

# Heavy-Quark Parton Distribution Functions and Their Uncertainties

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We investigate the uncertainties of the heavy-quark parton distribution functions in the variable flavor number scheme. Because the charm- and bottom-quark parton distribution functions (PDFs) are constructed predominantly from the gluon PDF, it is a common practice to assume that the heavy-quark and gluon uncertainties are the same. We show that this approximation is a reasonable first guess, but it is better for bottom quarks than charm quarks. We calculate the PDF uncertainty for  $t$ -channel single-top-quark production using the Hessian matrix method, and predict a cross section of  $2.12^{+0.32}_{-0.29}$  pb at run II of the Tevatron.

As a new run of the Fermilab Tevatron begins, there is a considerable interest in measuring fundamental parameters of the Standard Model and in looking for new particles. For some of these measurements the dominant uncertainty will come from parton distribution functions (PDFs). In particular, measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{tb}$  depend on an accurate prediction of the cross section for single-top-quark production [1] and will be limited by uncertainties in the  $b$ -quark PDF [2]. In a large  $\tan\beta$  model of supersymmetry, Higgs production from a  $b\bar{b}$  initial state is the dominant production mechanism [3].

In order to understand uncertainties of the heavy-quark parton distribution functions, it is necessary to remember where they arise. In the conventional QCD framework, heavy quarks are produced perturbatively from gluons splitting into heavy-quark pairs at energies above the heavy-quark mass threshold (we do not consider “intrinsic” heavy quarks [4] here). The heavy-quark PDF  $Q(x, \mu^2)$  is a formal object, which can be used reliably when the typical momentum scale  $\mu$  of the hard scattering is much larger than the mass of the heavy quark  $m_Q$ . This PDF resums nearly collinear singularities  $\alpha_s^n \ln^n(\mu^2/m_Q^2)/n!$  associated with propagation of the initial-state heavy quarks. Unlike the light-quark PDFs, the distributions for charm ( $c$ ) and bottom quarks ( $b$ ) are not independent functions that are determined by fitting to the hadronic data; instead, they are generated from the PDFs of light partons in the process of Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution.

The relationship between the PDFs for heavy and light partons is especially simple near the mass threshold of heavy quarks ( $\mu \sim m_Q$ ), where the DGLAP equations can be solved in the leading-logarithm approximation. Given the initial condition  $Q(x, \mu^2) = 0$  at  $\mu = m_Q$ , and neglecting gluon bremsstrahlung off heavy-quark lines and the scale dependence of the gluon PDF and  $\alpha_s$ , the leading-order solution can be found as [5, 6, 7]

$$Q(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_Q^2}\right) \int_x^1 \frac{dz}{z} P_{Qg}\left(\frac{x}{z}\right) g(z, \mu^2), \quad (1)$$

where the function  $P_{Qg}(y) \equiv [y^2 + (1-y)^2]/2$  describes the splitting of a gluon into a heavy-quark pair.

