Limit on the vertical beam size at linear colliders due to the crab crossing

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For suppression of multi-bunch instability and removal of backgrounds in linear e^+e^-, $\gamma\gamma$, γe colliders a crab-crossing scheme of collisions is foreseen. Radiation of electrons in the solenoid field of the detector put a limit on the minimum vertical beam size.

I. INTRODUCTION.

In an ideal case the vertical beam size at the interaction point (IP) at linear colliders (LC) is

$$\sigma_y = \sqrt{\frac{\epsilon_{ny}\beta_y}{\gamma}}, \tag{1}$$

where ϵ_{ny} is the normalize emittance, β_y the vertical beta-function which optimally is equal about the r.m.s. bunch length σ_z, $\gamma = E/mc^2$.

There are several limitations. The first one is due to radiation in the quads (Oide effect) [1]. In the limit of $\epsilon_n \to 0$ electrons travel near the axis, do not radiate and $\sigma_y \to 0$.

The second effect is due to appearance of beam-beam instability in e^+e^- or e^-e^- collisions. It occurs for flat beams at [2, 3]

$$D_y = \frac{2Nr_e\sigma_z}{\gamma\sigma_x\sigma_y} \sim 10\text{–}20. \tag{2}$$

To make smaller σ_y one has to decrease σ_z as well. For the e^+e^- linear collider CLIC [4] with $2E_0 = 3$–5 TeV beams with $\sigma_y = 0.7$–1 nm are considered.

At photon colliders [5] the minimum vertical beam size is about b/γ, where the distance between the interaction and conversion points b should be larger than the conversion length. One can show [6] that this requirements leads to minimum vertical size of the photon beam (electron beam size should be somewhat smaller)

$$\sigma_{\gamma,min} \sim \frac{b}{\gamma} \sim \frac{16r_e}{\alpha^2 x\xi^2}\left(\frac{\sigma_0}{\sigma_c}\right), \tag{3}$$

where $\xi^2 \sim 0.3$ is the parameter characterizing nonlinear effects in Compton scattering, σ_c is the Compton cross section and $\sigma_0 = \pi r_e^2$. For the typical case $x = 4E_0\omega_0/m^2c^4 = 4.8$, $\sigma_c/\sigma_0 = 0.7$, $\xi^2 = 0.3$, $\alpha = 1/137$, we get $\sigma_{\gamma,min} \sim 0.8$ nm.

So, one can see that at multi-TeV e^+e^- colliders and photon colliders electron beams with σ_y below 1 nm (10^{-7} cm) are under consideration.

Below we consider additional effect limiting σ_y caused by radiation in the solenoid detector field due to crab-crossing [7] which is foreseen in almost all LC for suppression of the multi-bunch instability and removal of the disrupted beams.

II. RADIATION IN THE SOLENOID FIELD

In the crab-crossing scheme electron beams collide at the angle $\theta_c \sim 20$–30 mrad. For preservation of the luminosity beams are tilted in respect to the direction of the beam motion by the angle $\theta_c/2$. As a result, electrons cross the transverse field $B = 0.5B_s\theta_c$. Due to emission of photons electrons loss energy and come to the IP with some spread in the vertical direction.

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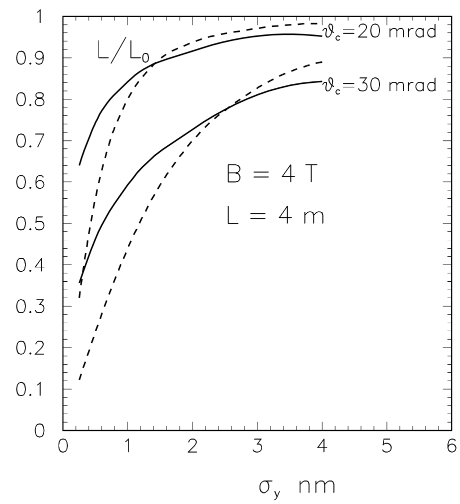

FIG. 1: The ratio of the e^+e^- luminosity with account of radiation in the detector field to the geometric luminosity as a function of the vertical beam size. Solid lines are results of exact simulation, dashed curves are obtained just by summing quadratically r.m.s spread given by (8) with initial beam size σ_y.

