High order corrections for top quark and jet production at the Tevatron

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An overview of the theoretical status of the top quark and single-jet inclusive production cross sections at the Tevatron is presented. NLO results as well as NNLO-NNLL and higher order threshold expressions derived from resummation calculations are discussed. Numerical results are presented for top quark production at the Tevatron, and it is shown that higher orders contribute sizable corrections to the total cross section and differential distributions, and they also dramatically reduce the factorization scale dependence of the cross section. For jet production, the scale dependence is also reduced but the NNLO threshold corrections are not very big.

1. Introduction

The calculation of cross sections in perturbative QCD relies on the factorization of long-distance and short-distance physics. The long-distance physics is described by parton distributions, ϕ, which are determined from experiment, while the short-distance physics is embodied in hard-scattering factors, which are perturbatively calculable. The cross section for $t\bar{t}$ production in hadronic collisions can then be written as $\sigma_{h_1 h_2 \to t\bar{t}} = \sum_f \phi_{f_i/h_1} \otimes \phi_{f_j/h_2} \otimes \hat{\sigma}_{f_i f_j \to t\bar{t}}$. A similar equation holds for jet production.

Near threshold for the production of the $t\bar{t}$ pair (or jet) there is restricted phase space for real gluon emission. Thus, there is an incomplete cancellation of infrared divergences between real and virtual graphs which results in large logarithms. At nth order in α_s, they are of the form $[(\ln^m(s_4/m^2))/s_4]_+, \quad m \le 2n-1$, with $s_4 \equiv s + t_1 + u_1$. Note that $s_4 \to 0$ at threshold. Here s, t_1, u_1 are the usual Mandelstam invariants. These logarithms can be resummed to all orders in α_s by factorizing soft gluons from the hard scattering [1, 2].

2. Top production

At the Born level, the partonic channel $q\bar{q} \to t\bar{t}$ is the dominant production channel at the Tevatron; it contributes over 90% of the $t\bar{t}$ Born cross section. The partonic channel $gg \to t\bar{t}$ contributes the remainder. At NLO we have contributions from real gluon emission diagrams and from one-loop virtual diagrams. We find sizable NLO corrections for the $q\bar{q}$ channel and relatively large corrections in the gg channel.

The resummed heavy quark cross section in moment space, defined as $\hat{\sigma}(N) = \int (ds_4/s) e^{-Ns_4/s} \hat{\sigma}(s_4)$, with N the moment variable, is derived by refactorizing the cross section into hard H and soft S functions that describe the hard scattering and soft gluon emission, respectively. We can then write the resummed cross section at NLL (next-to-leading logarithmic) or higher accuracy as

$$\hat{\sigma}_{f_a f_b \to t\bar{t}}(N) = \exp\left[E^{(f_a)}(N) + E^{(f_b)}(N)\right] \mathrm{Tr} \left\{ H \exp\left[\int_m^{m/N} \frac{d\mu}{\mu} \Gamma_S^\dagger\right] \tilde{S}\left(\alpha_s(m^2/N^2)\right) \exp\left[\int_m^{m/N} \frac{d\mu}{\mu} \Gamma_S\right] \right\},$$

where the incoming parton N-dependence is in $E^{(f_i)}(N)$, the process-dependent functions H and S are matrices in the space of color exchanges where the trace is taken, and Γ_S is the soft anomalous
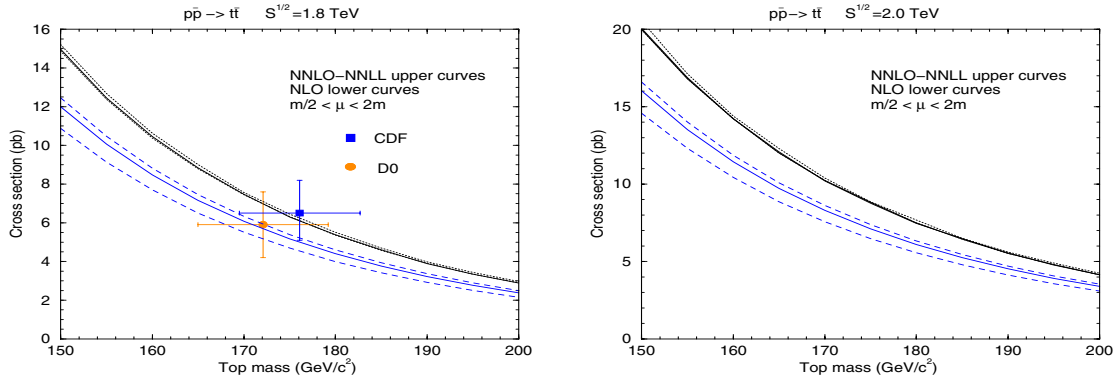
Figure 1: The total cross section for top quark production at the Tevatron with $\sqrt{S} = 1.8$ and 2.0 TeV.

