# Diquark Properties and the TOV Equations

*David Blaschke*[a,1], *Sverker Fredriksson*[b,2] *and Ahmet Mecit Öztaş*[b,c,3]

[a] *Department of Physics, University of Rostock*
[b] *Department of Physics, Luleå University of Technology*
[c] *Department of Physics, Hacettepe University*

# 1 Introduction: Diquarks

This is a status report of our work on quark/diquark effects inside compact astrophysical objects. It goes somewhat in excess of the results shown at the workshop.

The word diquark is due to Gell-Mann in 1964 [1]. The idea of a two-quark correlation has now spread to many areas of particle physics, motivated by phenomenology, lattice calculations, QCD or instanton theory. Now there are some 1200 papers on diquarks, some of which were covered by a review in 1993 [2]. The speaker (S.F.) has an updated database with diquark papers.

Although there is no consensus about diquark details, it seems certain that a $(ud)$ quark pair experiences some attraction when in total spin-0 and colour-**3***. Nuclear matter should also be subject to such pairing, maybe in some new way, *e.g.*, as in the nuclear EMC effect [3]. Quark pairing should also affect a (hypothetical) quark-gluon plasma (QGP), and by now there are more than 300 papers built on this idea. Many of these assume that the correlation is like the one between electrons in a superconductor, with diquarks being the Cooper pairs of QCD [4]. The notion of "superconducting quark matter" is due to Barrois in 1977 [5]. Current efforts owe much to the review paper by Bailin and Love [6], and to work by Shuryak, Wilczek and collaborators [7, 8]. Diquarks in a QGP have also been analysed "classically", with thermodynamics, or field theory [9, 10].

*Astrophysical* diquarks gained popularity about a decade ago, when they were suggested to influence the supernova collapse and "bounce-off" [11, 12, 13, 14], and to enhance the neutrino cooling of quark-stars. The latter effect is now subject to much research [15, 25].

[1]Postal address: D-18051 Rostock, Germany; E-mail: david@darss.mpg.uni-rostock.de
[2]Postal address: SE-97187 Luleå, Sweden; E-mail: sverker@mt.luth.se
[3]Postal address: TR-06532 Ankara, Turkey; E-mail: oztas@hacettepe.edu.tr

Here we study some features of compact objects that would be sensitive to diquark condensation in a QGP. The form factor of the diquark correlation and the quark isospin (a)symmetry due to presence of electrons will be given special attention.

There are many situations where a QGP with diquarks might play a role: (i) In a *quark star*, which might appear as dark matter [17, 18]; (ii) In a *hybrid/neutron star*, surrounded by a hadronic crust; (iii) In a *supernova*, or a *'hypernova' gamma-ray burster*, where diquarks might trigger neutrino bursts and the bounce-off; (iv) In the *primordial plasma* at the Big Bang, where diquarks might have delayed the hadronisation.

## 2 Formalism and Results

We use the BCS theory of colour superconductivity [6, 7, 8]. The gap $\Delta$ can be seen as the gain in energy due to the diquark correlation. Another gap, $\phi$, is related to the quark-antiquark condensate. The thermodynamical grand canonical potential, $\Omega$, is minimised in its variables, resulting in an equation of state (EOS) and other relations. We follow the approach of [19] for the $\Omega$ and generalise it for isospin asymmetry between $u$ and $d$ quarks [20]

$$\begin{aligned}
&\Omega(\phi,\Delta;\mu_B,\mu_I,\mu_e;T) = \\
&= \frac{\phi^2}{4G_1} + \frac{\Delta^2}{4G_2} - \frac{1}{12\pi^2}\mu_e^4 - \frac{1}{6}\mu_e^2 T^2 - \frac{7}{180}\pi^2 T^4 \\
&-2\int_0^\infty \frac{q^2 dq}{2\pi^2}(N_c-2) \times \Big\{ 2E_\phi + \\
&+ T\ln\left[1+\exp\left(-\frac{E_\phi-\mu_B-\mu_I}{T}\right)\right] + T\ln\left[1+\exp\left(-\frac{E_\phi-\mu_B+\mu_I}{T}\right)\right] \\
&+ T\ln\left[1+\exp\left(-\frac{E_\phi+\mu_B-\mu_I}{T}\right)\right] + T\ln\left[1+\exp\left(-\frac{E_\phi+\mu_B+\mu_I}{T}\right)\right]\Big\} \\
&-4\int_0^\infty \frac{q^2 dq}{2\pi^2} \times \Big\{ E_- + E_+ + \\
&+ T\ln\left[1+\exp\left(-\frac{E_- -\mu_I}{T}\right)\right] + T\ln\left[1+\exp\left(-\frac{E_- +\mu_I}{T}\right)\right] \\
&+ T\ln\left[1+\exp\left(-\frac{E_+ -\mu_I}{T}\right)\right] + T\ln\left[1+\exp\left(-\frac{E_+ +\mu_I}{T}\right)\right]\Big\} + C\ , \qquad (1)
\end{aligned}$$

where the subtraction $C = -\Omega((\phi_0^{vac})^2, 0; 0, 0, 0; 0)$ has been introduced to make (1) finite and to assure that pressure and energy density of the vacuum at $T = \mu = 0$ vanish. Thus at the boundary of a compact quark matter object, where the quark-condensate $\phi_0$ changes, a pressure difference arises, which is necessary for confining the system, at least for small masses.

