Color superconducting quark matter in compact stars

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1 Introduction

Matter just above nuclear density presents special challenges, both observational and theoretical. It is the densest state of which we have evidence in nature, but in terrestrial laboratories it can only be produced fleetingly and at high temperature over small volumes. We must therefore rely on astrophysical observations of compact stars to gather more indirect experimental information. Theoretically, it falls between the domain of nuclear physics, where models have been tested against scattering data, and the asymptotically high energy domain, where the relevant theory, QCD, becomes tractable.

We expect that somewhat above nuclear density, the nucleons will overlap so much as to lose their separate identities, and merge into quark matter. In this talk I will review some of theoretical expectations and speculations about quark matter, focusing on the phenomenon of quark pair condensation (color superconductivity).

indexFermi surface Since QCD is asymptotically free, one expects that at high enough densities and low temperatures, matter will consist of a Fermi sea of quarks, and the ones at the Fermi surface will be almost free. The residual gluon-mediated interaction is attractive in the color $\bar{\mathbf{3}}$ channel, so BCS quark pair condensation will take place, breaking the $SU(3)$ color gauge symmetry. his was first appreciated in the 1970s, and revived more recently [1, 2, 3, 4, 5]. It is discussed in more detail in the review articles [6]. The quark pairs play the same role here as the Higgs particle does in the standard model: the color-superconducting phase can be thought of as the Higgsed (as opposed to confined) phase of QCD.

t is important to remember from the outset that the breaking of a gauge symmetry cannot be characterized by a gauge-invariant local order parameter which vanishes on one side of a phase boundary. The superconducting phase can be characterized rigorously only by its global symmetries. In electromagnetism there is a non-local order parameter, the mass of the magnetic photons, that corresponds physically to the Meissner effect and distinguishes the free phase from the superconducting one. In QCD there is no free phase: even without pairing the gluons are not states in the spectrum. No order parameter distinguishes the Higgsed phase from a confined phase or a plasma, so we have to look at the global symmetries.

2 Patterns of color superconductivity

In the real world there are two light quark flavors, the up (u) and down (d), with masses $\lesssim$ 10 MeV, and a medium-weight flavor, the strange (s) quark, with mass $\sim$ 100 MeV. It is convenient to treat the u and d as massless, and study the effect of varying the s quark mass between zero and infinity.

2.1 Three flavors: color-flavor locking

In the three flavor case, $m_s = m_u = m_d = 0$, the structure of the quark pair condensate is particularly simple and elegant, because the number of flavors and colors is equal. The favored condensation pattern is "color-flavor locking",

$$\text{CFL phase:} \quad \Delta^{\alpha\beta}_{ij} = \langle q^\alpha_i q^\beta_j \rangle_{1PI} \propto C\gamma_5[(\kappa+1)\delta^\alpha_i\delta^\beta_j + (\kappa-1)\delta^\alpha_j\delta^\beta_i], \tag{1}$$

where color indices α, β and flavor indices i, j all run from 1 to 3, Dirac indices are suppressed, and C is the Dirac charge-conjugation matrix. The term multiplied by κ corresponds to pairing in the $(\mathbf{6}_S, \mathbf{6}_S)$, which although not highly favored energetically breaks no additional symmetries and so κ is in general small but not zero [7, 8, 9].

Eq. (1) exhibits the color-flavor locking property of this ground state. The Kronecker deltas connect color indices to flavor indices, so that the VEV is not invariant under color rotations, nor under flavor rotations, but only under simultaneous, equal and opposite, color and flavor rotations. Since color is only a vector symmetry, this VEV is only invariant under vector flavor rotations, and breaks chiral symmetry.

The pattern of symmetry breaking is therefore (with gauge symmetries in square brackets)

$$[SU(3)_{\text{color}}] \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset [U(1)_Q]} \times U(1)_B \longrightarrow \underbrace{SU(3)_{C+L+R}}_{\supset [U(1)_{\tilde{Q}}]} \times \mathbb{Z}_2 \tag{2}$$

Note that electromagnetism is not a separate symmetry, but corresponds to gauging one of the flavor generators.