This effect was taken into account in [8] and formula by J. Irwin for r.m.s size is given, but as it is presented without derivation therefore it worth to check it. Moreover, the number of emitted photons by each electrons is small, there are large fluctuation and therefore such description may by not valid.

The number of photons emitted by the electron in the transverse magnetic field B on the length L is [9]

$$N_\gamma = \frac{5\alpha eBL}{2\sqrt{3}mc^2} \sim 0.01\gamma\theta_d = 0.005\frac{eB_s\theta_c L}{mc^2}, \tag{4}$$

where $\alpha = 1/137$, $\theta_d = eBL/E_0$ is the deflection angle. For example, $B_s = 4$ T, $\theta_c = 30$ mrad, $L = 4$ m $\Rightarrow$ $N_\gamma \sim 1.4$ (does not depend on the energy).

If the electron loses the energy ΔE at the distance z from the interaction point, then its vertical deflection from the trajectory of the electron with the initial energy

$$\Delta y = \frac{z^2}{2\rho}\frac{\Delta E}{E_0}, \tag{5}$$

where $\rho = E/eB$ is the radius of the curvature. The largest deflections have electrons emitted far from the IP, the number of such electrons is less than one. So, the distribution on y has a long tail due to fluctuations in photon energies and distances and because the number of photons which cause large deflections is less than one. This means that the description of this effect by some additional r.m.s. spread on y is not adequate. Similar situation was for Oide effect [1].

Nevertheless, let us derive $\sigma_{y,r}$ due to emission of photons in the solenoid detector field. First of all, it should be noted that the displacement of the electron in the case of emission of several photons is just the sum of displacements due to emission of these photons (valid for $E_\gamma \ll E_0$), there is no interference effects. Therefore we can write the r.m.s spread of electrons at the IP as

$$\sigma_{y,r}^2 = \langle \int y^2\, dp \rangle = \left\langle \left(\frac{z^2}{2E_0\rho}\right)^2 \right\rangle \int E_\gamma^2\, dp. \tag{6}$$

The last integral per unit of length [10]

$$N_\gamma \langle E_\gamma^2 \rangle = \frac{55\hbar c e^2 \gamma^7}{24\sqrt{3}\,\rho^3}, \tag{7}$$

$\langle z^4 \rangle = L^4/5$. As result we get

$$\sigma_{y,r}^2 = \frac{55 r_e^2}{480\sqrt{3}\,\alpha}\left(\frac{eB_s\theta_c L}{2mc^2}\right)^5. \tag{8}$$

This formula agrees with [8]. For example, for $B_s = 4$ T, $L = 4$ m, we get $\sigma_{y,r} = 0.74$ nm and 2 nm for $\theta_c = 20$ and 30 mrad, respectively.

Fig.1 shows the ratio of the e^+e^- luminosity with account of radiation in the detector field to the geometric luminosity as a function of the vertical beam size. Solid curves are obtained by full simulation, dashed curves analytically using (8). It is taken into account that electron and positron are deflected (due to the radiation) in opposite directions. One can see that at small σ_y the difference between the simulation and analytical calculation using (8) is large. It is clear that for very small σ_y the luminosity is larger than gives r.m.s. approach because there are particles which cross the detector without photon emission.

I do not consider here the solenoid fringe field which can give comparable effect to the radiation of photons (or somewhat smaller if the the entrance point coincides with the magnet axis). Fringe field compensates partially the dispersion function at the IP (helps if photons are emitted before the solenoid).

Let me mention also more one fact. The deflection angle in the solenoid detector field is much larger than the vertical angular spread of the beams at the IP. Therefore the magnetic system in front of the detector (plus the solenoid fringe field) should provide the same position for both beams, zero dispersion at the IP and zero vertical collision angle.

In conclusion. In the case of the crab crossing scheme of beam collisions the radiation of particles in the solenoid detector field leads to the additional vertical beam size which is independent of the energy but strongly depends on the magnetic field and crab-crossing angle. For practical cases the limitation occurs at $\sigma_y \sim \mathcal{O}(1)$ nm. This effect is most important for multi-TeV linear colliders and photon colliders.

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