Instead of the chemical potentials $\mu_u$, $\mu_d$ of $u$ and $d$ quarks, one uses [20] those of baryon number, $\mu_B = (\mu_u + \mu_d)/2$, and isospin asymmetry, $\mu_I = (\mu_u - \mu_d)/2$. Then, if beta equilibrium with electrons holds, $\mu_e = -2\mu_I$. The particle densities are given by $n_B = n_u + n_d = -\partial\Omega/\partial\mu_B$, $n_I = n_u - n_d = -\partial\Omega/\partial\mu_I$ and $n_e = -\partial\Omega/\partial\mu_e = -\mu_e^3/(3\pi^2)$. Charge neutrality means $n_B + 3n_I - 6n_e = 0$, and isospin symmetry $n_I = 0$.

The dispersion relations are

$$E_\phi = \sqrt{q^2 + (m + F^2(q)\phi)^2} \quad \text{and} \quad E_\pm = (E_\phi \pm \mu_B)\sqrt{1 + \frac{F^4(q)\Delta^2}{(E_\phi \pm \mu_B)^2}}, \qquad (2)$$

where $F(q)$ is a form factor for two-quark correlations.

We use $T = 0$ and $m = 0$, and start by finding the gaps $\phi_0$ and $\Delta_0$ that minimise $\Omega$. These values give the EoS as

$$\Omega(\phi_0, \Delta_0; \mu_B, \mu_I, \mu_e; T = 0) = \epsilon - \mu_B n_B - \mu_I n_I - \mu_e n_e = -P, \qquad (3)$$

where $\epsilon$ is the energy density and $P$ the pressure.

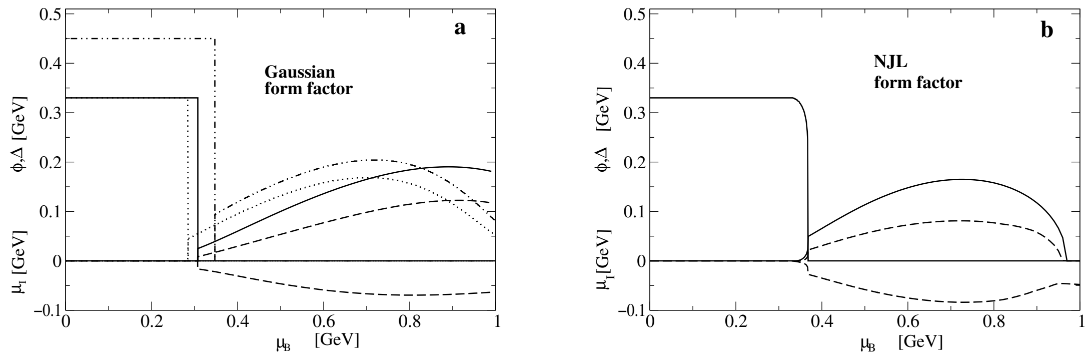


Figure 1: The gaps $\phi$, $\Delta$ and the isospin asymmetry chemical potential $\mu_I$ as functions of $\mu_B$ for (a) Gaussian, and (b) NJL form factors. *Solid lines* are with isospin symmetry (and no electrons), *Dashed lines* are with beta equilibrium. *Dotted lines* in (a) are with isospin symmetry for $\Lambda_G = 0.8$ GeV and $\phi_0^{vac} = 0.33$ GeV. *Dashed double-dotted lines* in (a) are with isospin symmetry for $\Lambda_G = 0.8$ GeV and $\phi_0^{vac} = 0.45$ GeV (giving the same vacuum pressure as with the solid lines).

We use the form factors suggested by Schmidt *et al.* [21], who fitted those to masses and decay properties of light mesons. The following special cases are consid-

