Signals for Supersymmetric Extra Dimensions

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We study the phenomenology of a supersymmetric bulk in the scenario of large extra dimensions. The virtual exchange of gravitino KK states in selectron pair production in polarized e^+e^- collisions is examined. The leading order operator for this exchange is dimension six, in contrast to that of graviton KK exchange which induces a dimension eight operator at lowest order. Some kinematic distributions for selectron production are presented. These processes yield an enormous sensitivity to the fundamental higher dimensional Planck scale.

One might wonder whether supersymmetry plays a role in the recently proposed ADD scenario [1] of large extra dimensions. Clearly, bulk supersymmetry is not in conflict with the basic assumptions of the model. In fact, various reasons exist for believing in a supersymmetric bulk, not least of which is the motivation of string theory. D-branes of string theory provide a natural mechanism for the confinement of SM fields. If string theory is the ultimate theory of nature then the proposal of ADD might be embedded within it with a bulk supporting a supersymmetric gravitational action, with supersymmetry serving as a mechanism for stabilizing the bulk radii. In [2], we investigated the consequences of a supersymmetric bulk in the ADD scenario. If bulk supersymmetry remains unbroken away from the brane, then it is natural to ask what happens to the superpartners of the bulk gravitons, the gravitinos. The bulk gravitinos must also expand into a Kaluza-Klein tower of states and induce experimental signatures.

We focus on the effects of the virtual exchange of the bulk gravitino and graviton KK tower states in the process $e^+e^- \to \tilde{e}^+\tilde{e}^-$ at a high energy Linear Collider (LC). This process is well-known as a benchmark for collider supersymmetry studies [3], as the use of incoming polarized beams enables one to disentangle the neutralino sector and determine the degree of mixing between the various pure gaugino states. The effects of the virtual exchange of a graviton KK tower in selectron pair production has been examined in [4] for the case of non-supersymmetric large extra dimensions. The introduction of gravitino KK exchange greatly alters the phenomenology of this process by modifying the angular distributions and by substantially increasing the magnitude of the cross section. We find that the leading order behavior for this process is given by a dimension-6 operator, in contrast to the dimension-8 operator corresponding to graviton KK exchange. This yields a tremendous sensitivity to the existence of a supersymmetric bulk, resulting in a search reach for the ultraviolet cut-off of the theory of order $20 - 25 \times \sqrt{s}$.

We assume that the Standard Model fields are confined to a brane. In string theory, D-branes are extended objects on which open strings terminate [5]. Only closed strings can propagate far away from the D-brane, which on a ten dimensional background is described locally by a type II string theory whose spectrum contains two gravitinos (their vertex operators carry one vector and one spinor index). The D-brane introduces open string boundary conditions which are invariant under just one supersymmetry, so only a linear combination of the two original supersymmetries survives for the open strings. Since open and closed strings couple to each other, the D-brane breaks the original $N = 2$ supersymmetry down to $N = 1$. The low energy effective theory will then be a $D = 10, N = 1$ supergravity theory with a single Majorana-Weyl gravitino which couples to a conserved space-time supercurrent.

We take the vacuum of space-time to be of the form $M^4 \times T^6$, where M^4 is four dimensional Minkowski space-time and $T^6 = S^1 \times \ldots \times S^1$, the direct product of six dimensions each compactified on a circle, insuring four dimensional Poincaré invariance. For simplicity, we take a common radius of compactification R_c for all extra dimensions. The gravitino kinetic term of the Lagrangian is

$$E^{-1}\mathcal{L} \;=\; \frac{i}{2}\,\bar{\Psi}_{\hat{\mu}}\,\Gamma^{\hat{\mu}\hat{\nu}\hat{\rho}}\,\nabla_{\hat{\nu}}\Psi_{\hat{\rho}}\,, \tag{1}$$

with E being the determinant of the vielbein in ten dimensions, $\Gamma^{\hat{\mu}\hat{\nu}\hat{\rho}}$ the antisymmetric product of three Γ matrices, and $\Psi_{\hat{\mu}}$ a Majorana-Weyl vector-spinor. Here the hatted indices range over all dimensions of the space-time.

