Large Extra Dimensions: Implications for the Strong CP Problem

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In this talk, we summarize some of the novel effects that arise when the QCD axion is placed in the "bulk" of large extra spacetime dimensions. First, we find that the mass of the axion can become independent of the energy scale associated with the breaking of the Peccei–Quinn symmetry. This implies that the mass of the axion can be adjusted independently of its couplings to ordinary matter, a feature which is not possible in four dimensions and which may contribute to axion invisibility. Second, we discuss the new phenomenon of laboratory axion oscillations (analogous to neutrino oscillations), and show that these oscillations cause laboratory axions to "decohere" extremely rapidly as a result of Kaluza–Klein mixing. This decoherence may also be a contributing factor to axion invisibility. Finally, we show that under certain circumstances, the presence of an infinite tower of Kaluza–Klein axion modes can significantly accelerate the dissipation of the energy associated with cosmological relic axion oscillations, thereby enabling the Peccei–Quinn symmetry-breaking scale to exceed the usual four-dimensional relic oscillation bounds. Together, these ideas provide new ways of obtaining an "invisible" axion within the context of higher-dimensional theories with large-radius compactifications.

Over the past few years, the possibility that large extra spacetime dimensions might exist has received considerable attention. The main attraction of this idea is the observation that large extra dimensions have the potential to lower the fundamental energy scales of physics such as the Planck scale [1], the GUT scale [2], and the string scale [3]. In this framework, one assumes that the observed four dimensions are merely a subspace within a p-dimensional membrane (or D-brane) which in turn floats in a higher-dimensional space. The extra compactified dimensions can therefore be of two types, either within the D-brane or transverse to it, and the phenomenology of both types of extra dimensions has been explored quite extensively in recent years.

One of the surprising aspects of extra dimensions is that they may provide a new approach towards solving the strong CP problem. One of the standard approaches to the strong CP problem is to introduce a Peccei-Quinn (PQ) symmetry [4] with a corresponding axion [5]. Unfortunately, the experimentally allowed parameter space for the mass and couplings of the axion have become very narrow [6], and it is not clear how to generate the high energy scale associated with the breaking of PQ symmetry or to explain the "invisibility" of the resulting QCD axion.

Within the framework afforded by large extra dimensions, however, this situation may be radically altered. The basic idea is that since the QCD axion is a singlet under all Standard-Model symmetries, it is free to leave the D-brane to which the Standard-Model particles are restricted and propagate into the higher-dimensional bulk. In other words, in theories with extra dimensions, the QCD axion can accrue an infinite tower of Kaluza-Klein axion states.

Can this be used to lower the fundamental PQ symmetry-breaking scale? This issue has been investigated in Refs. [7, 8]. As explicitly shown in Ref. [8] (and first proposed in Ref. [1]), it is possible to exploit the large volume factor associated with the extra dimensions in order to realize a large effective four-dimensional PQ scale from a smaller, higher-dimensional fundamental PQ scale. Specifically, if $f_{\rm PQ}$ is the underlying higher-dimensional PQ scale, the effective PQ scale $\hat{f}_{\rm PQ}$ in four dimensions [1, 8] is given by $\hat{f}_{\rm PQ} \equiv (VM_s)^{1/2} f_{\rm PQ} \gg f_{\rm PQ}$. It is the volume-renormalized scale $\hat{f}_{\rm PQ}$ which parametrizes the couplings between the four-dimensional axion and the PQ-

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charged Standard-Model particles on the brane. Therefore, as pointed out in Ref. [1], this volume-renormalization of the brane/bulk coupling can be used to obtain a sufficiently suppressed axion even if the underlying PQ scale is near the TeV range. This is therefore one method of generating an apparent high PQ symmetry-breaking scale in a natural way.

However, as discussed in Ref. [8], the presence of the Kaluza-Klein axions can have important and unexpected effects on axion phenomenology. In theories with extra dimensions, it turns out that the four-dimensional axion no longer is a mass eigenstate; instead, this axion a_0 mixes with the infinite tower of Kaluza-Klein axions a_n, with a mass mixing matrix given by [8]

$$\mathcal{M}^2 = m_{\rm PQ}^2 \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \dots \\ \sqrt{2} & 2+y^2 & 2 & 2 & \dots \\ \sqrt{2} & 2 & 2+4y^2 & 2 & \dots \\ \sqrt{2} & 2 & 2 & 2+9y^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{1}$$

where $m_{\rm PQ} \sim \Lambda_{\rm QCD}^2/\hat{f}_{\rm PQ}$ and $y \equiv (m_{\rm PQ}R)^{-1}$. This mixing has a number of interesting consequences.

