p + f(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s)\,, \tag{3}$$

where n_p is the proton density, determines the "filling fraction" f, i.e. the fraction of the volume filled by quark matter. For pure nuclear matter, $f=0$, the nuclear symmetry energy can force the electron chemical potential above ~ 100 MeV at a few times normal nuclear matter densities. With increasing filling fraction, however, negative charged droplets of quark matter replace some of the electrons and μ_e decreases. With increasing density and filling fraction it drops to its minimum value given $\mu_e = m_s^2/4\mu$ corresponding to pure quark matter, $f = 1$.

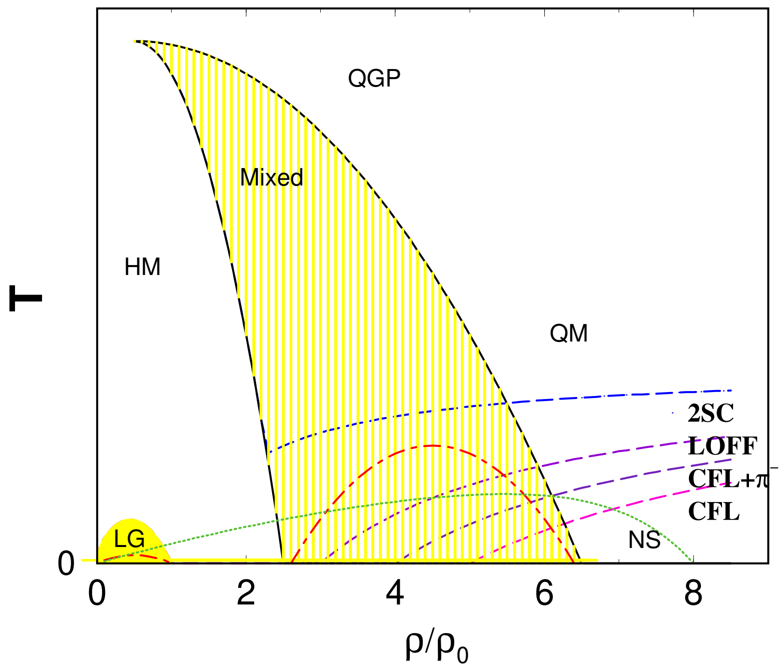

Figure 3: Sketch of the QCD phase diagram, temperature vs. baryon density *in neutron star matter*, i.e. charge neutral and in β-equilibrium containing electrons. Hatched areas indicate mixed phases of hadronic matter (HM) and quark matter (QM/QGP) as well as the nuclear liquid-gas. Dash-dotted lines indicate melting temperatures of the lattices in the mixed phase. Dashed lines separate CSC phases that may appear (see text). The trajectory of neutron star core densities during formation is shown by dotted line and densities below it exist inside neutron stars.

6 Color superconductivity

If quark matter exists in neutron stars or is produced in heavy-ion collisions, a condensate of quark Cooper pairs may appear at low temperatures characterized by a BCS gap Δ usually referred to as color superconductivity (CSC) [29]. The appearance of a gap through color-flavor locking (CFL) requires the gap to exceed the difference between the *u,d,s* quark Fermi momenta, which is not the case for sufficiently large strange quark masses or for an appreciable electron chemical potential, μ_e, which is present in neutron star matter as discussed in [32].

In neutron star matter β-equilibrium relates the quark and electron chemical potentials

$$\mu_d = \mu_s = \mu_u + \mu_e \,. \tag{4}$$

Temperatures are normally much smaller than typical Fermi energies in neutron stars.

The strange quark masses and electron chemical potentials stress the system in the direction of splitting the quark chemical potentials. The pairing interaction prefer overlapping quark Fermi surfaces. We shall investigate this competition in detail below. The case where interactions between quarks are strong and the effective pairing gaps is larger than $\Delta \gtrsim \sqrt{2}|\mu_e/2 - m_s^2/8\mu|$ was shown in [30] to favor the CFL phase.