Table I Top cross section at the Tevatron

$p\bar{p} \to t\bar{t}$	$\sqrt{S} = 1.8$ TeV		$\sqrt{S} = 2.0$ TeV	
$\mu = \mu_F = \mu_R$	NLO	NNLO	NLO	NNLO
$\mu = m/2$	5.4	6.4	7.4	8.9
$\mu = m$	5.2	6.3	7.1	8.8
$\mu = 2m$	4.7	6.3	6.5	8.8

dimension matrix [1, 2]. Note that the plus distributions $[(\ln^{2n-1}(s_4/m^2))/s_4]_+$ transform in moment space into $\ln^{2n} N$.

A prescription is needed to invert the moment-space resummed cross section back to momentum space. Alternatively we can expand the resummed cross section order by order in α_s and match to NLO to obtain NNLO-NNLL (next-to-next-to-leading order and next-to-next-to-leading logarithmic) and higher-order corrections [3, 4], thus avoiding prescription dependence and unphysical terms [3]. The problem with unphysical terms can be easily seen by studying the expansion at NLO. At NLO, the soft plus virtual $(S + V)$ corrections are $\hat{\sigma}_{q\bar{q}}^{(1)}(s_4) = \sigma^B(\alpha_s/\pi)\{c_1\delta(s_4) + c_2[1/s_4]_+ + c_3[(\ln(s_4/m^2))/s_4]_+\}$. This result comes from inverting the moment space expression $\hat{\sigma}_{q\bar{q}}^{(1)}(N) = \sigma^B(\alpha_s/\pi)\{c_1 - c_2\ln\tilde{N} + (c_3/2)(\ln^2\tilde{N} + \zeta_2)\}$, with $\tilde{N} = Ne^{\gamma_E}$. If we keep only NLL accuracy (as is the accuracy of the resummed cross section) in our results, i.e. only the $\ln^2 N$ and $\ln N$ terms, the inversion produces the following unphysical subleading terms: $\sigma^B(\alpha_s/\pi)[-(c_3/2)\gamma_E^2 + c_2\gamma_E - (\zeta_2/2)c_3]\delta(s_4)$. With $\sqrt{S} = 1.8$ TeV and $m = \mu = 175$ GeV/c^2, the full NLO $S + V$ corrections are 1.26 pb. If we keep only NLL accuracy they are 1.31 pb. If we keep NLL accuracy but also include the unphysical subleading terms above, we find 0.39 pb, a result very far from the full $S + V$ corrections. Thus at NLO, NNLO, and higher orders, we must not include unphysical subleading terms or use prescriptions that include such terms.

After expanding the resummed cross section in the $q\bar{q}$ channel in the $\overline{\text{MS}}$ scheme to two-loops, we find that the NLO threshold corrections agree with the exact NLO corrections in Ref. [5], while the new NNLO-NNLL corrections take the form

$$s^2\frac{d^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}^{(2)}(s_4, m^2, t_1, u_1)}{dt_1\,du_1} = \sigma_{q\bar{q}\to t\bar{t}}^B\left(\frac{\alpha_s(\mu_R^2)}{\pi}\right)^2\left\{8C_F^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + C_F\left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+\right.$$

$$\times\left[-\beta_0 + 12\left(\text{Re}\Gamma_{22}'^{q\bar{q}} - C_F + C_F\ln\left(\frac{sm^2}{t_1u_1}\right) - C_F\ln\left(\frac{\mu_F^2}{m^2}\right)\right)\right] + C(s_4)\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+\right\} + \mathcal{O}\left(\left[\frac{1}{s_4}\right]_+\right),$$

where $C(s_4)$ is a function that includes matching terms from the exact NLO calculation [3, 4]. Results have been derived through N^{4}LO in [3]. Analogous results have been derived for the gg channel [3, 4]. Note that in addition we have derived all NNLO scale-dependent terms for both channels [4].

In Table I and Fig. 1 we show values for the NLO and NNLO-NNLL $t\bar{t}$ cross section at the Tevatron for Run I and Run II. We find a sizable increase of the cross section, a dramatic decrease of the
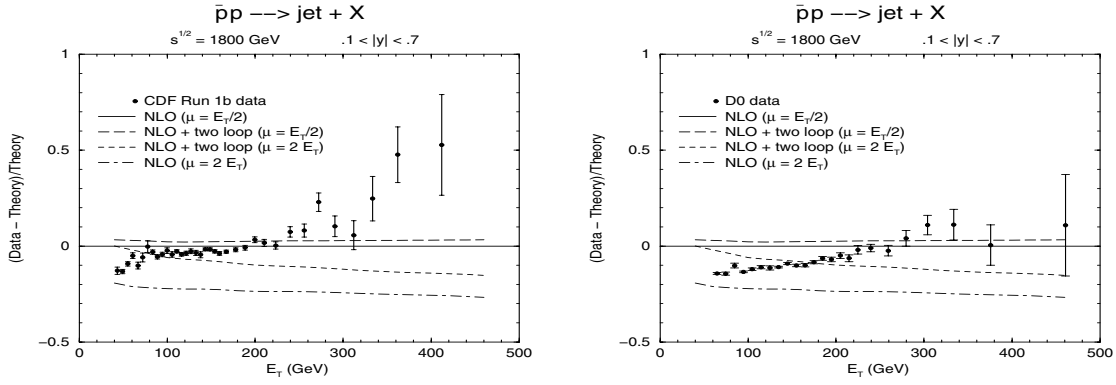
Figure 2: Jet production cross section at the Tevatron at Run I ($\sqrt{S}$ = 1.8 TeV) versus CDF and D0 data.

