Methods and Issues in Beam–Beam Simulation

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We will review the recent developments of the beam-beam simulation using the method of particle-in-cell (PIC). The simulation results are compared with both experimental measurements and the analytical theory. Finally the numerical noise in the PIC simulation is discussed.

1. Introduction

Since the discovery of the beam-beam interaction four decades ago [1], the beam–beam effects in e^+e^- colliders has remained as one of the most important phenomena in the beam physics. The limits imposed by the beam-beam interaction are the ultimate limitation on the luminosity in the colliders.

Historically, the computer simulation has played an important role to study the beam-beam phenomenon[2]. Due to the complexity of the interaction between the colliding beams, many approximations, for instance strong-weak [3] or soft-Gaussian [4], have been introduced in order to simulate the interaction within a reasonable computing time. As a result of dramatic increase of the computer speed in the recent years, it has become feasible to use the PIC method for the beam-beam simulation. This kind of simulation is self-consistent because the electromagnetic field is obtained by solving the Poisson equation with the updated charge distribution during the collision of the beams. This method was first applied to round beams by Krishnagopal and Siemann [5] and later to flat beams by Krishnagopal [6]. Recently, we found that the region of the mesh can be much reduced if an inhomogeneous potential is assigned on the boundary [7]. The smaller region of mesh allows denser mesh and therefore increases the resolution of the Poisson solver. We have applied the method to simulate the beam-beam effect in the PEP-II [8]. In this paper, we will show a few highlights of the simulation result and the comparisons to experimental observation and analytical calculation.

2. Luminosity

To make a direct comparison between simulation and experimental observation, we have recorded the luminosity and beam currents of the PEP-II during a period of four hours on October 1, 2000. The measured and simulated luminosities as a function of time are shown in Figure 1. Duration of each measurement was three minutes.

Additional to the beam currents, the parameters used in the simulation are the design beam energy and emittance, β function at the interaction point (IP), damping time, and measured working tunes. There are no fitting parameters in the simulation.

The agreement between the simulation and measurement is within 10% in a large range of the beam currents. Since the longitudinal effects of the beam-beam interaction are not yet included in the simulations, three-dimensional simulation could reduce the simulated luminosity. For example, the hourglass effect should reduce the simulated luminosity by 12% given the bunch length $\sigma_z = 1.3\,\text{cm}$ and the vertical β function at the IP $\beta_y^* = 1.25\,\text{cm}$.

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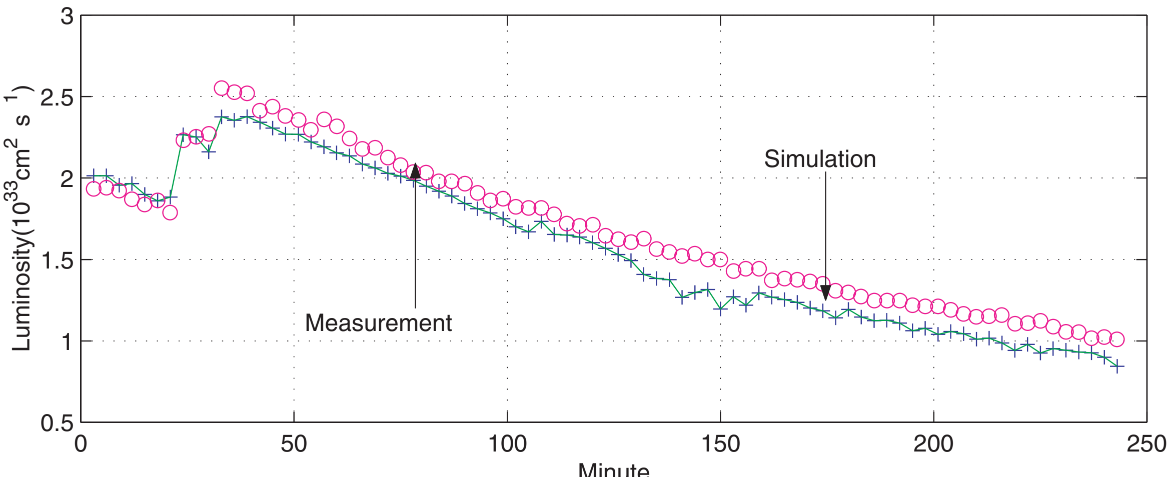

Figure 1: (color) Luminosity of PEP-II. The crosses represent measurement and the circles represent simulation. The number of bunches was 605.