If interactions are weak, the chemical potentials are then related to Fermi momenta by $\mu_i = \sqrt{m_i^2 + p_i^2}$. If the strange quark mass m_s is much smaller than the quark chemical potentials, Eq. (4) implies a difference between the quark Fermi momenta

$$p_u - p_d \;=\; \mu_e \,, \tag{5}$$

$$p_u - p_s \;\simeq\; \frac{m_s^2}{2\mu} - \mu_e \,, \tag{6}$$

$$p_d - p_s \;\simeq\; \frac{m_s^2}{2\mu} \,, \tag{7}$$

where μ is an average quark chemical potential. Strange quark masses are estimated from low energy QCD $m_s \simeq 150 - 200$ MeV and typical quark chemical potentials are typically $\mu \simeq 400 - 600$ MeV for quark matter in neutron stars [4]. Consequently, $m_s^2/2\mu \simeq 10 - 25$ MeV.

The BCS gap equation has previously been solved for pure *u,d* and *u,d,s* quark matter ignoring electrons and β-equilibrium and the conditions for condensates of dicolor pairs (2CS) and CFL respectively were obtained. There are three 2CS conditions

$$\Delta \gtrsim |p_i - p_j| \,, \quad i\widetilde{\chi}_j^0 = u, d, s \,. \tag{8}$$

In bulk quark matter total charge neutrality of quarks and electrons and β-equilibrium require that $\mu_e \simeq m_s^2/4\mu$. In a mixed phase of quark and nuclear matter the electron chemical potential is a decreasing function from the value in pure β-equilibrium nuclear matter $\mu_e \sim 100 - 200$ MeV down to that for bulk quark matter (if the cores of neutron stars are very dense) $\mu_e \simeq m_s^2/4\mu \simeq 5 - 10$ MeV.

We can now give a qualitative picture of the various phases that strongly interacting matter undergoes as the density increases towards the center of a cold neutron star (see Fig. 3). The inner crust will undergo several transitions as the nuclear matter and neutron gas mixed phase change dimensionality via nuclei, rods, plates, tubes, bubbles to nuclear matter in which the neutron and protons may be superfluid. If quark matter appears at higher densities in a mixed phase with nuclear matter those structures repeat again.

Inside the quark matter part of this mixed phase the CSC phase also changes with density or μ_e depending on the size of the effective pairing interaction Δ between two quarks *i,j* (see, e.g., [29, 33]).

- If the pairing is strong $\Delta \gtrsim \sqrt{2}|\mu_e/2 - m_s^2/8\mu|$ the CFL phase is favored and no electrons appear as was shown in [30]. This condition is not likely to be fulfilled in the mixed phase with low quark filling fraction, where μ_e may be of order $100 - 200$ MeV or larger.

- If $\mu_e/2 \gtrsim \Delta \gtrsim m_s^2/2\mu$ a number of CSC phases appear. In the beginning of the mixed phase ($f \sim 0$) only (7) fulfils (30) and we have a 2CS of *d,s* quarks. A 2CS of *u,s* quarks may, however, compete at larger filling. At the end of the mixed phase ($f \sim 1$) all (5,6,7) fulfil (30) resulting in CFL. In between a crystalline LOFF phase and a CFL phase with a π^- condensate (analogous to the CFL-K^0 phase in [29]) may appear.

- If the pairing interaction is weak or the strange quark mass large such that $\Delta \lesssim m_s^2/4\mu$ a number of different CSC phases such as a 2CS of *u,s* quarks or CFL-K may appear as has been discussed in [29, 31].

In the mixed phase gaps may be affected by the finite size of the quark matter structures as is the case for nuclei. Also surface and Coulomb energies generally disfavors mixed phases.

The finite temperature extension of the competing phases, calculations of the critical temperatures and densities, order of the transitions, etc. for neutron star matter should be investigated further. Probably, the superfluid phases undergo a second order phase transition to the normal phase with increasing temperature at constant baryon density. However, transitions between competing phases might also occur as indicated in Fig. 3.

