Extra dimensions vs. supersymmetric interpretation of missing energy events at a linear collider

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(Dated: December 10, 2001)

The photon plus missing energy signature is a primary handle on two important classes of theories. Theories with large extra dimensions predict the production of photons in association with Kaluza-Klein excitations of the graviton. In supersymmetric theories with superlight gravitinos, photons can be produced in association with gravitino pairs. The signatures for these two theories are compared, and it is found that they can be distinguished by studying the photon energy distributions and scaling of the cross section with center-of-mass energy. Both these methods fail, however, if there are six extra dimensions. In that case, additional phenomena predicted by the theories would be required to narrow down the underlying causes of the photon plus missing energy signal. We also study the ability of these measurements to determine the number of extra dimensions.

I. INTRODUCTION

In this report we assume that a signature for new physics is observed at a linear collider in the photon + missing energy channel,
\[ e^+e^- \rightarrow \gamma + \not{E}. \] \hspace{1cm} (1)

We analyze two possible interpretations of this signal. One possibility is in the context of the proposal of Arkani-Hamed, Dimopoulos and Dvali (ADD) \cite{ADD} that we live in a world with extra spatial dimensions. In this proposal, the gauge hierarchy problem is resolved by assuming that the fundamental, higher-dimensional gravitational scale is around a TeV. The apparent high value of the Planck scale in the effective four-dimensional theory can be explained if the volume of the extra dimensions is large. The Standard Model (SM) degrees of freedom are confined to a four-dimensional manifold (a “brane”) in the full space-time, while gravitons can propagate in the extra dimensions. If an extra-dimensional graviton is emitted in a particle collision, it would not be detected, leading to the signature of (1).

Another possibility is in the context of an effective supersymmetric theory obtained from a spontaneously broken supergravity theory. If the scale of supersymmetry breaking is sufficiently low such theories will contain an extremely light gravitino, \( m_{\tilde{G}} \sim 10^{-3} \text{ eV} \). Gravitino pair production in conjunction with a photon will again produce the signature of (1).

In this work we ask how well we can interpret a photon + missing energy signal. In particular, we would like to know whether one can distinguish between the two possibilities discussed above. We will also study if it is possible to distinguish between the ADD-type models with various numbers of extra dimensions.

II. GRAVITONS IN EXTRA DIMENSIONS

In theories with large extra dimensions, the missing energy in \( e^+e^- \rightarrow \gamma + \not{E} \) could be carried by the Kaluza-Klein (KK) excitations of the gravitons. The differential cross-section is given by \cite{Arkani-Hamed:1998rs}:[3]:
\[ \frac{d^2 \sigma}{dx_\gamma d \cos \theta}(e^+e^- \rightarrow \gamma G) = \frac{\alpha}{32} S_{n-1} \left( \frac{\sqrt{s}}{M_s^{(n)}} \right)^{n+2} \frac{1}{s} f(x_\gamma, \cos \theta) \] \hspace{1cm} (2)

\[ f(x, y) = \frac{2(1-x)^{\frac{n}{2}-1}}{x(1-y^2)} \left[ (2-x)^2(1-x+x^2) - 3y^2 x^2(1-x) - y^4 x^4 \right]. \] \hspace{1cm} (3)

Here \( M_s^{(n)} \) is the fundamental mass scale, \( n \) is the number of extra dimensions, \( S_{n-1} \) is the surface area of an \( n \)-dimensional sphere of unit radius, \( x_\gamma = 2E_\gamma/\sqrt{s} \), \( E_\gamma \) is the photon energy, and \( \theta \) is the angle between the photon and beam directions.
FIG. 1: The differential cross-section as a function of the fractional photon missing energy. The scales $M_{(n)}^*$ and $m_{\tilde{G}}$ are tuned so that the total cross-section is the same for each model. The three solid lines are for the graviton KK signal with 2, 4 and 6 extra dimensions. At $x_\gamma = 0.5$ the ordering of these solid lines is $n = 2$ highest, and $n = 6$ lowest. The dashed line is the gravitino signal.