First, as shown in Ref. [8], under certain circumstances the mass of the axion essentially *decouples* from the PQ scale, and instead is set by the radius of the extra spacetime dimension. This occurs because the eigenvalues λ of the matrix (1) are given as solutions to the transcendental equation $\pi R\lambda \cot(\pi R\lambda) = \lambda^2/m_{\rm PQ}^2$ where R is the radius of the extra compactified dimension. Note that the lightest eigenvalue necessarily satisfies $\lambda \leq 1/(2R)$; this result holds *regardless* of the value of $m_{\rm PQ}$. This implies that axions in the 10^{-4} eV mass range are consistent with (sub-)millimeter extra dimensions. More interestingly, however, this implies that in higher dimensions, the size of the axion mass is set by the radius R and not by the Peccei-Quinn scale $f_{\rm PQ}$ when $m_{\rm PQ} \gtrsim 1/(2R)$. This decoupling implies that it may be possible to adjust the mass of the axion independently of its couplings to matter. This is not possible in four dimensions.

Second, as discussed in Ref. [8], since the usual four-dimensional axion is no longer a mass

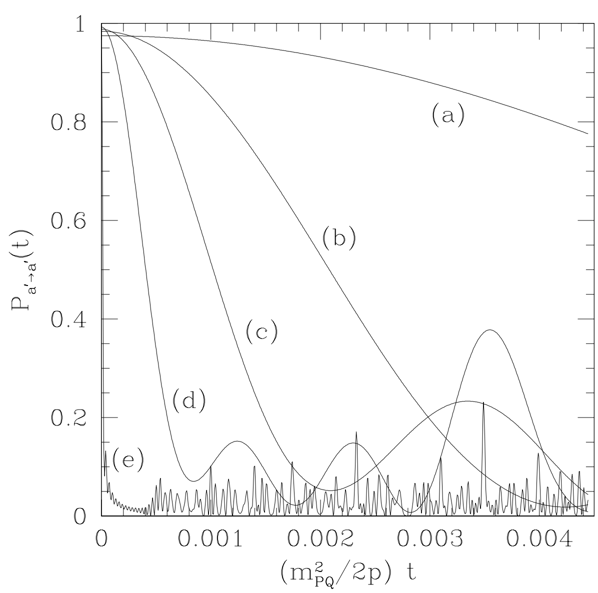

Figure 1: The axion preservation probability $P_{a'\to a'}(t)$ as a function of the number $n_{\rm max}$ of axion Kaluza-Klein states which are included in the analysis. In this plot we show the results for (a) $n_{\rm max} = 1$; (b) $n_{\rm max} = 2$; (c) $n_{\rm max} = 3$; (d) $n_{\rm max} = 5$; and (e) $n_{\rm max} = 30$. As $n_{\rm max}$ increases, the axion probability rapidly falls to zero as a result of the destructive interference of the Kaluza-Klein states, and remains suppressed without significant axion regeneration at any later times. This "decoherence" of the axion implies that there is negligible probability for subsequently detecting the original axion state at any future time. Further details can be found in Ref. [8].

eigenstate, it should undergo *laboratory oscillations* as it propagates. Such oscillations are entirely analogous to neutrino oscillations. Moreover, because the axion is now a higher-dimensional field, Standard-Model particles couple not only to the usual four-dimensional axion zero mode, but rather to the entire linear superposition $a' \sim \sum_n a_n$ of Kaluza-Klein modes. Therefore, the quantity of phenomenological interest is the probability $P_{a'\to a'}(t)$ that a' is preserved as a function of time. This probability [8] is shown in Fig. 1. Unlike the case of neutrino oscillations, we see that this probability drops rapidly from 1 (at the initial time $t = 0$) to extremely small values (expected to be $\approx 10^{-16}$ in realistic scenarios [8]). At no future time does this probability regenerate. Thus, we see that in higher dimensions, the axion state a' rapidly "decoheres" and becomes invisible with respect to subsequent laboratory interactions! This decoherence is therefore a possible higher-dimensional mechanism contributing to an invisible axion.