Temperatures in neutron stars, $T \lesssim 10^6 K \simeq 10^{-4}$ MeV after cooling, are typically much lower than lattice melting temperatures, $T_{melt} \simeq Z^2 e^2/170a$ where a is the lattice spacing (see Fig. 3). Thus the quark matter structures would be solid frozen and the cores of neutron stars would be crystalline and possibly also CSC. Lattice vibration will couple electrons at the Fermi surface with opposite momenta and spins via phonons and lead to a "standard" BCS gap for electrons. The isotopic masses are similar but as densities and Debye frequencies are larger, we can expect considerably larger BCS gaps for electrons. The electrical superconductivy affect magnetic fields through the Meissner effect and magnetic field decay.

7 Outlook

A number of promising developments have been mentioned above that may provide new information on neutron star masses, radii, EOS and possible phase transitions in high density matter.

On the observational front more and better masses and radii are determined by a number of different methods (thus with different systematic errors). On the theoretical front the uncertainties in the EOS are reduced by improved two- and three-body forces, relativistic effects and manybody calculations leading to socalled modern EOS. A number of phase transitions are still possible at high densities and can lead to marvelous structured phases and condensates.

We can hope for additional information from other directions as well:

Gamma ray bursters. The discovery of afterglow in *Gamma Ray Bursters* (GRB) allows determination of their very high redshifts ($z \geq 1$). They imply that GRB have an enormous energy output $\sim 10^{53}$ ergs which requires some central engine more powerful than ordinary supernovae [23]. These could be a special class of type Ic supernova (*hypernovae*) where cores collapse to black holes, or binary neutron stars merging, or some major phase transition to, e.g., quark matter [35]. Also soft GRB may be explained by accretion on strange stars [36].

Neutrinos from the formation of a "proto-neutron star" will be detected from the predicted 1-3 supernovae explosions in our and neighboring galaxy per century. In the case of the recent 1987A in LMC 19 neutrinos were detected on earth and with the upgraded neutrino detectors many thousand neutrinos are expected. The neutrinos can test the SN models, the neutron star EOS and early cooling. During the first second rapid cooling takes place and a phase transitions to, e.g., quark matter or CSC may occur. This may result in a delayed neutrino blimps [37].

Gravitational wave ground based interferometic detectors currently under commission will improve sensitivity significantly. Detectable candidate sources are inspiralling binary neutron stars or black holes merging which may be responsible for gamma ray bursts. R-mode instabilities in rapidly rotating neutron stars may also be detectable [38, 39].

Relativistic heavy-ion collisions are probing the high temperature and low baryon density part of the QCD phase diagram at the opposite end to neutron stars. The phase transition from hadronic matter to a quark-gluon plasma is searched for though the transition may well be a smooth cross over. Some "anomalous" effects, i.e. deviations from predictions based on hadronic theory, have been found and are currently studied intensively at RHIC. It has been speculated that one might find a critical point at high temperatures which would then indicate that the smooth cross over changes to a first order transition at higher baryon densities. That would be valuable information for neutron stars although it would not tell at which density the transition would occur at low temperatures.

Superfluidity in cold fermionic atomic systems will, due to the rapid progress in cooling and trapping techniques, soon be discovered. The great advantage of atomic traps is that one can vary the number of particles, their density and interaction strength as well as the number of spin states, masses, etc. Measuring the size of the pairing gap and varying these parameters provides a testing ground for analytical calculations in the dilute or weakly interacting limit [40]. This would be very useful for gap calculations in neutron and nuclear matter as well as quark matter.

The future of neutron star observations looks bright as new windows are about to open. A new fleet of X- and Gamma-ray satellites have and will be launched. With upgraded ground based observatories and detectors for neutrinos and gravitational waves [39] our knowledge of neutron star properties will be greatly improved. Heavy-ion physics at RHIC and LHC may add further to our understanding of the QCD phase diagram.

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