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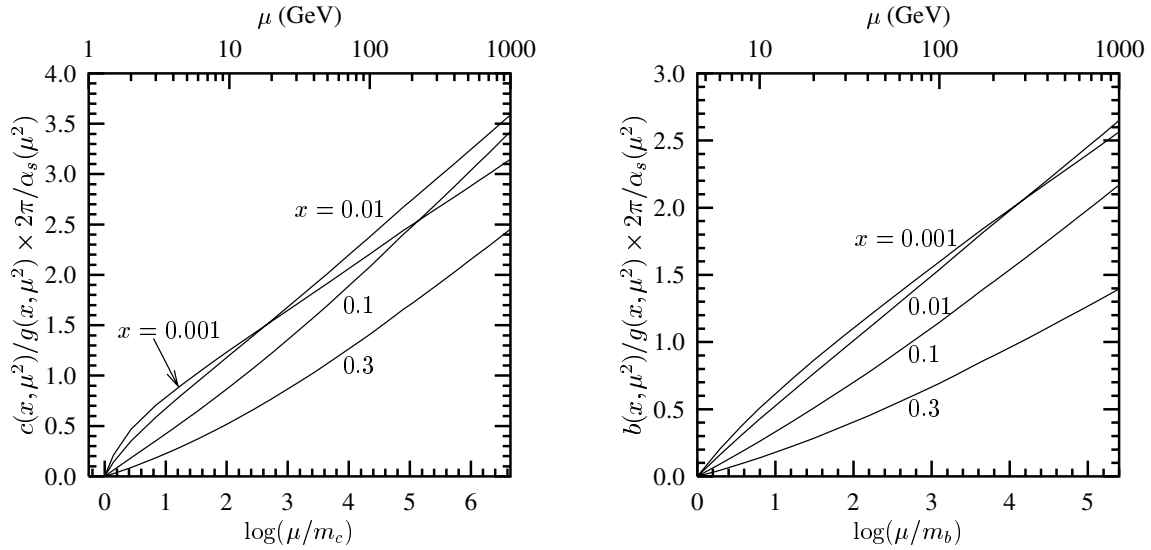


Figure 1: The ratio of the  $c$  and  $b$  distribution functions to the gluon distribution function, times  $2\pi/\alpha_s(\mu^2)$ , versus  $\ln(\mu/m_Q)$  for various fixed values of  $x$ . The curves are approximately linear, while the slope of the curves saturates at about 0.5 at small  $x$ , in agreement with the approximation of Eq. (2).

Since  $g(x)$  grows as  $x^{-n}$  ( $n \approx 1.4-1.5$ ), at sufficiently small  $x$  Eq. (1) may be rewritten as

$$\frac{Q(x, \mu^2)}{g(x, \mu^2)} \frac{2\pi}{\alpha_s(\mu^2)} \approx \left( \frac{1}{n} - \frac{2}{n+1} + \frac{2}{n+2} \right) \ln \left( \frac{\mu}{m_Q} \right) \approx 0.5 \ln \left( \frac{\mu}{m_Q} \right). \quad (2)$$

The approximation (2) works remarkably well over a wide range of  $x$  and  $\mu$ . In Fig. 1 we present an updated version of Fig. 5 of Ref. [1]. The ratios  $Q(x, \mu^2)/g(x, \mu^2) \times 2\pi/\alpha_s(\mu^2)$  are shown as functions of  $\mu$  for various fixed values of  $x$ , using the CTEQ5M1 parton distribution functions [8]. The dependence on  $\ln(\mu/m_Q)$  is approximately linear indicating that, even at next-to-leading order (NLO),  $Q(x, \mu^2) \propto [\alpha_s(\mu^2)/2\pi] \ln(\mu^2/m_Q^2) g(x, \mu^2)$ . Further, the constant of proportionality saturates at  $\sim 0.5$  for charm quarks when  $x \lesssim 0.1$ , and for bottom quarks when  $x \lesssim 0.05$ .

If the heavy-quarks PDFs are directly proportional to the gluon PDF, it is reasonable to expect that their uncertainties are approximately the same:

$$\frac{\delta Q(x, \mu^2)}{Q(x, \mu^2)} \approx \kappa_Q \frac{\delta g(x, \mu^2)}{g(x, \mu^2)}, \quad (3)$$

where  $\kappa_Q \approx 1$ . We test this relation by estimating the PDF uncertainties with the Hessian matrix method proposed J. Pumplin, *et al.* [9]. This paper contains 16 pairs of PDF sets, where each pair corresponds to varying one independent parameter  $z_i$  in the PDF fit such that the  $\chi^2$  of the fit changes by  $t^2 = (5)^2 = 25$ . We define the maximum positive and negative errors on an operator  $O$  by [9, 10]

$$\delta O_+ = \frac{T}{t} \sqrt{\sum_{i=1}^{16} \left( \max[O(z_i^0 + t) - O(z_i^0), O(z_i^0 - t) - O(z_i^0), 0] \right)^2}, \quad (4)$$

$$\delta O_- = \frac{T}{t} \sqrt{\sum_{i=1}^{16} \left( \max[O(z_i^0) - O(z_i^0 + t), O(z_i^0) - O(z_i^0 - t), 0] \right)^2}, \quad (5)$$

where the “tolerance”  $T$  is a scaling parameter that determines the overall range of allowed variation of  $\chi^2$ . In this study we use  $T = 10$ .