III. GRAVITINO SIGNAL

In supersymmetric theories with a superlight gravitino, photon + missing energy events could arise due to the pair-production of gravitinos, $e^+ e^- \rightarrow \tilde{G}\tilde{G} \gamma$. The differential cross-section is given by [4]:

$$\frac{d^2 \sigma}{dx_\gamma d\cos\theta} = \left(\frac{\alpha G_N}{45}\right) \frac{s^3}{m_{\tilde{G}}^4} f_{\tilde{G}\tilde{G}\gamma}(x_\gamma, \cos\theta)$$

(4)

$$f_{\tilde{G}\tilde{G}\gamma}(x, \cos\theta) = 2(1 - x)^2 \left[ \frac{(1 - x)(2 - 2x + x^2)}{x \sin^2\theta} + \frac{x(-6 + 6x + x^2)}{16} - \frac{x^3 \sin^2\theta}{32} \right]$$

(5)

where $\alpha$ is the fine structure constant, $G_N$ is the gravitational constant, $m_{\tilde{G}}$ is the gravitino mass, $x_\gamma$ is the photon scaled energy ($E_\gamma/E_{\text{beam}}$) and $\theta$ is the polar angle.

IV. OBSERVATIONS

Single photon plus missing energy events also occur within the SM due to escaping neutrinos. The SM background is $e^+ e^- \rightarrow \nu\bar{\nu}\gamma$, and can be obtained from numerous Monte Carlo packages, including Pandora [5]. Previous analyses [2–4] have carefully outlined the parameter spaces that produce a discernible excess of signal events over SM background in extra dimensional gravity models and in superlight gravitino models. We do not reproduce those results here, but are more interested in how similar or dissimilar the graviton-induced signal is to the gravitinos-induced signal.

To help us see the difference between the possible signal interpretations of single photon plus missing energy events at a linear collider, we have plotted in Fig. 1 the differential cross-section as a function of the photon missing energy, $x_\gamma = 2E_\gamma/\sqrt{s}$, obtained by integrating the cross-sections (2) and (4) with respect to $\cos\theta$. The three solid lines are for the graviton KK signal with 2, 4 and 6 extra dimensions. At $x_\gamma = 0.5$ the ordering of the solid lines is $n = 2$ highest, and $n = 6$ lowest. The dashed line is the gravitino signal. The scales ($M_{(n)}^*$ for the ADD-type models and $m_{\tilde{G}}$ for the supersymmetric model) have been chosen so that the total cross sections within the applied kinematic cuts, $x_\gamma > 0.05$ and $|\cos\theta| < 0.95$, are equal in all four cases. Similarly, Fig. 2 shows the normalized differential cross-sections of the four models as a function of $\cos\theta$, obtained by integrating with respect to $x_\gamma$. The ordering of the solid lines at $\cos\theta = 0$ is $n = 2$ highest, and $n = 6$ lowest.

It is clear from the figures that both the recoil energy and angular distributions of the events are quite similar in all four cases. It is still possible, however, to use the photon energy distribution to discriminate between the
FIG. 2: The differential cross-section as a function of the photon \( \cos \theta \). The scales \( M_\gamma^{(n)} \) and \( m_\tilde{G} \) are tuned so that the total cross-section is the same for each model. The three solid lines are for the graviton KK signal with 2, 4 and 6 extra dimensions. At \( \cos \theta = 0 \) the ordering of these solid lines is \( n = 2 \) highest, and \( n = 6 \) lowest. The dashed line is the gravitino signal.

ADD models with different values of \( n \). To do this, we perform a harder cut on the photon energy, \( x_\gamma > 0.2 \). This is advantageous because in this region, the differences between cross sections with different \( n \) do not change sign. We also perform an additional cut on \( x_\gamma \) to get rid of the peak in the Standard Model background associated with on-shell \( Z \) production. Assuming that electron and positron beam polarizations of 80 and 60\% respectively are achieved at a 500 GeV linear collider, and that the cross section can be measured with 1\% precision, we find that, for example, the case \( n = 2 \) can be distinguished from \( n = 3 \) for values of \( M_\gamma^{(2)} \) up to 4.6 TeV. (If the uncertainty is purely statistical, 1\% error on the cross section measurement corresponds to integrated luminosity of about 270 fb\(^{-1}\).) Note, however, that the photon energy distributions for the cases of gravitino emission and \( n = 6 \) ADD model are extremely close. It is not likely that the ambiguity between these two cases can be resolved by this measurement.