This pattern of condensation has many interesting features. (1) The color gauge group is completely broken, so all eight gluons become massive. This ensures that there are no infrared divergences associated with gluon propagators. (2) All the quark modes are gapped. The nine quasiquarks (three colors times three flavors) fall into a smaller-gap octet and a larger-gap singlet of the unbroken global $SU(3)$. (3) Electromagnetism is replaced by a "rotated electromagnetism", namely a linear combination $\tilde{Q}$ of the original photon and one of the gluon. 4) Two global symmetries are broken, he chiral symmetry and baryon number, so there are two gauge-invariant order parameters that distinguish the CFL phase from the QGP, with corresponding Goldstone bosons which are long-wavelength disturbances of the order parameter. (5) Quark-hadron continuity. It is striking that the symmetries of the 3-flavor CFL phase are the same as those one might expect for 3-flavor hypernuclear matter [10]. This means the spectrum may evolve continuously from hypernuclear matter to the CFL phase of quark matter—there need be no phase transition.

2.2 Two flavors

If the strange quark is heavy enough to be ignored, then the up and down quarks pair in the color $\bar{\mathbf{3}}$ flavor singlet channel, a pattern that we call the two-flavor superconducting (2SC) phase,

$$\text{2SC phase:} \quad \Delta_{ij}^{\alpha\beta} = \langle q_i^\alpha q_j^\beta \rangle_{1PI} \propto C\gamma_5 \varepsilon_{ij} \varepsilon^{\alpha\beta 3}, \tag{3}$$

where color indices α, β run from 1 to 3, flavor indices i, j run from 1 to 2. Four-fermion interaction calculations agree on the magnitude of Δ: around 100 MeV. This is found to be roughly independent of the cutoff, although the chemical potential at which it is attained is not. Such calculations are based on calibrating the coupling to give a chiral condensate of around 400 MeV at zero density, and turning μ up to look for the maximum gap.

As with any spontaneous symmetry breaking, one of the degenerate ground states is arbitrarily selected. In this case, quarks of the first two colors (red and green) participate in pairing, while the third color (blue) does not. The ground state is invariant under an $SU(2)$ subgroup of the color rotations that mixes red and green, but the blue quarks are singled out as different. The pattern of symmetry breaking is therefore

$$\begin{array}{rl} & [SU(3)_{\text{color}}] \times [U(1)_Q] \times SU(2)_L \times SU(2)_R \\ \longrightarrow & [SU(2)_{\text{color}}] \times [U(1)_{\tilde{Q}}] \times SU(2)_L \times SU(2)_R \end{array} \tag{4}$$

The features of this pattern of condensation are (1) The color gauge group is broken down to $SU(2)$, so five of the gluons will become massive, with masses of order $g\mu$. The remaining three gluons are associated with an unbroken $SU(2)$ red-green gauge symmetry, whose confinement distance scale rises exponentially with density [11]. (2) The red and green quark modes acquire a gap Δ. There is no gap for the blue quarks in this ansatz, and it is an interesting question whether they find some other channel in which to pair. The available attractive channels appear to be weak so the gap will be much smaller, perhaps in the keV range [4]. We will ignore such pairing here. (3) A rotated electromagnetism ("$\tilde{Q}$") survives unbroken. It is a combination of the original photon and one of the gluons. 4) No global symmetries are broken, although additional condensates that break chirality have been suggested [12], so the 2SC phase has the same global symmetries as the quark-gluon plasma (QGP).

3 Two massless + one massive quark flavors

A nonzero strange quark mass explicitly breaks the flavor $SU(3)_L \times SU(3)_R$ symmetry down to $SU(2)_L \times SU(2)_R$. If the strange quark is heavy enough then it will decouple, and 2SC pairing will occur. For a sufficiently small strange quark mass we expect a reduced form of color-flavor locking in which an $SU(2)$ subgroup of $SU(3)_{\text{color}}$ locks to isospin, causing chiral symmetry breaking and leaving a global $SU(2)_{\text{color}+V}$ group unbroken.