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	$\tilde{e}_L^- \tilde{e}_L^+$	$\tilde{e}_R^- \tilde{e}_L^+$	$\tilde{e}_L^- \tilde{e}_R^+$	$\tilde{e}_R^- \tilde{e}_R^+$
$e_L^- e^+$	s-channel $\gamma\,, Z\,, G_n$ t-channel $\tilde{W}\,, \tilde{B}\,, \Psi_n$		t-channel $\tilde{B}\,, \Psi_n$	s-channel $\gamma\,, Z\,, G_n$
$e_R^- e^+$	s-channel $\gamma\,, Z\,, G_n$	t-channel $\tilde{B}\,, \Psi_n$		s-channel $\gamma\,, Z\,, G_n$ t-channel $\tilde{B}\,, \Psi_n$

TABLE I: The diagrammatic contributions to individual scattering processes for polarized electron beams. A blank box indicates that there are no contributions for that polarization configuration.

The derivation of an effective four dimensional action using the Kaluze-Klein reduction of (1) is presented in [2]. Here we present a summary of the results. The 4-d effective Lagrangian can be written as

$$e^{-1}\,\mathcal{L}^{\vec{s}}_{eff}(x) \;=\; \sum_{j=1}^{4}\left\{\frac{i}{2}\,\bar{\omega}^{\vec{s},j}_m(x)\gamma^{mnp}(\partial_n\omega^{\vec{s},j}_p(x)) \;+\; i\,\bar{\omega}^{\vec{s},j}_m(x)\sigma^{mp}m^j_{\vec{s}}\omega^{\vec{s},j}_p(x)\right\}\,, \tag{2}$$

with the mass given by $m^j_{\vec{s}} = (-1)^j\frac{\sqrt{\vec{s}\cdot\vec{s}}}{R_c}$. The fields associated with the negative mass eigenvalues can be redefined to remove this sign, however, care must be taken with the Feynman rule for the coupling of the gravitino to matter. The sum in (2) runs over the four Majorana vector-spinors in four dimensions. We have applied the Majorana-Weyl condition in ten dimensions, which yields four Majorana spinors after the decomposition into four dimensions. Generally, the masses of the four gravitinos at each Kaluza-Klein level can be shifted by supersymmetry breaking effects on the brane. When studying the phenomenology of such models, we will assume that the $N = 4$ supersymmetry is broken at scales near the fundamental scale M_D, with only $N = 1$ supersymmetry surviving down to the electroweak scale. The phenomenological contributions from the heavy gravitinos associated with the breaking of the extended supersymmetry near the fundamental scale will be highly suppressed, due to the large mass of these individual excitations.

In [2], we derive the coupling of fermions and scalars to the gravitinos. The term coupling scalars, spinors and gravitinos, and minimally coupled to gravity, is

$$\mathcal{L}_I \;=\; -\frac{\kappa}{\sqrt{2}}|e|\left\{(\partial_\mu\Phi_L)\,\bar{\Omega}_\nu\gamma^\mu\gamma^\nu\psi_L \;+\; (\partial_\mu\Phi_R)\,\bar{\Omega}_\nu\gamma^\mu\gamma^\nu\psi_R\right\} \;+\; h.c.\,, \tag{3}$$

with $\bar{\Omega}_\nu$ a Majorana vector-spinor. Expanding $|e|$ to leading order in the vierbein yields the appropriate Feynman rules.

The unpolarized matrix element for the case of massive gravitino KK exchanges is

$$\mathcal{M} = \frac{\kappa^2}{2}\sum_{\vec{n}}\frac{k_1^\nu k_2^\rho}{t-m^2_{\vec{n}}}\bar{e}(p_1)\gamma_\mu\gamma_\nu P^{\vec{n},\mu\tau}\gamma_\rho\gamma_\tau e(p_2)\,, \tag{4}$$

where the sum extends over the gravitino KK modes and $\kappa = \sqrt{8\pi G_N} = \bar{M}_P^{-1}$ is the reduced Planck scale. $P^{\vec{n},\mu\tau}$ represents the numerator of the propagator for a Rarita-Schwinger field of mass $m_{\vec{n}}$ and is given by

$$P^{\vec{n},\mu\nu} \;=\; i\,(\not{k}+m_{\vec{n}})\left(\frac{k^\mu k^\nu}{m^2_{\vec{n}}}-\eta^{\mu\nu}\right)-\frac{i}{3}\left(\gamma^\mu+\frac{k^\mu}{m_{\vec{n}}}\right)(\not{k}-m_{\vec{n}})\left(\gamma^\nu+\frac{k^\nu}{m_{\vec{n}}}\right) \tag{5}$$