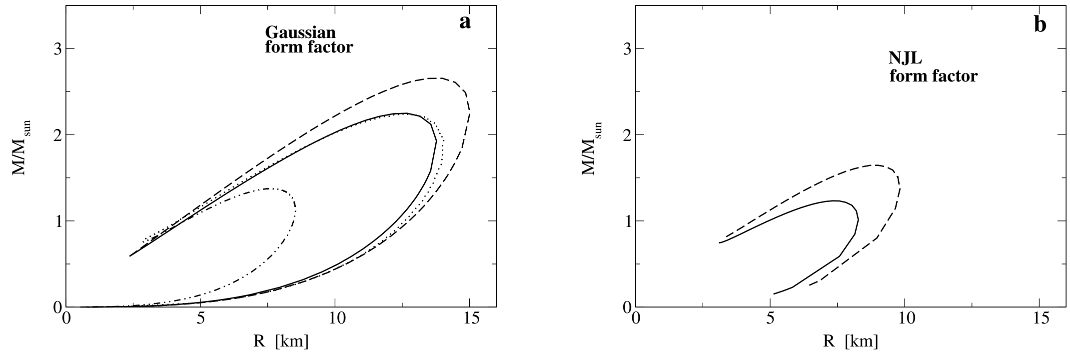


Figure 2: a) The mass ratio $M/M_{sun}$ *vs.* the radius $R$ of a compact object according the the TOV equation. The curves correspond to the same situations as in Fig. 1.

ered: (i) Gaussian: $F^2(q) = \exp(-q^2/\Lambda_G^2)$; (ii) Lorentzian: $F^2(q) = 1/[1 + (q/\Lambda_L)^4]$; (iii) cutoff NJL: $F^2(q) = \Theta(1 - q/\Lambda_{NJL})$. Here $\phi_0 = 0.33$ GeV, $\Lambda_G = 1.025$ GeV, $\Lambda_L = 0.8937$ GeV and $\Lambda_{NJL} = 0.9$ GeV [21]. We contrast this to $\Lambda_G = 0.8$ GeV used in [19]. Fig. 1 shows our results for the Gaussian and NJL form factors, and for two extreme cases: (i) isospin symmetry (equal numbers of $u$ and $d$ quarks), and (ii) beta equilibrium and charge neutrality. In the former case charge is balanced *outside* the object. In the latter, the electron fraction is small, and $n_d \approx 2n_u$. Here $\mu_I$ is given in the lower part of the figures. We also test the sensitivity on the parameter $\Lambda_G$. There is now the choice to either keep $\phi_0^{vac}$ fixed, or to change it so that the vacuum pressure $(\phi_0^{vac})^2/(4G_1)$ is kept fixed.

The EoS is used as input to the standard Tolman-Oppenheimer-Volkoff equations [22] for an equilibrium (spherical) fluid (see also [23]):

$$\frac{dP(r)}{dr} = -\frac{[\epsilon(r) + P(r)][m(r) + 4\pi r^3 P(r)]}{r[r - 2m(r)]}, \tag{4}$$

$$m(r) = 4\pi \int_0^r \epsilon(r') r'^2 dr'. \tag{5}$$

The equations are iterated in $r$, starting with some value $\epsilon_0$ at $r = 0$. The procedure stops when $P = 0$, which defines the radius $R$ of the object, and $M = m(R)$ is plotted *vs.* $R$, see Fig. 2. Each $\epsilon_0$ value results in one point in the plot, with unique values also of $\mu_B$ and $\mu_I$. The graph becomes a backbending spiral, with $\epsilon_0$ and $\mu_B$ increasing along the curve. Solutions on left-going parts of the curve are unstable.

# 3 Conclusions and Outlook

The diquark form factor and the isospin (a)symmetry are important inside a quark/diquark star. They can make a difference of up to two solar masses, as for its maximal mass.

We now have results also for $0 < T < \Delta$, where condensate-free states can occur in intervals of $\mu_B$ [24]. These correspond to an interaction-free plasma, and appear as "bumps" in the TOV curves. They might be relevant as phase transitions inside a collapsing system (supernova). The QGP might suddenly enter a diquark phase, whereafter it again becomes a free-quark state just before the bouncing [31]. We also intend to study such events for the Big Bang plasma. Maybe a transition into a diquark phase resulted in a release of neutrinos, just as with the cosmic microwave background. The neutrino energy would have been of the order of $\Delta$, but now cooled to keV energies.

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# Discussion

**A. Thampan:** Should one not see a signature (say in terms of phase transitions or so) in the $P$ *vs.* $\rho$ (EoS) relationships?

**Fredriksson:** For some extreme parameter values bumps occur in, *e.g.*, $M$ *vs.* $R$ and $P$ *vs.* $\epsilon$, and certainly also in a $P$ *vs.* $\rho$, although we did plot the latter. For the case of beta equilibrium, these transitions are smoother in the TOV graphs. The bumps are clearer for $T > 0$, as mentioned above.

**J.E. Horvath:** Do diquarks disappear suddenly in this model? Could you identify what makes the matter self-bound?

**Fredriksson:** Not suddenly in time, because these are equilibrium solutions. But diquarks disappear "suddenly" when other parameters change, *e.g.*, $\mu_B$ (see "NJL"). A strange star is self-bound by gravity at high masses, but also due to our "vacuum pressure" $(C)$.

**Unknown:** Are the stars beyond the mass peak stable?

**Fredriksson:** Our approach does not tell, but states with $\frac{\partial M}{\partial \epsilon} < 0$ are usually prescribed to be unstable [23].