As m_s is increased from zero to infinity, there has to be some critical value at which the strange quark decouples, color and flavor rotations are unlocked, and the full $SU(2)_L \times SU(2)_R$ symmetry is restored [13]. On the way, however, other interesting phenomena may occur. Where differing masses or chemical potentials obstruct the pairing of one species of quark with another (eg in the 2SC+s phase) we expect regions of crystalline superconductivity [14], which grow larger at high chemical potential where forward scattering dominates the quark-quark interactions [15]. This could lead to interesting phenomena such as glitches in quark stars [14]. It is also possible that the CFL condensate responds to the strange quark mass by rotating in a direction that reduces the strangeness content. This corresponds to a condensation of K_0 mesons [16] yielding a "CFL-K^0" phase.

symmetry:	gauged	baryon number	hyper-charge	isospin	chiral	axial $U(1)$
broken by:			weak	electromag $m_u \neq m_d$	$m_{u,d} \neq 0$	instanton
QGP	$[SU(3) \times U(1)_Q]$	$U(1)_B$	$U(1)_Y$	$SU(2)_V$	$SU(2)_A$	$U(1)_A$
2SC	$[SU(2) \times U(1)_{\tilde{Q}}]$	$U(1)_B$	$U(1)_Y$	$SU(2)_V$	$SU(2)_A$	$U(1)_A$
CFL	$[U(1)_{\tilde{Q}}]$	$\{1\}$	$U(1)_Y$	$SU(2)_V$	$\{3\}$	$\{1\}$
CFL-K^0	$[U(1)_{\tilde{Q}}]$	$\{1\}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1\}$

Table 1: Symmetry breaking of the high-density QCD phases in various approximations. For broken symmetries, the number of Goldstone bosons is given in curly brackets. Gauged symmetries are in square brackets. The strongest explicit breaking of each of the global symmetries is given: only baryon number is a global symmetry of the real world. The QGP is assumed to have the symmetries of the Lagrangian.

The CFL-K^0 phase is expected to occur if the strange quark mass is large enough, and the up and down are light enough. Calculations at asymptotically high density,

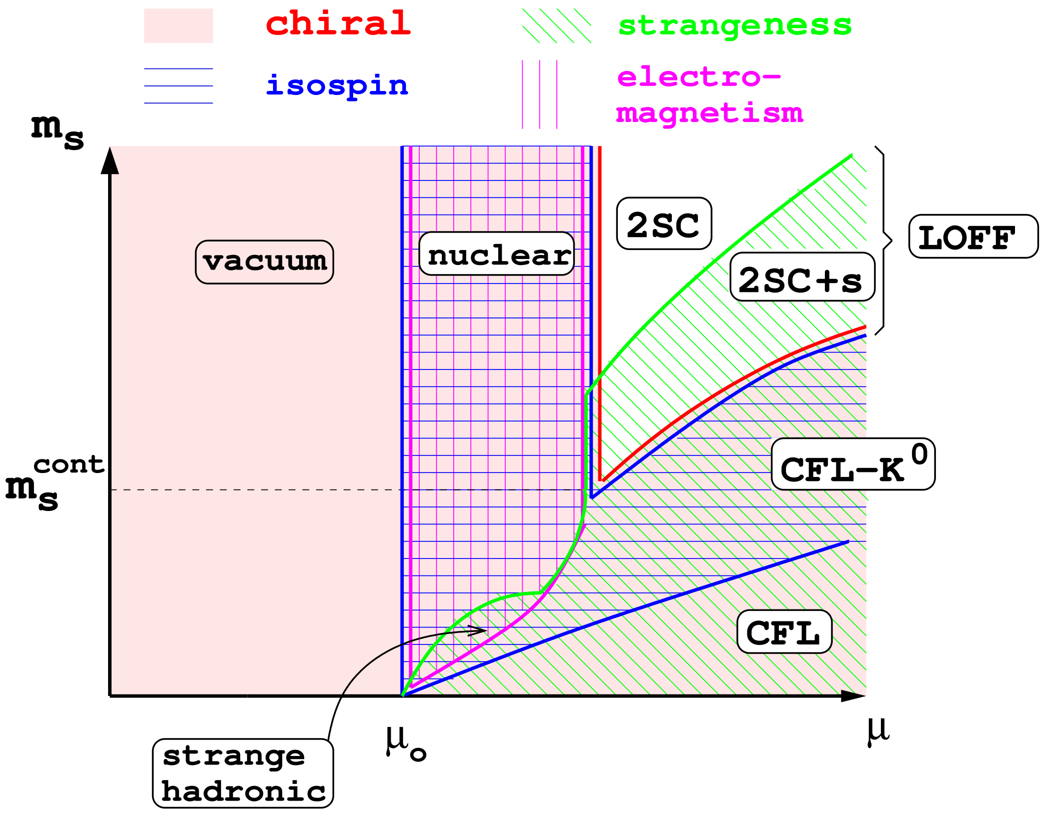

Figure 1: Possible phase diagram for 2+1 flavor QCD at $T = 0$. In the blue (horizontal) shaded area, isospin is broken. In the magenta (vertical) shaded area, electromagnetism is broken. In the green (top-left to bottom-right) shaded area, strangeness is broken. Chiral symmetry is broken in the whole densely (pink) shaded area, i.e. everywhere except in the wedge at top right. Small print: most of these symmetries are approximate, and potentially important physics has been ignored—see the text.