3. Coherent Oscillation

Studying the power spectrum of colliding beams is a powerful way to investigate and understand the beam-beam interaction. Historically, in symmetric colliders where two beams are identical, the tune shift of the coherent π mode has provided many useful insights into the dynamics of the beam-beam interaction. It has been shown analytically that this tune shift is proportional to the beam-beam parameter ξ, namely $\delta\nu_\pi = \lambda\xi$ [9, 10, 11, 12]. The coefficient λ is between 1 and 2 depending on the beam distribution. For a self-consistent beam distribution [12],

$$\delta\nu_{x,\pi} = \Lambda\xi_x, \delta\nu_{y,\pi} = \Lambda(1-r)\xi_y \tag{1}$$

where $\Lambda = 1.330 - 0.370r + 0.279r^2$, $r = \sigma_y/(\sigma_x + \sigma_y)$, and σ_x and σ_y are the horizontal and vertical beam size respectively.

Experimentally, this relation has been observed in many different colliders [13, 14]. The results of measurements are consistent with the calculation based on the Vlasov theory. Here, we simulated the π shifts as a function of the beam-beam parameter, which is computed using the beam size at the equilibrium. The results are summarized in Figure 2.

The predicted linear relation based on Equation 1 is also plotted in the figure. One can see that the agreement between the theory and simulation is rather good even at very high beam–beam parameter.

4. Numerical Noise

Despite of these achievements, the resolution of the PIC simulation is still limited by the numerical noise generated by a finite number of macro particles and finite mesh size. The convergence of the numerical solution provides a practical way to check the effects of the numerical noise and to optimize the choice of mesh parameters and number of macro particles.

Here, we study a case of symmetric collider using the parameters of the High Energy Ring of the PEP-II. The beam intensity $N = 10^{11}$. First, to check the noise due to finite macro particles, we vary the number of macro particles: $N_b = 10^4, 10^5$, and 10^6 while fixing the mesh to 64×128 and five divisions per sigma in both horizontal and vertical plane. The result is shown in Figure 3a. It is clear that the equilibrium solution is convergent rather well when N_b is larger than 100,000.

Second, to check the noise due to finite mesh size, we make the mesh twice denser by changing the mesh from 64×128 to 128×128 with ten divisions per sigma in the horizontal plane while keeping $N_b = 10^5$. The result is shown in Fig. 3b. It is clear that in this case the denser mesh does not make any difference.

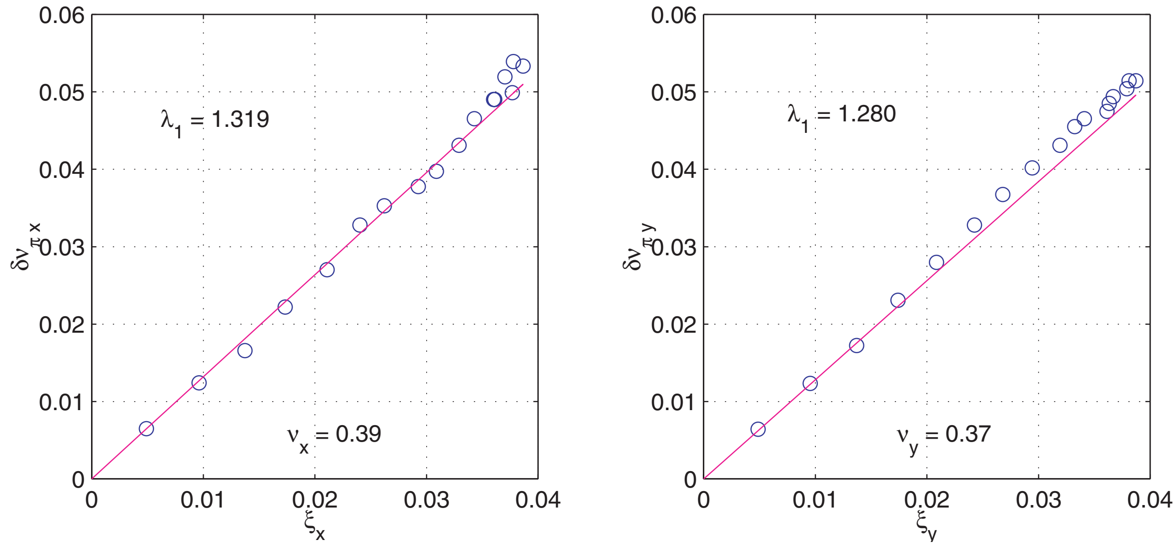

Figure 2: (color) The tune shift of coherent π mode as a function of the beam-beam parameter. The left plot is for the horizontal plane and right plot is for the vertical plane. The circles represent the simulated tune shifts. The solid lines represent $\delta\nu_\pi = \lambda\xi$ derived from the Vlasov theory.