The mass splitting between the evenly spaced bulk gravitino KK excitations is given by $1/R_c$, which lies in the range 10^{-4} eV to few MeV for $\delta = 2$ to 6 assuming $M_D \sim 1$ TeV; their number density is thus large at collider energies. The sum over the KK states can then be approximated by an integral which is log divergent for $\delta = 2$ and power divergent for $\delta > 2$. We employ a cut-off to regulate these ultraviolet divergences, with the cut-off being set to Λ_c, which in general is different from M_D, to account for the uncertainties from the unknown ultraviolet physics. This approach is the most model independent and is that generally used in the case of virtual graviton exchange [6]. In practice, the integral over the gravitino KK states is more complicated than that in the case with spin-2 gravitons due to the dependence of the gravitino propagator on $m_{\vec{n}}$. We find that the leading order term for $\sqrt{|t|} \ll \Lambda_c$ results in the replacement (in the case of $\delta = 6$)

$$\frac{\kappa^2}{2}\sum_{\vec{n}}\frac{P^{\vec{n},\mu\tau}}{t-m^2_{\vec{n}}} \rightarrow \frac{-i8\pi}{5\Lambda_c^3}\left(\eta^{\mu\nu}-\frac{1}{3}\gamma^\mu\gamma^\nu\right) \tag{6}$$

in the matrix element; the structure of the summed gravitino propagator is thus altered from that of a single massive state. Hence the leading order behavior for gravitino KK exchange results in a dimension-6 operator! This is in stark contrast to graviton KK exchange which yields a dimension-8 operator at leading order. We thus expect an increased sensitivity to the fundamental scale M_D in the case of a supersymmetric bulk.

In order to perform a numerical analysis of this process, we need to specify a concrete supersymmetric model. We choose that of Gauge Mediated Supersymmetry Breaking (GSMB) as it naturally contains a light zero-mode gravitino. We specify a sample set of input parameters at the messenger scale, where the supersymmetry breaking is mediated via the messenger sector, and use the Renormalization Group Equations (RGE) to obtain the low-energy sparticle spectrum. We choose two sets of sample input parameters describing the messenger sector which are consistent with our model. The RGE evolution of these parameter sets results in the sparticle spectrum

$$\begin{aligned} \text{Set I}: \; m_{\tilde{e}_L} &= 217.0\,\text{GeV}, \quad m_{\tilde{e}_R} = 108.0\,\text{GeV}, \quad \chi_i^0 = (76.5,\, 141.5,\, 337.0,\, 367.0)\,\text{GeV}\,, \\ \text{Set II}: \; m_{\tilde{e}_L} &= 210.5\,\text{GeV}, \quad m_{\tilde{e}_R} = 104.5\,\text{GeV}, \quad \chi_i^0 = (110.5,\, 209.6,\, 322.5,\, 324.0)\,\text{GeV}\,, \end{aligned}$$

where χ_i^0 with $i = 1, 4$ corresponds to the four mixed neutralino states. The first set of parameters yields a bino-like state for the lightest neutralino, whereas the second set results in a Higgsino-like state for χ_1^0. These input parameters were selected in order to obtain a sparticle spectrum which is kinematically accessible to the Linear Collider; our results are essentially insensitive to the exact details of the spectrum.

Figure 1 shows the angular distributions with 100% electron beam polarization for each helicity configuration listed in Table I for the two sets of parameters discussed above, with and without the contributions from supersymmetric extra dimensions. In each case, the solid curve corresponds to the bino-like case and the dashed curve represents the Higgsino-like scenario. The top set of curves are those for a supersymmetric bulk with $\Lambda_c = 1.5$ TeV, while the bottom set corresponds to our two $D = 4$ supersymmetric models, *i.e.*, without the graviton and gravitino KK contributions. We note that the $D = 4$ results (i.e. MSSM) agree with those in the literature [3]. We see from the figure that in the process where the gravitino contributions are dominant, $e^-_{L,R}e^+ \to \tilde{e}^\pm_L \tilde{e}^\mp_R$, there is little difference in the shape or magnitude between the two χ_1^0 compositions. The use of selectron pair production in polarized e^+e^- collisions as a means of determining the composition of the lightest neutralino is thus made more difficult in the scenario with supersymmetric large extra dimensions. In what follows, we present results only for the bino-like χ_1^0 as a sample case; our conclusions will not be dependent on the assumptions of the composition of the lightest neutralino.