extrapolated back to around nuclear density, indicate that it is favored for quark matter at a few times nuclear density [17]. This estimate ignores terms that split the gaps corresponding to different flavor pairings [13] and $U(1)_A$-breaking terms induced by instantons that are expected to become important in that regime. In table 1 we show the symmetries broken by the various phases. In the real world, the only relevant true global symmetry is baryon number, and both CFL and CFL-K^0 break it, so they both have one massless superfluid mode. However, we expect a number of pseudo-Goldstone bosons arising from spontaneously broken near-symmetries. These are different in the CFL and CFL-K^0 phases. It has recently been noted that the breaking of isospin in the CFL-K^0 phase leads to 2 not 3 pseudo-Goldstone bosons, [18], and that the breaking of $U(1)_Y$ by the K^0-condensate may lead to long-lived axion-type domain walls [19].

In Fig. 1 we give one possibility for the resultant QCD phase diagram in the μ-m_s plane. The complications relating to the CFL-K^0 phase have been ignored, and most of the symmetries shown in the figure are approximate, so the "symmetry breaking" shaded areas actually correspond to regions where some pseudo-Goldstone states become very light, and the border lines are crossovers. The exceptions are electromagnetism, which is a gauge symmetry, with the mass of the photon as an order parameter, and baryon number. Electromagnetism is broken in the magenta (vertical) shaded region. Baryon number is a combination of electric charge, isospin, and strangeness, and is broken in all the shaded regions, leading to an exactly massless superfluid mode.

The low-μ part of the CFL phase is the region of quark-hadron continuity [10], where the whole baryon octet self-pairs in an isospin-respecting way [13]. The low-μ part of the CFL-K^0 phase can also be given a hadronic interpretation, in which there is n-n and p-Σ^- pairing, which breaks isospin amd strangeness, but leaves electromagnetism unbroken.

4 The transition to color superconducting quark matter in compact stars

The only place in the universe where we expect very high densities and low temperatures is compact stars (for a recent review, see [20]). These typically have masses close to $1.4M_\odot$, and are believed to have radii of order 10 km. Their density ranges from around nuclear density near the surface to higher values further in, although uncertainty about the equation of state leaves us unsure of the value in the core.

Color superconductivity gives mass to excitations around the ground state: it opens up a gap at the quark Fermi surface, and makes the gluons massive. One would therefore expect its main consequences to relate to transport properties, such as mean free paths, conductivities and viscosities. he influence of color superconductivity on the equation of state is an $\mathcal{O}((\Delta/\mu)^2)$ (few percent) effect, which is probably not phenomenologically interesting given the existing uncertainty in the equation of state at the relevant densities.

4.1 The transition region

There are two possibilities for the transition from nuclear matter to quark matter in a neutron star: a mixed phase, or a sharp interface. The surface tension of the interface determines which is favored. To be concrete, we will consider the case where the strange quark is light enough so that quark pairing is always of the CFL type. Figure 2 shows the μ_B-$\mu_e^{\rm eff}$ phase diagram, ignoring electromagnetism. The lightly (yellow) shaded region is where nuclear matter (NM) has higher pressure. The darker

