Color Superconductivity and Blinking Proto-Neutron Stars

Greg W. Carter
Department of Physics and Astronomy
State University of New York
Stony Brook, NY 11794-3800

1 Introduction

Due to instabilities at the Fermi surface, matter composed of deconfined quarks is believed to be a color superconductor at low temperatures (see Refs. [1] and other reviews). While the precise value of the energy gap at astrophysical densities is yet indeterminate, calculations using models of vacuum physics generally predict a gap in the neighborhood of 100 MeV for densities at and above three times that of equilibrium nuclear matter. This implies that quark matter with a chemical potential fixed around 400 MeV undergoes a phase transition between a quark-gluon plasma state and one of paired quarks at a critical temperature near 50 MeV. In this talk I discuss how this could lead to a variation in the neutrino signal emanating from a proto-neutron star formed in the wake of a supernova.

Our current understanding of Type II (core collapse) supernovae, based on a handful of observations and quite a bit of theoretical modeling, begins with the implosion of the inner core of star of mass 8–20 solar masses (see Ref. [2] for a recent review). The evolution of the remnant proto-neutron star is driven by the diffusion of neutrinos, which make their way through the hot ($T \sim 25$ MeV) and dense ($n_B \sim 3n_0$) core matter before a few are eventually detected on Earth. The temporal characteristics of this signal are determined for the most part by the neutrino mean free path through this core.

For temperatures much lower than the superconducting energy gap, $T \ll \Delta$, quark states are replaced by diquark quasi-particles and neutrino-quark cross sections are exponentially suppressed. Thus at late times, when $T \sim 1$ KeV, the neutrino mean free path in quark matter is practically infinite. But within the first minute after core collapse, deleptonization heats the dense core to $T \simeq 50$ MeV before thermal neutrino emission cools the system. It is during this time that transitions between states of strongly-interacting matter would occur. While the phase transition from hadronic matter to a quark plasma is likely first order (but poorly understood), a slightly later

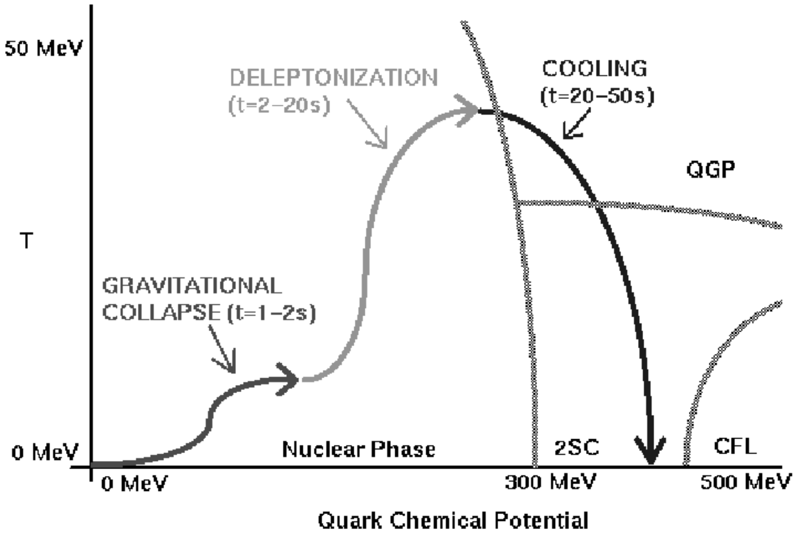

Figure 1: A tour of the phase diagram via a proto-neutron star.

transition from quark to superconducting quark matter might be second order, as in BCS theory. The associated critical behavior of the latter would lead to a slowdown in cooling, followed by a burst of streaming neutrinos as the temperature eventually falls below the quark energy gap. In what follows I will discuss inelastic neutrino-quark scattering in a simplified proto-neutron star environment in order to quantify this possible phenomena and speculate on an observable signal.

We are concerned here with the period in the proto-neutron star's evolution during which the matter is at its hottest and most dense, immediately following the Joule heating due to primordial lepton release. The ambient conditions are a temperature near 50 MeV and a density around four times that of nuclear matter, or equivalently a quark chemical potential of 400 MeV. This is the final period (Cooling) in the evolution mapped on the generic phase diagram for nuclear matter in Figure 1.