Finally, one can investigate the effects of Kaluza-Klein axions on cosmological relic axion oscillations. Recall that as the universe cools and passes through the QCD phase transition at $T \approx \Lambda_{\rm QCD}$, instanton effects suddenly establish a non-zero axion potential where none previously existed. In the usual four-dimensional situation, the axion can therefore find itself displaced relative to the newly-established minimum of the potential, and begin to oscillate around it with a "damping" term due to the expansion of the universe (the Hubble term). Some of the most severe upper bounds on the PQ scale come from demanding that the energy trapped in these oscillations today be less than the critical energy density to overclose the universe.

In higher dimensions, by contrast, all of the axion Kaluza-Klein modes are coupled together via the mixing matrix (1). Thus, even if we assume that only the zero-mode axion starts out initially displaced when the axion potential is generated, this coupling is sufficient to trigger the excited Kaluza-Klein modes into oscillation. This situation is illustrated in Fig. 2(a).

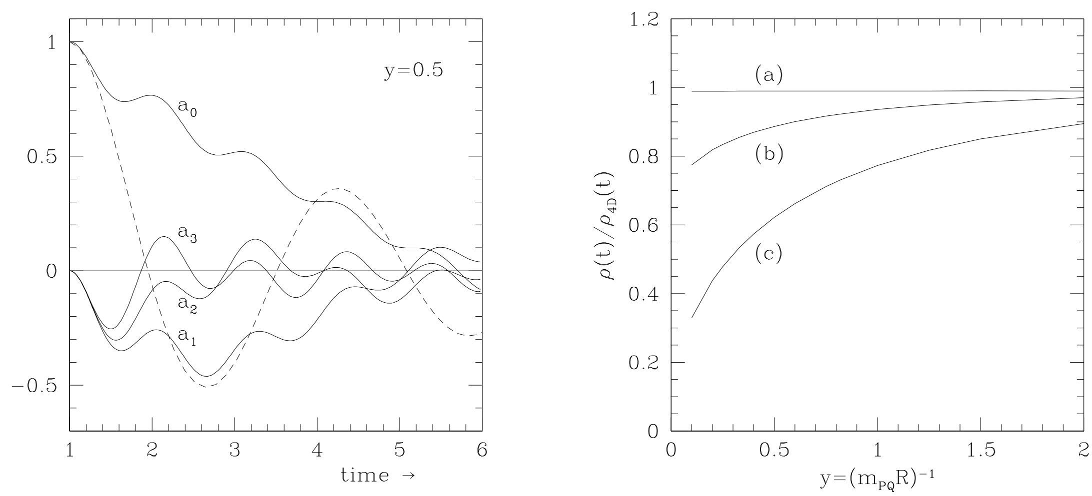

Figure 2: (a) Coupled Kaluza-Klein cosmological relic axion oscillations. Although the excited Kaluza-Klein modes have vanishing initial displacements, they are triggered into oscillation as a result of the initial displacement of the zero-mode. The superimposed dashed line shows the behavior of the usual four-dimensional axion zero-mode when no Kaluza-Klein modes are present. (b) The energy-dissipation ratio factor $\rho(t)/\rho_{4D}(t)$ where $\rho_{4D}(t)$ is the normalized energy density today in the usual four-dimensional case and $\rho(t)$ is the analogous quantity in the higher-dimensional case. The different curves correspond to different input parameters discussed in Ref. [8]. As $y \to 0$ (implying a large extra dimension), we see that the cosmological relic energy density is dissipated more rapidly than in four dimensions.

It is therefore important to understand whether the infinite towers of Kaluza-Klein axion states accelerate or retard the dissipation of the cosmological energy density associated with these relic oscillations. Remarkably, one finds [8] that the net effect of these coupled Kaluza-Klein axions is always either to *preserve* or to *enhance* the rate of energy dissipation. This behavior is shown in Fig. 2(b). Because these Kaluza-Klein states accelerate the rate at which this relic oscillation energy is dissipated, the usual relic oscillation bounds are loosened in higher dimensions. This suggests that it may be possible to raise the effective PQ symmetry-breaking scale beyond its usual four-dimensional value, thereby further contributing to axion invisibility.

Together, these results suggest that it may be possible to develop a new, higher-dimensional approach to axion phenomenology. Further details concerning these ideas can be found in Ref. [8], and other recent ideas can be found in Refs. [7, 9].

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