Extrapolation of Supersymmetry-Breaking Parameters to High Energy Scales

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I study how well one can extrapolate the values of supersymmetry-breaking parameters to very high energy scales using future data from the Large Hadron Collider and an e^+e^- linear collider. I consider tests of the unification of squark and slepton masses in supergravity-inspired models. In gauge-mediated supersymmetry breaking models, I assess the ability to measure the mass scales associated with supersymmetry breaking. I also show that it is possible to get good constraints on a scalar cubic stop-stop-Higgs couplings near the high scale. Different assumptions with varying levels of optimism about the accuracy of input parameter measurements are made, and their impact on the extrapolated results is documented.

After the discovery of supersymmetry, the most important goal of experimental investigations in high energy physics will be to discern the type of supersymmetry breaking. In almost all imaginable scenarios, this will involve evaluating the supersymmetry breaking parameters at some very high input scale (perhaps the Planck mass scale, the GUT scale, or the scale of new extra dimensions) where they hopefully take a simple form. In practice, one must measure the parameters of the supersymmetric model at the weak scale, and then evolve them according to the renormalization group (RG) to the high scale where various proposed organizing principle can be tested.

In this note, I will examine the question of how accurately this can be done. In doing so, it is crucial to assign uncertainties to *all* parameters of the model. Even quantities which do not enter directly into the RG running of quantities of interest at one-loop can affect them indirectly but significantly, by 2-loop effects and/or by the effects of enforcing proper electroweak symmetry breaking. Rather than attempting to argue in detail for how well the LHC and a future linear collider can measure the relevant quantities, I will simply present results for rough guesses of experimental uncertainties, labelled as "optimistic," "pessimistic," and "intermediate." My choices for these assumed uncertainties, in per cent, are shown in Table 1. Of course, experimental realities can (and probably will) turn out to be quite different. History has shown that estimates of obtainable uncertainties concocted before experiments are done can be considerably different from those eventually obtained.

The methods used here are as follows. A template set of model parameters at the input scale is picked, and evolved to the electroweak scale to obtain central values for MSSM observables. For each individual observable uncertainty, a one-sigma range for all model parameters is RG-evolved back up to the input scale. The one-sigma ranges for all parameters at all RG scales Q up to the high scale are then obtained by adding the individual one-sigma ranges in quadrature. My conventions for model parameters and signs are those in [1]. I use the 2-loop MSSM RG-equations found in [2]. Electroweak symmetry breaking and sparticle pole masses are computed using [3]. Previous works have conducted similarly-motivated analyses; c.f. [4]-[5]

A test of the ability to verify squark and slepton mass unification at the input scale is shown in figures 1-3. The template model in this case is an mSUGRA model with input parameters $m_{1/2} = 240$ GeV, $m_0 = 120$ GeV, $A_0 = -120$ GeV, $\tan\beta = 10$, and $\mu > 0$. Figure 1 shows the one-sigma range for the running sfermion masses $\tilde{e}_R$, $\tilde{e}_L$, and $\tilde{Q}_L$ using the "pessimistic" set of uncertainties. While the results at the GUT scale are consistent with scalar mass unification, they are hardly definitive. The improved situation for the case of "intermediate" uncertainties is shown in Figure 2. I find that further improvement to the case of "optimistic" uncertainties does not actually help much, because the uncertainties are now dominated by that of the gluino

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Table I Assumed uncertainties, in per cent, for various model parameters at the weak scale.

Parameter	"optimistic"	"intermediate"	"pessimistic"
α_1, α_2	0.2	0.2	0.2
α_3	1	1	1
y_t	1	1	1
y_b	5	5	5
M_1, M_2	0.2	0.5	1
M_3	2	2	4
$m_{\tilde{e}}, m_{\tilde{\mu}}$	0.1	0.5	1
$m_{\tilde{\tau}}$	0.2	1	2
$m_{\tilde{q}}, m_{\tilde{t}}, m_{\tilde{b}}$	1	1	3
m_{H_u,H_d}	1	1	5
a_t	2	2	5
a_b	10	10	100
a_τ	10	10	100