Figure 2 shows the angular distribution for the process $e^-_R e^+ \to \tilde{e}^+_R \tilde{e}^-_R$ with $\sqrt{s} = 500$ GeV assuming 100% polarization of the electron beam, detailing the effects of each class of contributions to selectron pair production. The bottom curve represents the full contributions (s- and t-channel) from the 4-dimensional standard gauge-mediated supersymmetric model discussed above in the case where the χ_1^0 is bino-like, corresponding to parameter set I. The middle curve displays the effects of adding only the s-channel contributions of the bulk graviton KK tower in the scenario of a non-supersymmetric bulk with $\Lambda_c = 1.5$ TeV. We see that there is little difference in the distribution between the $D = 4$ supersymmetric case and with the addition of the graviton KK tower, in either shape or magnitude. It would hence be difficult to disentangle the effects of graviton exchange from an accurate measurement of the underlying supersymmetric parameters using this process alone. The top curve corresponds to the full set of contributions from a supersymmetric bulk, *i.e.*, our standard supersymmetric model plus KK graviton and KK gravitino tower exchange for the case of six extra dimensions with $\Lambda_c = 1.5$ TeV. Here we see that the exchange of bulk gravitino KK states yields a large enhancement in the cross section and a substantial shift in the shape of the angular distribution, particularly at forward angles, even for $\Lambda_c = 3\sqrt{s}$. This provides a dramatic signal for a supersymmetric bulk!

We now compute the potential sensitivity to the cut-off scale from selectron pair production using our sample case with a bino-like lightest neutralino state. We employ the usual χ^2 procedure, including statistical errors only. We sum over both initial left- and right-handed electron polarization states, assuming $P_{e^-} = 80\%$. The resulting 95% C.L. search for Λ_c from each final state, $\tilde{e}^+_L \tilde{e}^-_L$, $\tilde{e}^+_R \tilde{e}^-_R$, and $\tilde{e}^\pm_L \tilde{e}^\mp_R$, is given as a function of integrated luminosity in Fig. 3 for $\sqrt{s} = 0.5$ and 1.0 TeV. We see that for 500 fb^{-1} of integrated luminosity, corresponding to design values, the search reach in the left- and right-handed selectron pair production channels is given roughly by $\Lambda_c \simeq 6 - 10 \times \sqrt{s}$, which is essentially what is achievable for bulk graviton KK exchange in the reaction $e^+e^- \to f\bar{f}$ [6]. However, the $\tilde{e}^\pm_L \tilde{e}^\mp_R$ production channel yields an enormous search capability with a 95% C.L. sensitivity to Λ_c of order $25 \times \sqrt{s}$ for design luminosity. This process thus has the potential to either discover a supersymmetric bulk, or eliminate the possibility of supersymmetric large extra dimensions as being relevant to the hierarchy problem. We stress that there is nothing special about our choice of supersymmetric parameters; our results will hold as long as selectrons are kinematically accessible to high energy e^+e^- colliders.

We conclude that selectron pair production provides a very powerful tool in searching for a supersymmetric bulk.

In summary, we have examined the phenomenological consequences of a supersymmetric bulk in the scenario of large extra dimensions. We assumed that supersymmetry is unbroken in the bulk, with gravitons and gravitinos being free to propagate throughout the higher dimensional space, and that the SM and MSSM gauge and matter fields are confined to a 3-brane. Motivated by string theory, we worked in the framework of $D = 10$ supergravity, and found that the KK reduction of the bulk gravitinos yields four Majorana spinors in four dimensions. We then assumed that the residual $N = 4$ supersymmetry is broken near the fundamental scale M_D, with only $N = 1$ supersymmetry surviving at the electroweak scale.