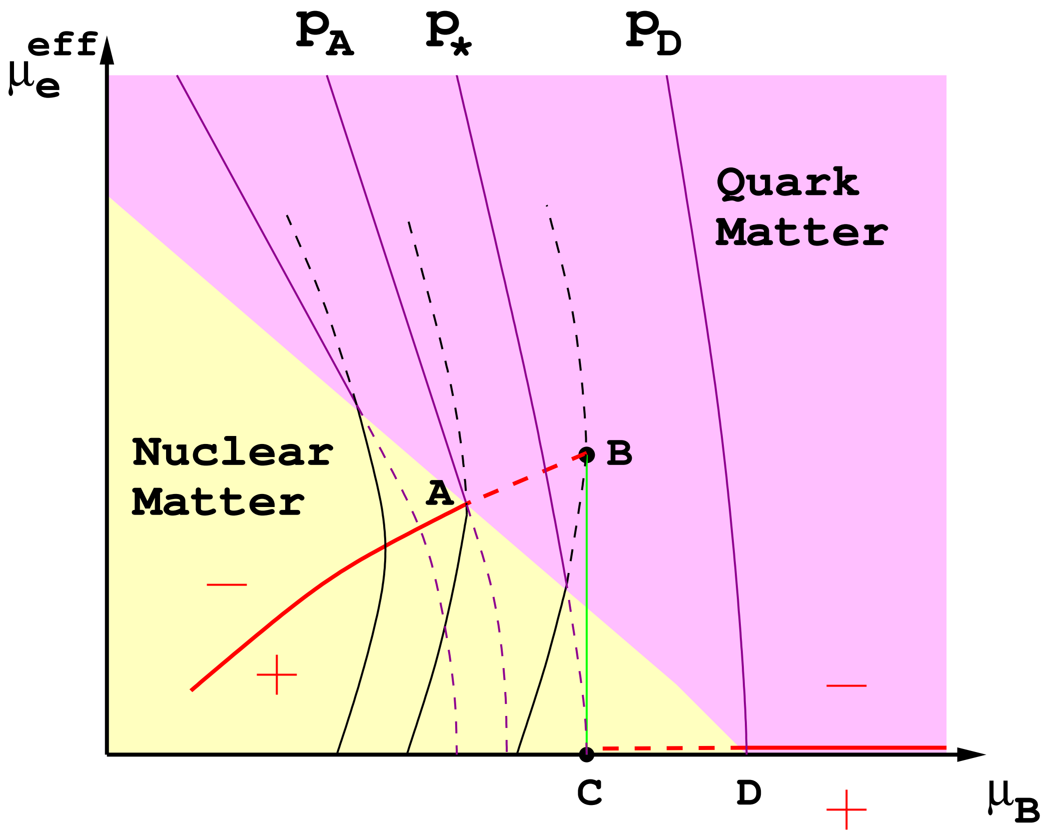

Figure 2: A schematic form of the μ_B-μ_e phase diagram for nuclear matter and CFL quark matter, ignoring electromagnetism. For an explanation see the text.

(magneta) region is where quark matter (QM) has higher pressure. Where they meet is the coexistence line. The medium solid lines labelled by values of the pressure are isobars. Below the coexistence line they are given by the NM equation of state, above it by the QM equation of state. he thick (red) lines are the neutrality lines. Each phase is negatively charged above its neutrality line and positively charged below it. Dotted lines show extensions onto the unfavored sheet (NM above the coexistence line, QM below it).

The electric charge density is

$$Q = -\left.\frac{\partial p}{\partial \mu_e}\right|_{\mu_B} \tag{5}$$

so the neutrality line goes through the right-most extremum of each isobar, since there the derivative of pressure with respect to μ_e is zero. For the CFL phase, the neutrality line is $\mu_e = 0$ [21].

Two possible paths from nuclear to CFL matter as a function of increasing μ are shown. In the absence of electromagnetism and surface tension, the favored option is to progress along the coexistence line from A to D, giving an overall neutral phase made of appropriate relative volumes of negatively charged CFL matter and positively charged nuclear matter.

If, on the other hand, Coulomb and surface energies are large, then the system remains on the nuclear neutrality line up to B, where there is a single interface between nuclear matter at B and CFL matter at C. This minimal interface, with its attendant charged boundary layers [22], occurs between phases with the same μ_e, $\mu = \mu_B = \mu_C$, and pressure P_*. The effective chemical potential μ_e^{eff} changes across the interface, though, as a result of the presence of the electric field. For more details see Ref. [22].

As yet, not much work has been done on signatures related to these features. The single interface creates a dramatic density discontinuity in the star: CFL quark matter at about four times nuclear density floats on nuclear matter at about twice nuclear density. This may affect the mass vs. radius relationship for neutron stars with quark matter cores. It may also have qualitative effects on the gravitational wave profile emitted during the inspiral and merger of two compact stars of this type. The mixed phase has distinctively short neutrino mean free paths, due to coherent scattering [23]. Also, the droplets form a crystal lattice that could pin vortices, leading to glitches.