We have also studied whether performing a more restrictive angular cut could help in distinguishing between the cases of gravitino and \( n = 6 \) graviton emission. From Fig. 2 it appears that the two signals have discernible differences at small values of \( \cos \theta \). We have found, however, that with the assumed 1\% precision on the cross section measurement, this analysis will not be able to conclusively separate these two cases.

Another way to distinguish between models is by varying the collider center of mass energy \( \sqrt{s} \). The TESLA study [6] has shown that measuring cross sections at two design center of mass energies, 500 GeV and 800 GeV, allows to determine the number of extra dimensions in the ADD models for a wide range of parameters. For example, if there are two extra dimensions, the \( n = 3 \) case can be excluded at 99\% CL for values of \( M_\gamma^{(2)} \) up to 6.1 TeV. The beam polarizations assumed in [6] coincide with the values chosen for our analyses here.

While upgrading the linear collider energy from 500 GeV to 800 GeV, for example, will take much time, effort and money, the beam energy can be changed by as much as 5\% without significantly disrupting the operation of the collider. Therefore, it is interesting to study if different models can be distinguished by performing measurements at two center of mass energies separated by 5\%. We have performed such a study using Pandora. We have chosen the scales of each model so that their cross sections (with the same cuts and polarizations as before) are identical at \( \sqrt{s} = 500 \) GeV, and evaluated their differences at \( \sqrt{s} = 475 \) GeV. Taking the Standard Model background into account and assuming that the cross sections at each energy can be measured with 1\% precision, we find that the \( n = 2 \) and \( n = 3 \) cases can be distinguished at the 3\( \sigma \) level for \( M_\gamma^{(2)} \lesssim 3.9 \) TeV. The sensitivity of this method is thus somewhat lower than for studying photon energy distributions. Surprisingly, the scaling of the gravitino pair production cross section with \( \sqrt{s} \) is again identical to the \( n = 6 \) ADD model. Changing collider energy does not help in separating these two cases.

We conclude that while the ADD models with \( n \neq 6 \) can be distinguished from the supersymmetric models with light gravitino by carefully studying the photon + missing energy signal alone, this cannot be done for the case of 6 extra dimensions. (Let us note in passing that the missing energy signal in the \( n = 6 \) ADD model is also identical to the one due to the emission of scalar states associated with the brane coordinates [7].) In this case, distinctions between the models will only occur with additional phenomena. The next-best observables in
the extra-dimensional graviton model include the dimension-eight contact interactions \[2, 8\] induced by
\[
e^+e^- \rightarrow G^{(n)} \rightarrow \gamma\gamma, f\bar{f}, \text{ etc.,}
\]
\[e^+e^- \rightarrow G^{(n)} \rightarrow \gamma\gamma, f\bar{f}, \text{ etc.,} (6)\]
or from string Regge states \[9, 10\].

The superlight gravitino models are supersymmetric models, and the superpartner spectrum must satisfy the same naturalness criteria as other supersymmetry ideas. The expectation is that some superpartner states, most notably charginos and neutralinos, should be accessible and well-studied at a 500 GeV linear collider \[11\]. The kinematic reach for superpartners then approaches \[\sqrt{s}/2 = 250 \text{ GeV}\] in \(e^+e^-\) collisions. An increase of the reach of selectrons is possible in \(e^-\gamma \rightarrow \tilde{G}\tilde{e}_{R,L}\) collisions, since selectron masses approaching \[\sqrt{s} = 500 \text{ GeV}\] are kinematically accessible \[12\]. The full complement of observables will be needed to distinguish the precise underlying theory being discovered at the colliders.

V. ACKNOWLEDGEMENTS

We would like to thank the organizers of the Snowmass 2001 workshop where this work was initiated. We are grateful to many of the workshop’s participants and especially to the members of the P3 and E3 working groups for interesting discussions. We are indebted to Michael Peskin for his invaluable help with the Pandora event generator.

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

\[6\] G.W. Wilson, LC-PHSM-2001-010, \texttt{http://www.desy.de/~lcnotes}.