Signals for Non-Commutative QED in $e\gamma$ and $\gamma\gamma$ Collision

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We study the effects of non-commutative QED (NCQED) in fermion pair production, $\gamma+\gamma \to f+\bar{f}$ and Compton scattering, $e+\gamma \to e+\gamma$. Non-commutative geometries appear naturally in the context of string/M-theory and gives rise to 3- and 4-point photon vertices and to momentum dependent phase factors in QED vertices which will have observable effects in high energy collisions. We consider e^+e^- colliders with energies appropriate to the TeV Linear Collider proposals and the multi-TeV CLIC project operating in $\gamma\gamma$ and $e\gamma$ modes. Non-commutative scales roughly equal to the center of mass energy of the e^+e^- collider can be probed, with the exact value depending on the model parameters and experimental factors. However, we found that the Compton process is sensitive to Λ_{NC} values roughly twice as large as those accessible to the pair production process.

Although string/M-theory is still developing, and the details of its connection to the Standard Model are still unclear, numerous ideas from string/M-theory have affected the phenomenology of particle physics. The latest of these ideas is non-commutative quantum field theory (NCQFT) [1, 2]. NCQFT arises through the quantization of strings by describing low energy excitations of D-branes in background EM fields. NCQFT generalizes our notion of space-time, replacing the usual, commuting, space–time coordinates with non-commuting space–time operators. Testable differences exist between QFT with commuting space–time coordinates and NCQFT.

At this time, the details of a general NCQFT model to compare to the Standard Model are just emerging [3]. However, NCQED does exist and can be studied. NCQED modifies QED, with the addition of a non-Lorentz invariant, momentum dependent phase factor to the normal $ee\gamma$ vertex, along with the addition of cubic ($\gamma\gamma\gamma$) and quartic ($\gamma\gamma\gamma\gamma$) coupling, also, with non-Lorentz invariant momentum dependent phase factors. The Feynman rules for NCQED are given in [4, 5]. Although the momentum dependent phase factors and higher dimensional operators in the Lagrangian arise naturally in NCQFT, the modifications, although similar, will in general, take on a different form than those for NCQED. We will see that the modifications of NCQFT to QED can be probed in $\gamma\gamma \to f\bar{f}$ and $e\gamma \to e\gamma$ collisions. For full details of our analysis, please see [6].

The essential idea of NCQFT is that in the non-commuting space time the conventional coordinates are represented by operators which no longer commute:

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu} \equiv \frac{i}{\Lambda_{NC}^2} C_{\mu\nu} \tag{1}$$

Here we adopt the Hewett-Petriello-Rizzo parametrization [5] where the overall scale, Λ_{NC}, characterizes the threshold where non-commutative (NC) effects become relevant and $C_{\mu\nu}$ is a real antisymmetric matrix whose dimensionless elements are presumably of order unity. One might expect the scale Λ_{NC} to be of order the Planck scale. However, given the possibility of large extra dimensions [7, 8] where gravity becomes strong at scales of order a TeV, it is possible that NC effects could set in at a TeV. We therefore consider the possibility that Λ_{NC} may lie not too far above the TeV scale.

The C matrix can be parameterized, following the notation of [9], as

$$C_{\mu\nu} = \begin{pmatrix} 0 & C_{01} & C_{02} & C_{03} \\ -C_{01} & 0 & C_{12} & -C_{13} \\ -C_{02} & -C_{12} & 0 & C_{23} \\ -C_{03} & C_{13} & -C_{23} & 0 \end{pmatrix} \tag{2}$$

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where $\sum_i |C_{0i}|^2 = 1$. Thus, the C_{0i} are related to space-time NC and are defined by the direction of the background **E**-field. Likewise, the C_{ij} are related to the space-space non-commutativeness and are defined by the direction of the background **B**-field.

NCQED is beginning to attract theoretical and phenomenological interest [5, 10, 11, 12]. Hewett, Petriello and Rizzo [5] have performed a series of phenomenological studies of NCQED at high energy, linear, e^+e^- colliders. They analyzed diphoton production ($e^+ + e^- \to \gamma + \gamma$), Bhabha scattering ($e^+ + e^- \to e^+ + e^-$) and Moller scattering ($e^- + e^- \to e^- + e^-$). There are striking differences between QED and NCQED for all three processes; most interesting is significant structure in the ϕ angular distribution.