2 Neutrino Mean Free Path in a Color Superconductor

Thermal neutrino diffusion is the primary means of heat escape from the dense core. While noting that the neutrino production rate will also differ from that of normal matter [3], in the diffusive regime the dominant critical behavior will be a change in the *inelastic* quark-neutrino cross section, since here neutrino production rates decouple from the transport equation and depend only on the neutrino mean free path. We thus calculated the differential and total cross sections in order to compute the neutrino

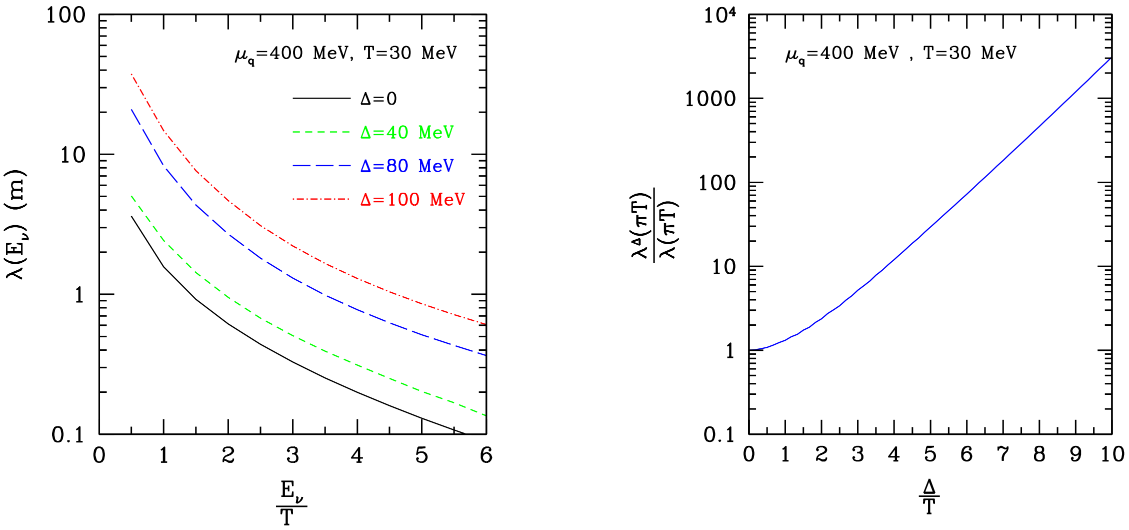

Figure 2: Left Panel: Neutrino mean free path as a function of neutrino energy. Right Panel: The neutrino mean free path for neutrino energy $E_\nu = \pi T$, plotted as a function of the gap Δ.

mean free path in two-flavor quark matter. The magnitude of the superconducting gap, Δ, was taken as arbitrary within a range of values found in recent literature. Closely following BCS theory, we assumed the gap to be a constant, and calculated the response functions and neutrino cross sections in the weak coupling approximation.

Details, including polarization operators, differential and total cross sections, and limiting behavior can be found in Ref. [4]. The essential physical difference between neutrinos scattering in superconducting quark matter as opposed to free quark matter is that, in the quasi-particle phase, the incident neutrino must transfer energy greater than twice the gap, or $E_\nu \geq 2\Delta$, to excite a quark pair. Stated equivalently, when the gap becomes large ($\Delta \gtrsim T \sim 30$ MeV) the spacelike response functions ($q_0 < |\vec{q}|$) are substantially suppressed, and the inelastic cross section is greatly diminished.

The mean free path, $\lambda = (\sigma/V)^{-1}$, therefore grows with energy gap. This main result is shown in Figure **??**. The left panel assumes a constant energy gap Δ at a constant temperature $T = 30$ MeV. As expected, the mean free path grows substantially when $T < \Delta$ for all neutrino energies. In the right panel an average thermal neutrino energy of $E_\nu = \pi T$ has been assumed, and the superconducting gap is varied for fixed T. The expected exponential behavior is clear for $\Delta \gtrsim 3T$ and, combining information from the two plots, one can deduce that for low temperatures the neutrinos are able to pass through kilometers of superconducting quark matter with only an occasional collision. These results are essentially independent of temperature for all $T \lesssim 50$ MeV.

3 Cooling of an Idealized Quark Star

The theoretical state of the art in modeling proto-neutron star cooling involves complex computer simulations of multi-phase environments. To explore the consequences of the onset of superconductivity in a macroscopic core of deconfined quark matter, we will simply assume the existence of such a system and approximate it by a sphere of quark matter. We furthermore consider the relatively simple case of two massless flavors with identical chemical potentials and disregard for now any non-superconducting quarks which might be present in the system. We also make the (safer) assumption that the neutrino mean free path is much smaller than the dimensions of the astrophysical object and several orders of magnitude greater than the quark mean free path, meaning that the system cools by neutrino diffusion.