scale dependence μ, and good agreement with experiment, when we add the NNLO threshold corrections.

We have also investigated the uncertainties due to subleading logarithms (beyond NNLL accuracy) [3]. Including them as the largest error source in the calulation, we find for m = 175 GeV/c^2,

$$\sigma_{p\bar{p}\to t\bar{t}}(1.8 \text{ TeV}) = 6.3^{+0.1}_{-0.4} \text{ pb}; \qquad \sigma_{p\bar{p}\to t\bar{t}}(2.0 \text{ TeV}) = 8.8^{+0.1}_{-0.5} \text{ pb}.$$

The above results are all for single-particle-inclusive (1PI) kinematics. In pair-inclusive (PIM) kinematics the NNLO-NNLL corrections are smaller [4]. If we take the average of the 1PI and PIM results, we find

$$\sigma^{av}_{p\bar{p}\to t\bar{t}}(1.8 \text{ TeV}) = 5.8 \pm 0.4 \pm 0.1 \text{ pb}; \qquad \sigma^{av}_{p\bar{p}\to t\bar{t}}(2.0 \text{ TeV}) = 8.0 \pm 0.6 \pm 0.1 \text{ pb},$$

where here the first error is due to the kinematics uncertainty and the second due to the scale dependence.

Top transverse momentum distributions [6] at NNLO-NNLL are given in [3, 7]. We also note that similar methods can be used for single-top production (including FCNC effects [8]).

3. Jet production

The invariant single-jet inclusive cross section involves contributions from several parton-parton scattering subprocesses at lowest order: $q_j\bar{q}_j \to q_j\bar{q}_j$, $q_j\bar{q}_j \to q_k\bar{q}_k$, $q_j\bar{q}_k \to q_j\bar{q}_k$, $q_jq_j \to q_jq_j$, $q_jq_k \to q_jq_k$, $q\bar{q} \to gg$, $gg \to q\bar{q}$, $qg \to qg$, and $gg \to gg$.

We resum QCD corrections in high-E_T jet production to NLL accuracy [2] and expand the resummed cross section to NNLO [9]. There are additional complications relative to $t\bar{t}$ production because of the final-state jets.

We have calculated the NNLO-NLL corrections for all partonic subprocesses. For example the NNLO threshold corrections for $q_j\bar{q}_k \to q_j\bar{q}_k$ are

$$E_J\frac{d^3\hat{\sigma}^{(2)}_{q_j\bar{q}_k\to q_j\bar{q}_k}}{d^3p_J} = \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^B_{q_j\bar{q}_k\to q_j\bar{q}_k} \left\{ 2C_F^2 \left[\frac{\ln^3(s_4/p_T^2)}{s_4}\right]_+ + 3C_F \left[\frac{\ln^2(s_4/p_T^2)}{s_4}\right]_+ \right.$$
$$\times \left[-2C_F \ln\left(\frac{\mu_F^2}{p_T^2}\right) - \frac{3}{2}C_F - \frac{(N_c^2+1)}{N_c}\ln\left(\frac{-t}{s}\right) - \frac{(N_c^2-5)}{N_c}\ln\left(\frac{-u}{s}\right) - \frac{\beta_0}{12} \right] \right\} + \mathcal{O}\left(\left[\frac{\ln(s_4/p_T^2)}{s_4}\right]_+\right),$$

with p_T the jet's transverse momentum. Numerical results for jet production at the Tevatron are given in Fig. 2. The NNLO threshold corrections decrease the scale dependence, but they do not provide a sizable increase of the cross section, especially for smaller values of the scale.

4. Conclusions

Threshold resummation for heavy quark and jet production is a very powerful formalism and it allows us to derive high-order corrections to the cross sections for these processes. The resummed cross section has been expanded analytically through N^4LO and care has been taken to exclude unphysical subleading terms.

The NNLO-NNLL (and higher-order) corrections provide new analytical and numerical predictions for the total cross section and differential distributions for both top quark and jet production at the Tevatron. We note a significantly reduced scale dependence at higher orders for both processes, and a significant increase of the $t\bar{t}$ cross section after resummation.

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