Starting with the $D = 10$ action for this scenario, we expanded the bulk gravitino into a KK tower of states, and determined the 4-d action for the spin-3/2 KK excitations. We then presented the coupling of the bulk gravitino KK states to fermions and their scalar partners on the brane. We applied these results to a phenomenological analysis by examining the effects of virtual exchange of the gravitino KK tower in superparticle pair production. We focused on the reaction $e^+e^- \to \tilde{e}^+\tilde{e}^-$ as this process is a benchmark for collider supersymmetry studies. Our numerical analysis was performed in the framework of gauge mediated supersymmetry breaking as it naturally affords a light zero-mode gravitino. However, our results do not depend on the specifics of this particular model, with the exception of the existence of a light zero-mode gravitino state.

Performing the sum over the KK propagators, we found that the leading order contribution to this process arises from a dimension-6 operator, and is independent of the zero-mode mass. This is in stark contrast to the virtual exchange of spin-2 graviton KK states, which yields a dimension-8 operator at leading order. We thus found that the gravitino KK contributions substantially alter the production rates and angular distributions for selectron pair production, and may essentially be isolated in the $\tilde{e}^\pm_L\tilde{e}^\mp_R$ channel. The resulting sensitivity to the cut-off scale is tremendous, being of order $20 - 25 \times \sqrt{s}$.

We expect that the virtual exchange of gravitino KK states in hadronic collisions will have somewhat less of an effect in squark and gluino pair production than what we have found here. The reason is that these processes are initiated by both quark annihilation and gluon fusion sub-processes, only one of which will be sensitive to tree-level gravitino exchange for a given production channel. The sensitivity to the cut-off scale will then depend on the relative weighting of the quark and gluon initial states. In addition, t-channel gravitino contributions will only be numerically relevant for up- and down-squark production due to flavor conservation; hence their effect will be diluted by the production of the other degenerate squark flavors and the relative weighting of the parton densities.

We note also that virtual exchange of gravitino KK states may also have a large effect on selectron pair production in e^-e^- collisions, which are tailor-made for t-channel Majorana exchanges. High energy Linear Colliders thus provide an excellent probe for the existence of supersymmetric large extra dimensions, and have the capability of discovering this possibility or eliminating it as being relevant to the hierarchy problem.

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[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999) [arXiv:hep-ph/9807344]; N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, Phys. Rev. D **63**, 064020 (2001) [arXiv:hep-th/9809124]; N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B **480**, 193 (2000) [arXiv:hep-th/0001197]. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398]. N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, Phys. Rev. D **63**, 064020 (2001) [arXiv:hep-th/9809124].

[2] The details of this work are presented in: J. L. Hewett and D. Sadri, where more extensive references can be found. [arXiv:hep-ph/0204063];

[3] A. Bartl, H. Fraas and W. Majerotto, Z. Phys. C **34**, 411 (1987); H. Baer, A. Bartl, D. Karatas, W. Majerotto and X. Tata, Int. J. Mod. Phys. A **4**, 4111 (1989); T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi and Y. Okada, Phys. Rev. D **51**, 3153 (1995).

[4] T. G. Rizzo, Phys. Rev. D **60**, 075001 (1999) [arXiv:hep-ph/9903475].

[5] J. Polchinski, arXiv:hep-th/9611050.

[6] J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999) [arXiv:hep-ph/9811356].