Figure 2 shows the dependence of the ratios  $\kappa_c$  and  $\kappa_b$  on  $x$  and  $\mu$ . We see that these ratios are quite close to unity for  $x < 0.05$ , but climb to about 2 at larger  $x$ . The approximation (3) holds better for  $b$  quarks:  $\kappa_b$  is less than 1.4 at  $x \leq 0.1$  and the whole range of  $\mu$ , while  $\kappa_c$  can reach

up to 1.75 in this region. The values  $\kappa_Q$  are close to 1 as  $x \rightarrow 0$ , which supports the validity of Eq. (2) in the small- $x$  region.

Figure 3 shows the relative uncertainties  $\delta c(x, \mu^2)/c(x, \mu^2)$  and  $\delta b(x, \mu^2)/b(x, \mu^2)$  as functions of  $x$  at various values of  $\mu$ . Both uncertainties have minima around  $x \approx 0.01$ , where the PDFs are best constrained by the existing data. At small  $x$ , the uncertainties grow because the small- $x$  region is covered only by DIS experiments, which do not constrain well the gluon PDF. At  $x \gtrsim 0.1$ , the heavy-quark PDFs become negligible compared to the valence quark PDFs, so that  $c(x, \mu^2)$  and  $b(x, \mu^2)$  are practically unconstrained at  $x \gtrsim 0.3$ .

Single-top-quark production via  $t$ -channel  $W$ -exchange probes the  $b$ -quark PDF at  $\mu \sim m_t \simeq 175$  GeV. The range of  $x$  probed at the run II of the Tevatron for accepted events is 0.06–0.5, with the bulk of the cross section coming from the region around  $x \simeq 175/2000 \simeq 0.09$ . Using this value of  $x$  and Figure 3b, we can estimate the uncertainty of the total cross section at run II to be  $\sim \pm 15\%$ . We have explicitly re-calculated the NLO cross section in [1] and its uncertainty with the method described above and  $T = 10$ . The result is  $2.12^{+0.32}_{-0.29}$  pb, or  $^{+15\%}_{-14\%}$ , in excellent agreement with Figure 3b. For this choice of  $T$ , and using CTEQ5M1 PDFs, the PDF uncertainties will be larger than all other theoretical or experimental errors once  $2 \text{ fb}^{-1}$  of integrated luminosity is accumulated. One positive note is that Figure 3b predicts an uncertainty of around  $\pm 7\%$  at the LHC, which will be comparable to experimental systematics.