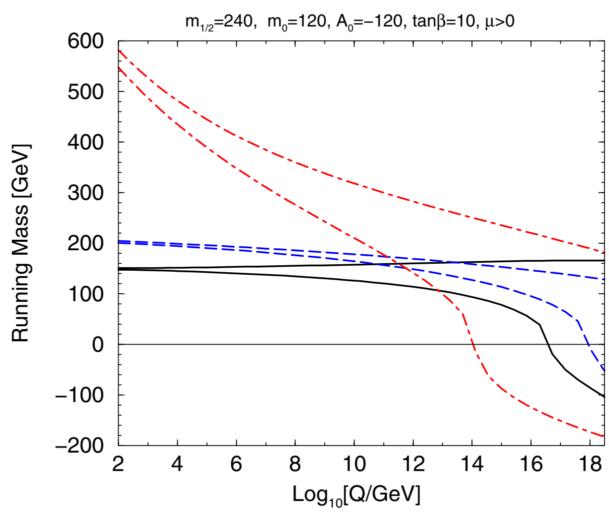

Figure 1: RG running of the one-sigma range of values for $m_{\tilde{e}_R}$ (solid), $m_{\tilde{e}_L}$ (dashed), and $m_{\tilde{Q}_L}$ (dot-dashed), using "pessimistic" uncertainties for all weak scale parameters as in Table 1. The underlying model parameters are $m_{1/2} = 240$ GeV, $m_0 = 120$ GeV, $A_0 = -120$ GeV, $\tan\beta = 10$, $\mu > 0$.

mass. So measuring slepton masses at the 0.1% level does not represent much improvement over the 0.5% level, unless the uncertainty in the gluino mass can be improved significantly. A different way of showing the results is presented in figure 3, which plots the one-sigma allowed range for $[m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R}]^{1/2}$, which should be consistent with 0 at the GUT scale. Even in the case of "optimistic" uncertainties, there is almost a 100 GeV one-sigma uncertainty in this quantity at the GUT scale. This illustrates a limitation on our future ability to distinguish D-term or other sources of scalar mass non-universality.

Another interesting question is whether one can distinguish scalar cubic couplings at the high scale. In most models of SUSY breaking, these couplings are assumed to be proportional to the corresponding Yukawa coupling, so for example the Lagrangian contains a coupling $\mathcal{L} = -a_t \tilde{t}^*_R \tilde{t}_L H^0_u$ with $a_t = A_0 y_t$ at the input scale. A principle goal in understanding SUSY breaking is then to measure A_0, and especially its ratio (including sign) to $m_{1/2}$. In general, this is made more difficult by the fact that a_t has some fixed-point-like behavior. Nevertheless, I find that it may be possible to determine at least the sign of A_0 and possibly get some limits on its magnitude. In practice, at the weak scale one can obtain a_t from the stop masses and mixing angle and μ

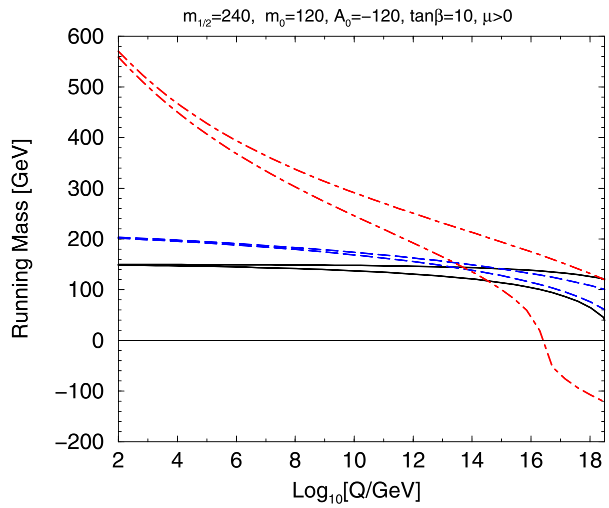

Figure 2: As in Figure 1, but using "intermediate" uncertainties for all weak scale parameters as in Table 1. The results for "optimistic" uncertainties do not differ very much from these.