Mathews [11] and Baek, Ghosh, He and Hwang [12] have also studied NCQED at high energy e^+e^- linear colliders. In the former case Mathews studied high energy Compton scattering while Baek *et al.,* studied fermion pair production in $\gamma + \gamma \to e^+ + e^-$. Independently of the aforementioned studies we studied Compton scattering and lepton pair production. Our study uses angular distributions to enhance the sensitivity of measurements to Λ_{NC}, uses more realistic acceptance cuts, *etc*.

We consider linear e^+e^- colliders operating at $\sqrt{s} = 0.5$ and 0.8 TeV appropriate to the TESLA proposal, [13] $\sqrt{s} = 0.5$, 1.0 and 1.5 TeV as advocated by the NLC proponents [14], and $\sqrt{s} = 3.0$, 5.0 and 8.0 TeV being considered in CLIC studies [15]. In order to estimate event rates, we assume an integrated luminosity of $L = 500$ fb^{-1} for all cases. We impose acceptance cuts on the final state particles of $10^o \le \theta \le 170^o$ and $p_T > 10\ GeV$. Furthermore, all exclusion limits given below are for unpolarized electron and photon beams; the helicity structure of the NCQED cross section is identical to that in the SM, *i.e.* the fermion-photon couplings are vector-like, so polarization will not lead to an improvement in the exclusion limits.

In the pair production case, where only space-time NC enters, only one parameter, α, remains in addition to Λ_{NC}. We report exclusion limits for $\alpha = 0$, $\pi/4$ and $\pi/2$. In the Compton scattering case, both space-space and space-time NC enter, leaving the two parameters, α and γ, in addition to Λ_{NC}. We examine the two values $\gamma = 0$ and $\pi/2$, and for each value of γ give exclusion limits for $\alpha = 0$, $\pi/4$ and $\pi/2$. We remind the reader that α relates to the direction of **E**, whereas γ determines the orientation of **B.**

For the pair production process, the differential cross section for this process is given by:

$$\frac{d\sigma(\gamma\gamma \to f\bar{f})}{d\cos\theta\, d\phi} = \frac{\alpha^2}{2s}\left\{\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - 4\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\sin^2\left(\frac{k_1 \cdot \theta \cdot k_2}{2}\right)\right\}. \tag{3}$$

The first two terms in the expression are the standard QED contributions, while the last term is due to the Feynman diagram with the cubic $\gamma\gamma\gamma$ coupling. The phase factor, $\sin^2\left(\frac{k_1 \cdot \theta \cdot k_2}{2}\right)$ only appears in this new term. p_1 and p_2 are the momentum of the electron and positron, respectively, while k_1 and k_2 are the momenta of the incoming photons. $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_1 - p_1)^2$ and $\hat{u} = (k_1 - p_2)^2$. The bilinear product in eqn. 6 simplifies to

$$\frac{1}{2} k_1 \cdot \theta \cdot k_2 = \frac{\hat{s}}{4\Lambda_{NC}^2} C_{03}. \tag{4}$$

The expression for the cross section is not Lorentz invariant due to the presence of the phase factor. Note that only space-time non-commutativity contributes and there is no ϕ dependence in this case. As $C_{03} = \cos\alpha$, NCQED reproduces QED for pair production when $\alpha = \pi/2$, and also as $\Lambda_{NC} \to \infty$.

The exclusion limits based on lepton pair production in $\gamma\gamma$ collisions and assuming an integrated luminosity of $L = 500 fb^{-1}$ are summarized in Table I of Ref. [6] for $\alpha = 0$ and $\pi/4$. The values range from 220 GeV ($\sqrt{s} = 500\ GeV$) to 2.7 TeV ($\sqrt{s} = 5.0\ TeV$). These are based on the angular distribution which, as already noted, gives the highest limits. These limits could be improved by including three lepton generations in the final state and assuming some value for the lepton detection efficiency.