Based on these results, we conclude that a reasonable choice of the mesh should be 64×128 with five divisions per sigma in the both planes and number of macro particle is 100,000. The average number of particles per cell is about twelve.

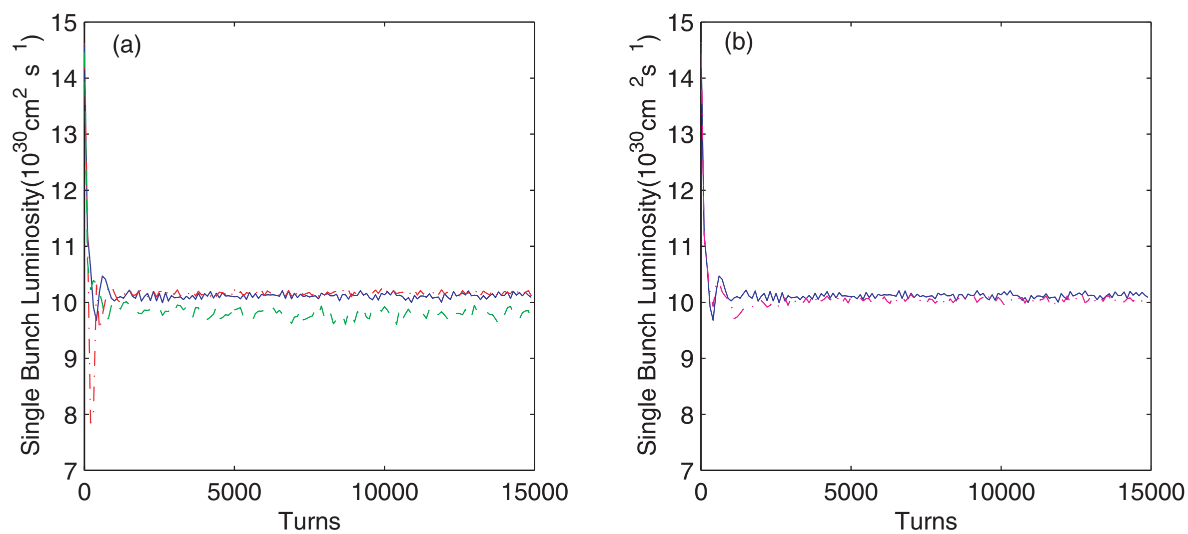

Figure 3: (color) (a) The dashed green curve is the case when $N_b = 10^4$, the solid blue curve is for 10^5, and the dash-dotted red curve is for 10^6. (b) The solid blue curve is for the mesh 64×128 and the dash-dotted magenta curve is for the mesh 128×128.

Since the initial noise presents a deviation from a Gaussian distribution, different noise should evolve different at the beginning as shown in Figure 3. As the solution approaches its equilibrium, its initial memory is washed away completely by the damping and quantum excitation. This result indicates that the convergence of the solution is much harder to achieve in a hatron collider than electron one.

5. Conclusion

The agreement of luminosity between the simulation and measurement is surprising and remarkable given the simplicity of the two-dimensional model. The success is largely because the operating tunes of the PEP-II are well optimized and many resonances including the synch-betatron resonances are carefully avoided. In general, the three-dimensional effects such as the hourglass effects should be included in the beam-beam simulation. Unfortunately, in order to achieve the required convergence, the three-dimensional simulations have to be performed on parallel supercomputers.

It is clear from these examples in the paper that many progresses in the beam-beam simulation have been made in recent years. These improvements allow us to attempt to make some reliable predictions of how to choose operating parameters of the collders. Of course, a success of prediction remains to be demonstrated.

6. ACKNOWLEDGMENTS

I would like to thank A. Chao, S. Tzenov, and T. Tajima for their collaboration. I would also like to thank W. Kozanecki, M. Minty, J. Seeman, R. Warnock, and U. Wienands for many helpful discussions.

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