In a such a sphere of quark matter, the diffusion equation for energy transport is

$$C_V \frac{dT}{dt} = -\frac{1}{r^2}\frac{\partial L_\nu}{\partial r}\,, \tag{1}$$

where C_V is the specific heat per unit volume of quark matter, T is the temperature, and r is the radius. The neutrino energy luminosity for each neutrino type, L_ν, depends on the neutrino mean free path and the spatial gradients in temperature. It is approximated by

$$L_\nu \cong -6\int dE_\nu\,\frac{c}{6\pi^2}\,E_\nu^3 r^2 \lambda(E_\nu)\frac{\partial f(E_\nu)}{\partial r}\,, \tag{2}$$

where c denotes the speed of light in vacuum. In our analysis we assume that neutrino interactions are dominated by the neutral current scattering common to all neutrino types. Consequently, we take the same neutrino and anti-neutrino mean free path for every neutrino flavor, giving rise to the factor of six in Eq. (2). The equilibrium Fermi distribution, $f(E_\nu)$, and the (scattering) mean free path, $\lambda(E_\nu)$, are integrated over all neutrino energies, E_ν.

The solution to the diffusion equation will also depend on the size of the system, the radius $R \sim 10$ km. The temporal behavior is characterized by a time scale τ_c, which is proportional to the inverse cooling rate and can hence be deduced from Eq. (1). The characteristic time

$$\tau_c(T) = C_V(T)\frac{R^2}{c\langle\lambda(T)\rangle}\,, \tag{3}$$

is a strong function of the ambient matter temperature since it depends on the matter's specific heat and the neutrino mean free path. This applies to a system characterized by the radial length R and the energy-weighted average of the mean free path, $\langle\lambda(T)\rangle$. Following our general treatment of the superconducting gap, we assume that the temperature dependence of the specific heat is described by BCS theory. We will then use the scattering results to calculate $\langle\lambda(T)\rangle$. Furthermore, since neutrinos are in thermal equilibrium for the temperatures of interest, we may assume $\langle\lambda(T)\rangle \simeq \lambda(E_\nu = \pi T)$. Finally, we note that the diffusion approximation is only valid when $\lambda \ll R$ and will thus fail for very low temperatures, when $\lambda \sim R$.

4 A Signal of Color Superconductivity?

The results of our calculations are summarized in Figure **??**. We have taken the temperature dependence of the gap (long dashed curve) from BCS theory, $\Delta(T)/\Delta_0 = \sqrt{1-(T/T_c)^2}$, where Δ_0 is the zero-temperature gap at a given density. The specific heat (dot-dashed curve) is also taken from the mean-field result, and is peaked around $T = T_c$ as expected in a second-order phase transition. With our result for the neutrino mean free path (short dashed curve), we combine factors to obtain the characteristic diffusion time (solid line), shown as a ratio to that found in a system of free quarks.

The consequences of these results are interpreted as follows. The early cooling rate, around T_c, is influenced mainly by the peak in the specific heat associated with the second order phase transition (the neutrino mean free path is not strongly affected when $\Delta \ll T$). Subsequently, as the matter cools, both C_V and λ^{-1} decrease in a non-linear fashion for $\Delta \sim T$. Upon further cooling, when $\Delta \gg T$, both C_V and λ^{-1} decrease exponentially. Both of these effects accelerate the cooling process at later times.

We conclude that if the core of a neutron star formed in the aftermath of a supernova contains a large amount of quark matter at sufficiently high temperature and density, and if the associated neutrino signal can be detected on Earth, a unique temporal profile would be observed. Specifically, we expect a brief interval during which the cooling would slow around $T \sim T_c$, signified by a period of reduced neutrino detection. This would be followed by a brief burst of neutrinos after the temperature falls well below T_c and the quasi-quarks decouple from the thermal neutrinos.

We note that many real-world complications would weaken or disrupt this signal, as explained in Ref. [4]. Primary among them are the presence of strange quarks and the likely existence of a hadronic shell of nuclear matter covering the core, opaque to neutrinos, which would even out a sharp signal. Furthermore, ours is in fact a generic prediction for any second-order phase transition, applied here to color superconductivity given our understanding of proto-neutron star evolution and the phase diagram of QCD. However, given the real prospect of detecting neutrinos emitted from a future supernova event, transport processes like the ones we discuss here might someday serve as a reliable probe of the properties of extremely dense matter.

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