For the Compton scattering process, we find:

$$\frac{d\sigma(e^-\gamma \to e^-\gamma)}{d\cos\theta\, d\phi} = \frac{\alpha^2}{2s}\left\{-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + 4\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\sin^2\left(\frac{k_1 \cdot \theta \cdot k_2}{2}\right)\right\}. \tag{5}$$

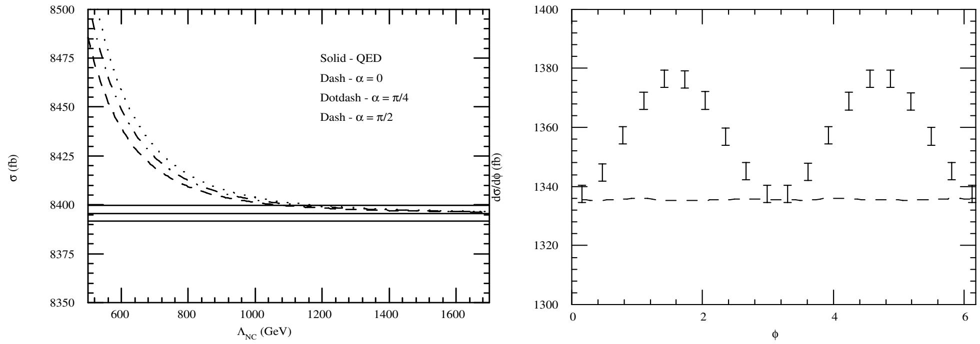

Figure 1: (a) σ vs. Λ_{NC} for the Compton scattering process with $\sqrt{s} = 500$ GeV for $\gamma = 0$. The horizontal band represents the SM cross section $\pm$ 1 standard deviation (statistical) error. (b) $d\sigma/d\phi$ for the Compton scattering process with $\sqrt{s} = 500$ GeV and for $\Lambda = 500$ GeV, $\alpha = \pi/2$ and $\gamma = 0$. The dashed curve corresponds to the SM angular distribution and the points correspond to the NCQED angular distribution including 1 standard deviation (statistical) error.

The first two terms in the expression are the standard, QED contribution, while the last term is due to the Feynman diagram with the cubic $\gamma\gamma\gamma$ coupling. As before, the phase factor only appears in this new term.

Here, p_1 and k_1 are the momenta of the initial state electron and photon, respectively, while p_2 and k_2 are the momenta of the final state electron and photon, respectively. $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables. In this case the phase factor simplifies to

$$\frac{1}{2}k_1 \cdot \theta \cdot k_2 = \frac{xk\sqrt{s}}{4\Lambda_{NC}^2}[(C_{01} - C_{13})\sin\theta\cos\phi + (C_{02} + C_{23})\sin\theta\sin\phi + C_{03}(1 + \cos\theta)]. \quad (6)$$

Compton scattering is sensitive to both γ and α, so it is complimentary to pair production studied here.

Fig. 1a shows the cross section σ vs. Λ_{NC} for QED and NCQED with $\alpha = 0$, $\pi/4$ and $\pi/2$, for a $\sqrt{s} = 0.5$ TeV e^+e^- collider operating in $e\gamma$ mode. The QED (solid) curve includes the central QED value and $\pm 1\sigma$ bands (assuming 500 fb^{-1} of integrated luminosity). Fig. 1b shows the angular distribution, $d\sigma/d\phi$, for QED and NCQED with $\alpha = \pi/2$, and $\sqrt{s} = \Lambda_{NC} = 500$ GeV. The error bars in Fig. 1b assume 500 fb^{-1} of integrated luminosity. Note that there is no ϕ dependence for $\alpha = 0$ since for this case both **E** and **B** are parallel to the beam direction. In contrast, when $\alpha = \pi/2$, **E** is perpendicular to the beam direction which is reflected in the strong oscillatory behavior in the ϕ distribution. The exclusion limits obtainable from Compton scattering are summarized in Table II of Ref. [6] for $L = 500 fb^{-1}$, with and without a 2% systematic error. The exclusion limits range from 545 GeV ($\sqrt{s} = 500$ GeV) to 7.4 TeV ($\sqrt{s} = 5.0$ TeV).

The pair production process is only sensitive to space-time NC and is therefore insensitive to γ. As α increases towards $\pi/2$ the deviations from SM decrease towards zero, with $\alpha = \pi/2$ being identical to the SM. On the other hand, the Compton scattering process is sensitive to both space-space and space-time NC as parametrized by γ and α. On the whole, we found that the Compton scattering process is superior to lepton pair production in probing NCQED. Despite significantly smaller statistics, the large modification of angular distributions (see Fig. 1b) leads to higher exclusion limits, well in excess of the center of mass energy for all colliders considered.

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