4.2 Latent heat

If there is a first-order phase transition between nuclear matter and quark matter, then there is the possibility that heat could be released when the center of a gravitationally compressed neutron star converts to quark matter [24].

A proper treatment requires the construction of solutions to the TOV equations, and a comparison of the masses of the resultant stars. To get an idea of the energy scales involved, however, one can study the statistical mechanics of a simplified system consisting of a lump of nuclear matter, slowly being compressed in a piston. We assume the system remains at zero temperature throughout. Since the compression is slow, the system will remain in equilibrium at all times, which means it can be characterized by a chemical potential μ for quark number. This is true in spite of the fact that the system as a whole has a fixed quark number: since it is in equilibrium, any subsystem has the same intensive properties as the whole system, and the rest of the system acts as a particle reservoir for the subsystem.

Under slow compression of nuclear matter, the pressure p and chemical potential μ rise, until we reach a point in phase space where quark matter with the same chemical potential μ_* would have the same pressure p_*.

$$
\begin{aligned}
-p_* &= \frac{F_{NM}}{V_{NM}} = \frac{E_{NM}}{V_{NM}} - \mu_* \frac{N_{NM}}{V_{NM}} \\
-p_* &= \frac{F_{QM}}{V_{QM}} = \frac{E_{QM}}{V_{QM}} - \mu_* \frac{N_{QM}}{V_{QM}}
\end{aligned}
\tag{6}
$$

Under continued compression, the system contracts and μ and p remain constant for a while, as the nuclear matter is converted to the denser quark matter phase. When the conversion is complete, further compression causes μ and p to start rising again. From (6) we see that the quark matter phase has higher energy density, so the latent heat per unit volume is negative,

$$
\frac{E_{NM}}{V_{NM}} - \frac{E_{QM}}{V_{QM}} = \mu_* \Big(\frac{N_{NM}}{V_{NM}} - \frac{N_{QM}}{V_{QM}} \Big) \tag{7}
$$

The relevant quantity, however, is the amount of energy liberated per quark, since the quark number $N = N_{NM} = N_{QM}$ is constant, and the two phases have different volumes $V_{NM} > V_{QM}$. From (6) the latent heat per quark is

$$
\Delta E/N = \frac{E_{NM}}{V_{NM}} \frac{V_{NM}}{N} - \frac{E_{QM}}{V_{QM}} \frac{V_{QM}}{N} = p_* \Big(\frac{V_{QM}}{N} - \frac{V_{NM}}{N} \Big) \tag{8}
$$

which is again negative. The energy required comes from the mechanism that maintains the pressure by doing work on the piston. In the case of a compact star, this is the gravitational field, and the transition to quark matter is driven by the higher gravitational binding energy of the denser quark matter phase. The work done per quark is

$$
\Delta W/N = -p dV/N = \frac{p_*}{N}(V_{NM} - V_{QM}) \tag{9}
$$

which is just the negative of (8): the work done by the gravitational field provides exactly the energy needed to convert the nuclear matter to quark matter.

It is clear that as long as the microscopic processes that convert quark matter into nuclear matter occur on a much faster timescale than the compression, no energy will be liberated. It is possible for energy to be liberated if the compression to happens fast enough that the center goes out of equilibrium and becomes supercompressed, i.e. metastable.

4.3 Mixed phase vs sharp interface

If the surface tension σ_{QCD} and the electrostatic forces are ignored, then a mixed phase is favored over a sharp interface. [25, 26, 27]. If we treat $\sigma_{\rm QCD}$ as an independent parameter, we can estimate the surface and Coulomb energy cost of the mixed phase