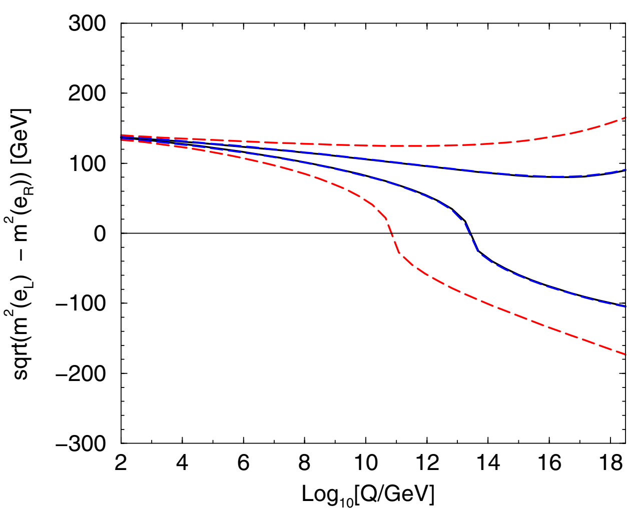

Figure 3: RG running of the one-sigma range of values for $[m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R}]^{1/2}$, using "optimistic"(solid) and "pessimistic" (dashed) uncertainties for all weak scale parameters as in Table 1. The underlying model parameters are as in Figures 1,2. The results for "intermediate" uncertainties do not differ much from those shown for the "optimistic" case.

according to:

$$a_t = \frac{1}{v \sin\beta}\left[(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}) \cos\theta_{\tilde{t}} \sin\theta_{\tilde{t}}\right] + y_t \mu \cot\beta \tag{1}$$

where $v = 175$ GeV. The 1-sigma results of running a_t up are shown in figure 4 for an mSUGRA model with $m_{1/2} = 240$ GeV, $m_0 = 120$ GeV, $\tan\beta = 10$, $\mu > 0$, and $A_0 = \pm m_{1/2}$. Even with the case of pessimistic assumptions with an assumed uncertainty in a_t at the weak scale of 5%, one can evidently determine the sign of A_0 at the GUT scale. The two possible signs for A_0 can be clearly separated from each other and perhaps from the case that $A_0 = 0$.

Finally, I turn to the case of determining SUSY breaking parameter in GMSB models. It has been shown that a global fit to these model parameters using only LHC data can determine them with very high accuracy [5]. It is also useful to examine the consistency of the global fit, using several different variables to get at the same underlying parameter. For example, in the minimal GMSB model with a single $\mathbf{5} + \overline{\mathbf{5}}$ messenger sector, one can consider running quantities defined

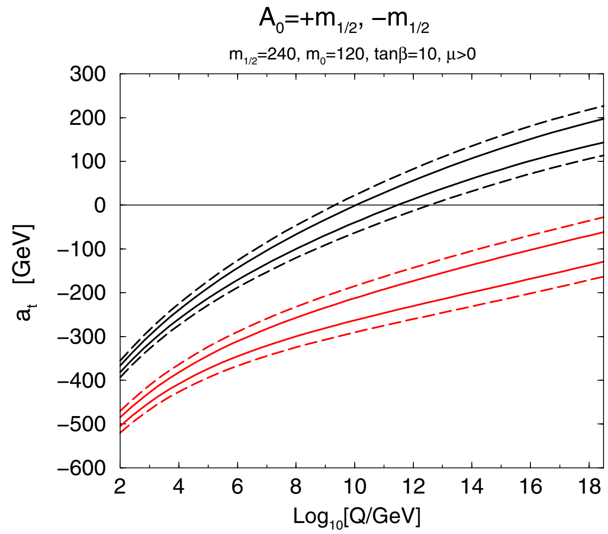

Figure 4: RG running of the one-sigma range of values for the Higgs-stop-stop coupling, using "optimistic" (solid) and "pessimistic" (dashed) uncertainties at the weak scale. The upper set of lines corresponds to $A_0 = +m_{1/2}$, and the lower set to $A_0 = -m_{1/2}$ at the GUT scale. The other underlying model parameters are as in Figures 1,2,3.