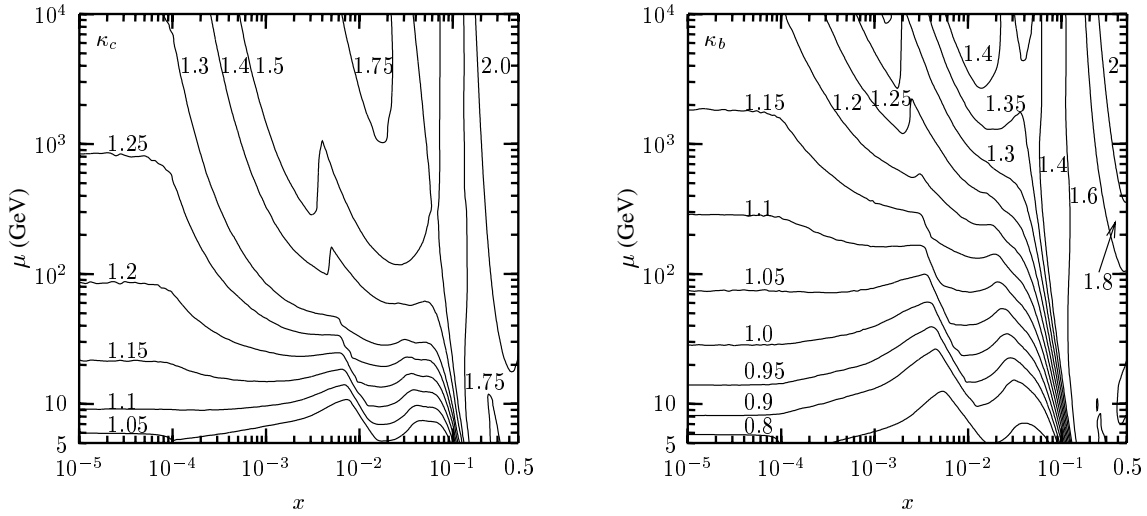


Figure 2: The ratios  $\kappa_Q \equiv (\delta Q/Q)/(\delta g/g)$  for  $c$  and  $b$  quarks as a function of  $x$  and  $\mu$ .

The approximation that the heavy-quark uncertainties are the same as the gluon uncertainty is a good guess for smaller  $x$  and  $\mu$ . We calculate these uncertainties using a “tolerance”  $T = 10$  in a Hessian matrix method [9]. We show that the heavy-quark uncertainties strongly depend on  $x$  and relatively weakly on the scale  $\mu$ . We calculate the PDF uncertainty for  $t$ -channel single-top-quark production, and predict a cross section of  $2.12^{+0.32}_{-0.29}$  pb at run II of the Tevatron.

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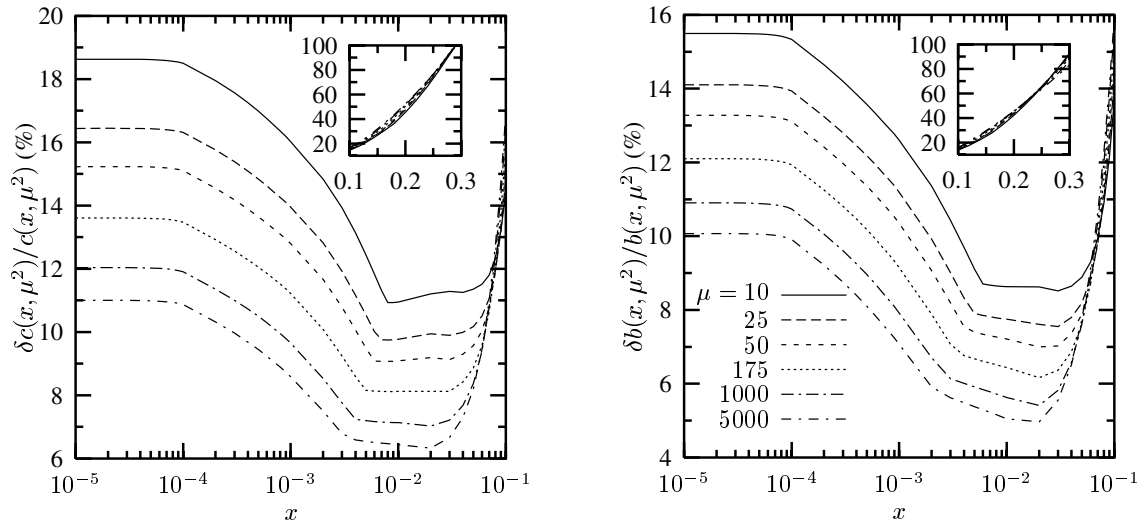


Figure 3: The uncertainties of the  $c$  and  $b$  distribution functions as a function of  $x$  for various values of scale  $\mu$ .

[10] (2001), hep-ph/0110378.