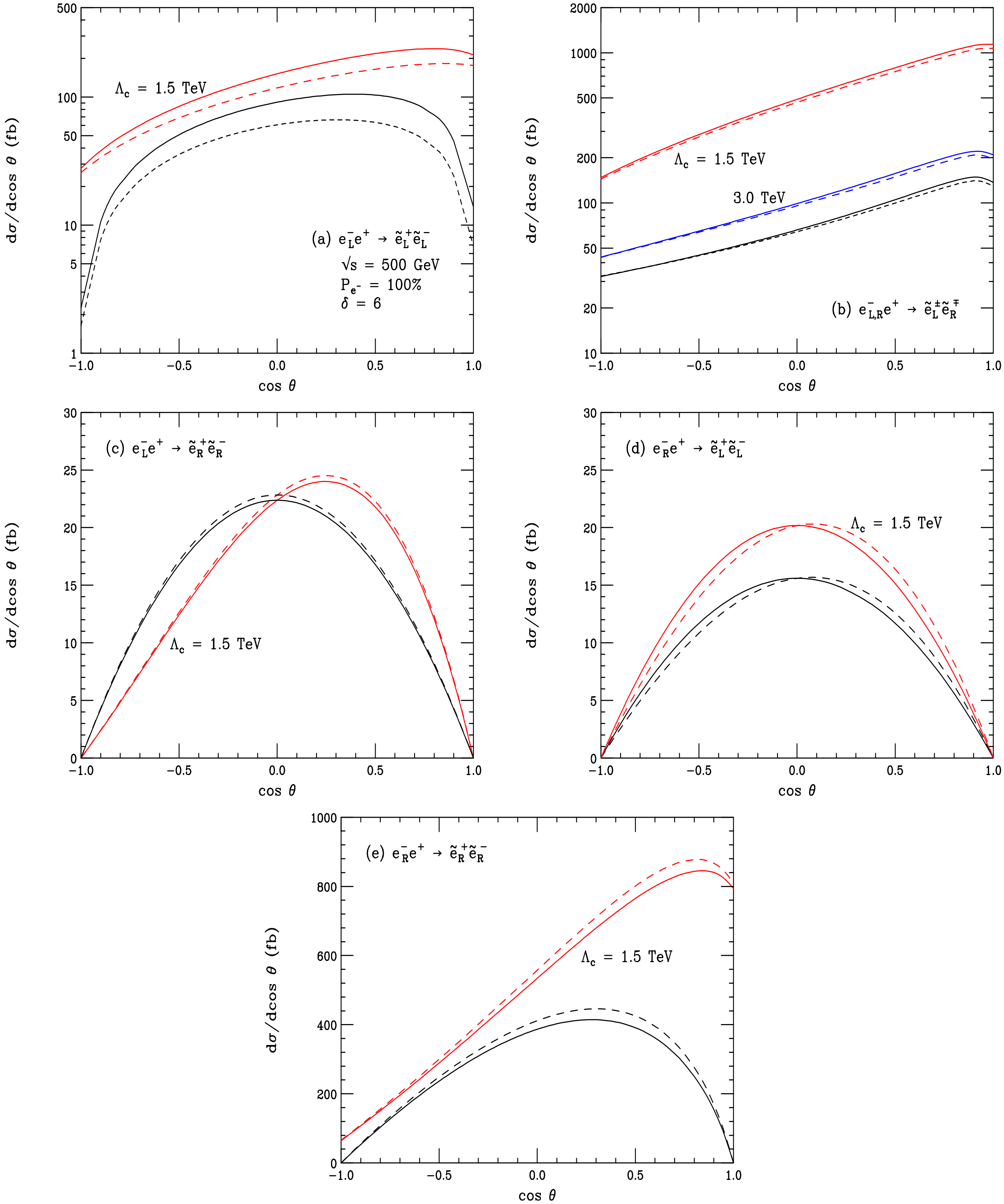

FIG. 1: Angular distributions for each helicity configuration with supersymmetric bulk contributions for $\Lambda_c = 1.5$ TeV (top curves), and for the $D = 4$ supersymmetric models (bottom curves). The solid (dashed) curves correspond to a bino-like (Higgsino-like) composition of the lightest neutralino.