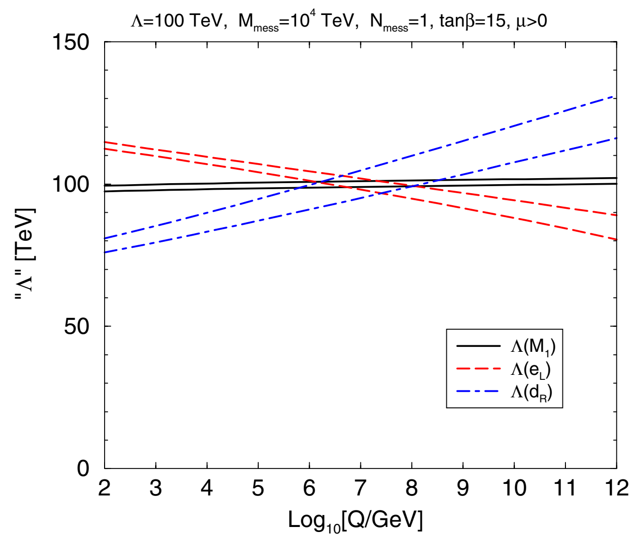

Figure 5: RG running of the quantities Λ_{M_1}, $\Lambda_{\tilde{e}_L}$, and $\Lambda_{\tilde{d}_R}$ using "pessimistic" uncertainties for all weak scale parameters as in Table 1. The intersection gives the values and uncertainties for the minimal GMSB model parameters Λ on the vertical axis and $M_{\rm mess}$ on the horizontal axis.

as follows:

$$\Lambda_{m^2_{\tilde{e}_R}} = m^2_{\tilde{e}_R}/[\frac{6}{5}\left(\frac{\alpha_1}{4\pi}\right)^2] \tag{2}$$

$$\Lambda_{m^2_{\tilde{e}_L}} = m^2_{\tilde{e}_L}/[\frac{3}{2}\left(\frac{\alpha_2}{4\pi}\right)^2 + \frac{3}{10}\left(\frac{\alpha_1}{4\pi}\right)^2] \tag{3}$$

$$\Lambda_{m^2_{\tilde{d}_R}} = m^2_{\tilde{d}_R}/[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 + \frac{2}{15}\left(\frac{\alpha_1}{4\pi}\right)^2] \tag{4}$$

$$\Lambda_{m^2_{\tilde{u}_R}} = m^2_{\tilde{u}_R}/[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 + \frac{8}{15}\left(\frac{\alpha_1}{4\pi}\right)^2] \tag{5}$$

$$\Lambda_{m^2_{\tilde{Q}_L}} \;=\; m^2_{\tilde{Q}_L}/[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 + \frac{3}{2}\left(\frac{\alpha_2}{4\pi}\right)^2 + \frac{1}{30}\left(\frac{\alpha_1}{4\pi}\right)^2] \tag{6}$$

from 1st and 2nd family sfermion masses, and

$$\Lambda_{M_a} = (4\pi/\alpha_a)\, M_a \qquad (a = 1, 2, 3) \tag{7}$$

from gaugino masses. Evaluated at an RG scale equal to the common messenger mass $M_{\rm mess}$, all of these quantities should be equal to the parameter Λ of minimal GMSB. A test of this using one-sigma allowed ranges is shown in figure 5 for a model with $\Lambda = 100$ TeV, $M_{\rm mess} = 10^4$ TeV, $N_{\rm mess} = 1$, $\tan\beta = 15$, and $\mu > 0$. For simplicity, the graph only shows the determination of Λ_{M_1}, $\Lambda_{\tilde{e}_L}$, and $\Lambda_{\tilde{d}_R}$, using the "pessimistic" set of assumptions regarding weak-scale uncertainties. As can be seen, this analysis can provide a non-trivial check of the assumptions going into the model parameterization, as well as determining the model parameters themselves.

In general, improvements in the accuracy with which superpartner masses, mixing angles, and couplings can be measured will help us to understand better the mode of supersymmetry breaking. As the above results help to illustrate, one can produce strong constraints and consistency checks on the model of supersymmetry breaking. However, it is also clear that the uncertainties on high-scale parameters are often not reduced by arbitrarily accurate measurements of the most well-measured parameters; because the most poorly measured parameters come to dominate the uncertainties. In particular, an accurate measurement of the gluino mass may be the bottleneck to accurate determinations of the SUSY breaking mechanism.

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