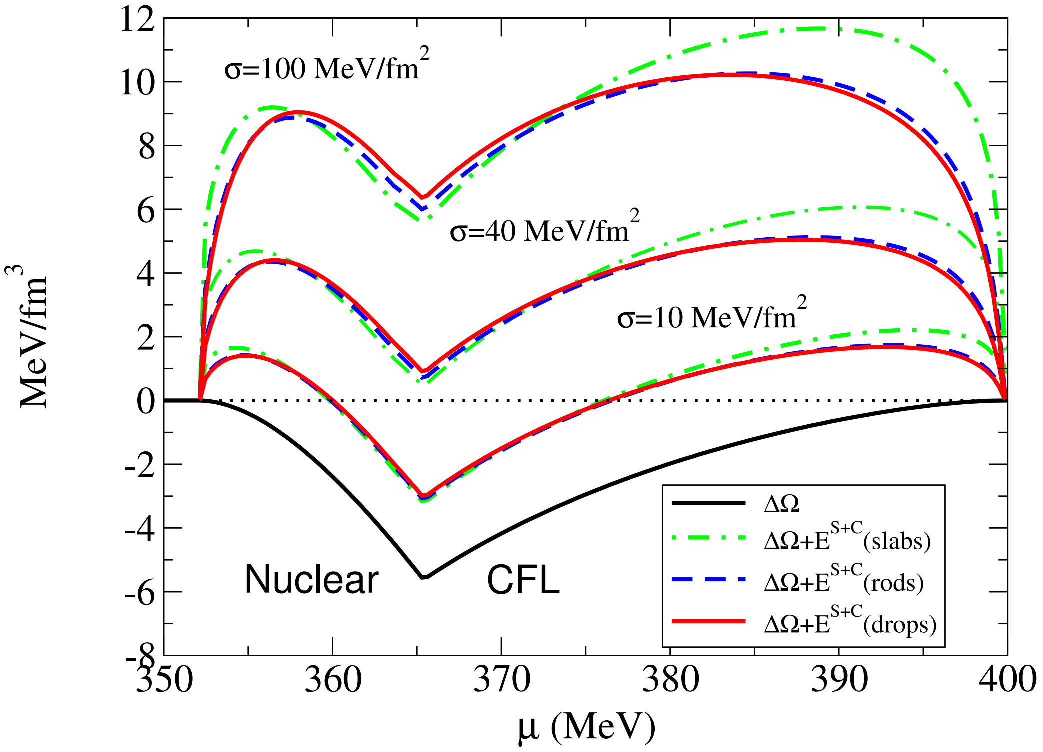

Figure 3: The free energy difference between the mixed phase and the homogeneous neutral nuclear and CFL phases. In the lowest curve, the surface and Coulomb energy costs of the mixed phase are neglected, do the mixed phase always has the lower free energy. Other curves include surface and Coulomb energy for different values of $\sigma_{\rm QCD}$ and different mixed phase geometry. As $\sigma_{\rm QCD}$ increases, the surface and Coulomb price paid by the mixed phase increases.

[27, 22]. In Fig. 3 we see how the competition between sharp interface and mixed phase depends on the $\sigma_{\rm QCD}$. The curves show the difference of free-energy between various kinds of mixed phase and the sharp interface (which occurs at $\mu = 365$ MeV, hence the kink there). For any value of $\sigma_{\rm QCD}$, the mixed phase progresses from drops to rods to slabs of CFL matter within nuclear matter to slabs to rods to drops of nuclear matter within CFL matter.

For any given $\sigma_{\rm QCD}$, the mixed phase has lower free energy than homogeneous neutral CFL or nuclear matter wherever one of the curves in Fig. 3 for that $\sigma_{\rm QCD}$ is negative. We see that much of the mixed phase will survive if $\sigma_{\rm QCD} \simeq 10~\mathrm{MeV/fm^2}$ while for $\sigma_{\rm QCD} \gtrsim 40~\mathrm{MeV/fm^2}$ the mixed phase is not favored for any μ. This means that if the QCD-scale surface tension $\sigma_{\rm QCD} \gtrsim 40~\mathrm{MeV/fm^2}$, the single sharp interface with its attendant boundary layers, described in previous sections, is free-energetically favored over the mixed phase.

5 Conclusions

The reawakening of interest in the color superconducting nature of quark matter has led us to appreciate that the QCD phase diagram is much richer than previously thought. I have outlined some of the structure in this paper. There has naturally been much speculation about observable consequences in compact star phenomenology. In spite of the limited nature of our observational knowledge of compact stars, there have been many suggestions, and these are discussed in the review articles [6] and in other contributions to these proceedings. Although we are still in the early stages of such phenomenology, we can take inspiration from high-temperature QCD, where great progress has been made in overcoming similar obstacles. At the same time as heavy-ion colliders map the high-temperature region of the QCD phase diagram, we hope that astrophysical observations and calculations will complement it by filling in details of the high-density region.

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