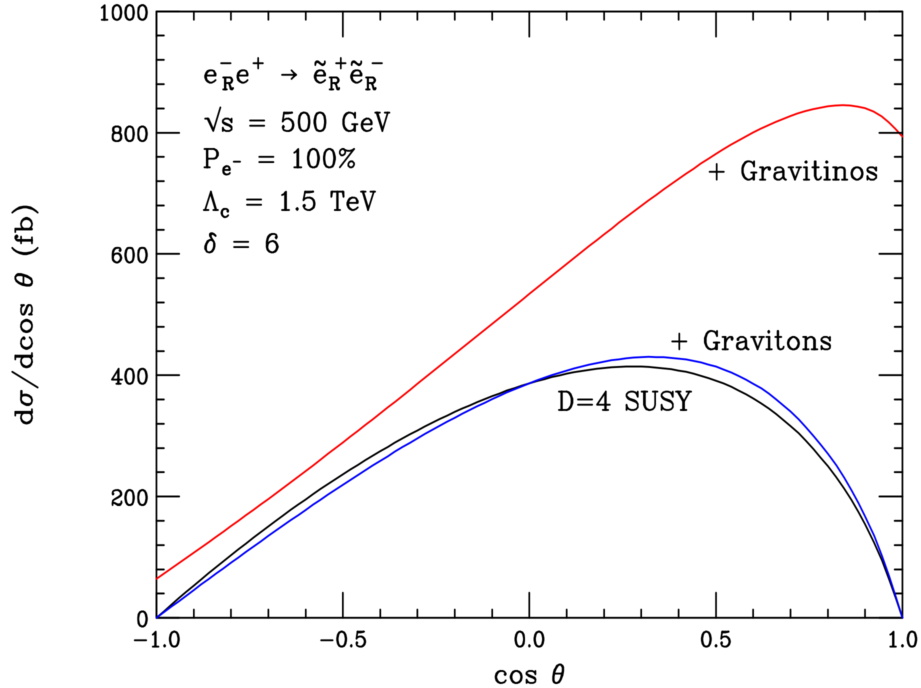

FIG. 2: The angular distribution for $e^-_R e^+ \rightarrow \tilde{e}^+_R \tilde{e}^-_R$ from the $D = 4$ supersymmetric model I, plus the addition of bulk graviton KK tower exchange, and with bulk gravitino KK tower exchange, corresponding to the bottom, middle, and top curves, respectively.

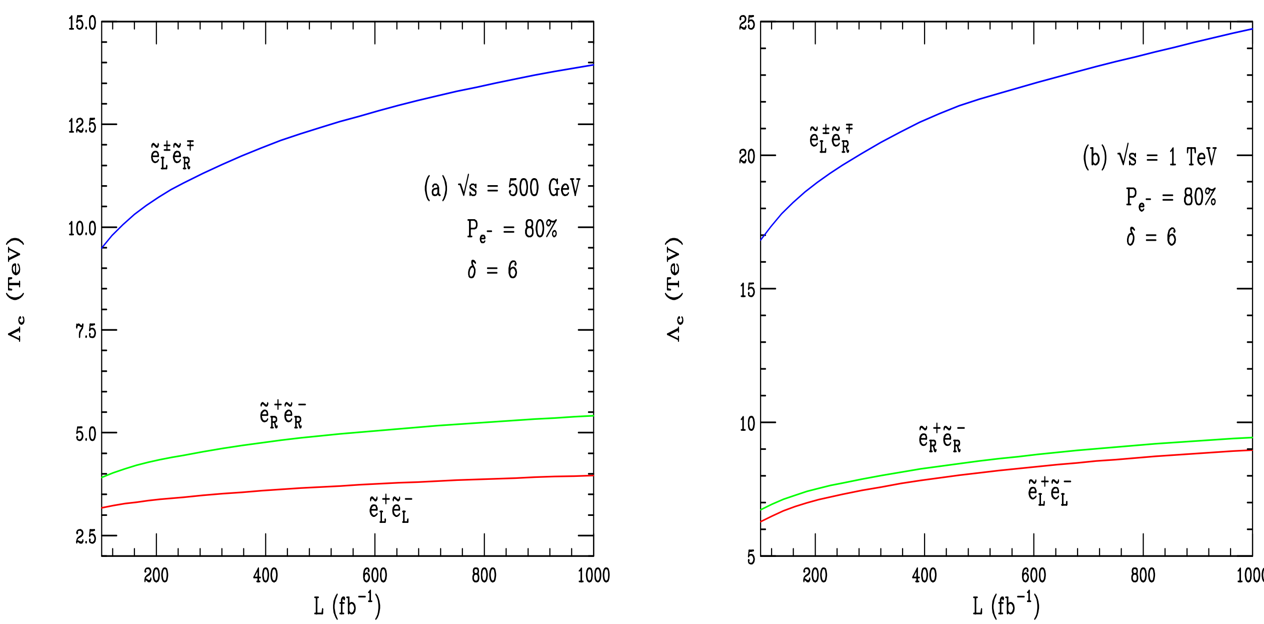

FIG. 3: 95% C.L. search reach for Λ_c in each production channel as a function of integrated luminosity for $\sqrt{s} = 0.5$ and 1.0 TeV.