The current world average of the muon’s anomalous magnetlc moment $a_\mu$ differs from the standard model prediction by 2.6σ: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (43 \pm 16) \times 10^{-10}$ [1]. The reported deviation is about three times larger than the standard model’s electroweak contribution [2], and so deviations of roughly this order are expected in many models motivated by attempts to understand electroweak symmetry breaking. Its interpretation as supersymmetry is particularly attractive, as supersymmetry naturally provides electroweak scale contributions that are easily enhanced (by large tan β) to produce deviations larger than the standard model’s electroweak corrections. In addition, $a_\mu$ is both flavor- and CP-conserving. Thus, while the impact of supersymmetry on other low energy observables can be highly suppressed by scalar degeneracy or small CP-violating phases in simple models, supersymmetric contributions to $a_\mu$ cannot be. In this sense, $a_\mu$ is a uniquely robust probe of supersymmetry, and an anomaly in $a_\mu$ is a natural place for the effects of supersymmetry to appear.

We begin with the most model-independent analysis possible consistent with slepton flavor conservation. (For more details, see Ref. [3].) In general, the reported deviation may be explained entirely by new physics in the muon’s electric dipole moment [4]. However, this possibility is not realized in supersymmetry, and we assume that the deviation arises solely from contributions to $a_\mu$. We then consider uncorrelated values of the relevant SUSY parameters, allowing arbitrary gaugino mass parameters and slepton masses and left-right mixings. Despite the generality of this framework, we find stringent upper bounds on charged superpartners, with strong implications for future collider searches. We then consider minimal supergravity, and find remarkable consistency of the $a_\mu$ deviation with all present constraints from colliders, dark matter searches, and precision observables, such as $B \to X_s \gamma$. This is not to be taken for granted: as an illustration, we show that the current $a_\mu$ result disfavors anomaly-mediated supersymmetry breaking.

Supersymmetric contributions to $a_\mu$ have been explored for many years [5]. Following the recent $g_\mu - 2$ result, the implications for supersymmetry have been considered in many studies [6-30] in a variety of frameworks, including minimal supergravity [8, 9, 12, 13, 16, 20, 28], no-scale supergravity [14, 17, 20], models with non-universality [13, 22, 29, 30], inverted hierarchy [20], gauge-mediation [9, 14, 16, 20], anomaly-mediation [14, 20], R-parity violation [15, 26], and CP-violation [11], as well as flavor models [10, 19, 21, 23, 24], and string-inspired frameworks [14, 16, 18, 24].

The anomalous magnetic moment of the muon is the coefficient of the operator $a_\mu \bar{\mu}_\mu \mu^\alpha \mu F^\alpha_{mn}$, where $F^\alpha_{mn} = \frac{i}{2} \{ \gamma^m, \gamma^n \}$. The supersymmetric contribution, $a_\mu^{\text{SUSY}}$, is dominated by diagrams with neutralino-stau and chargino-sneutrino loops. In the absence of significant slepton flavor violation, these diagrams are completely determined by only seven supersymmetry parameters: $M_1, M_2, \mu, \tan \beta, m_{\tilde{\mu}_L}, m_{\tilde{\mu}_R}$, and $A_\mu$. In general, $M_1, M_2, \mu,$ and $A_\mu$ are complex. However, bounds from electric dipole moments generally require their phases to be very small. In addition, $|a_\mu^{\text{SUSY}}|$ is typically maximized for real parameters. In deriving model-independent upper bounds on supersymmetric parameters below, we assume real parameters, but consider all possible sign combinations; these results are therefore valid for arbitrary phases.

To determine the possible values of $a_\mu^{\text{SUSY}}$ without model-dependent biases, we have calculated $a_\mu^{\text{SUSY}}$ in a series of high statistics scans of parameter space, requiring only consistency with collider bounds and a neutral stable lightest supersymmetric particle (LSP). (LSPs that decay visibly in collider detectors are examined in Ref. [3].) We begin by scanning over the parameters $M_2, \mu, m_{\tilde{\mu}_L},$ and $m_{\tilde{\mu}_R}$, assuming gaugino mass unification $M_1 = M_2/2, A_\mu = 0,$ and $\tan \beta = 50$. The free parameters take values up to 2.5 TeV. The resulting values in the $(M_2, a_\mu^{\text{SUSY}})$ plane are given by the points in Fig. 1. We then consider arbitrary (positive and negative) values of $M_2/M_1$, leading to possibilities bounded by the solid curve. Finally, we allow any $A_{\mu}$ in the interval $[-100 \text{ TeV}, 100 \text{ TeV}]$.  

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which extends the envelope curve to the dashed contour of Fig. 1. As can be seen, allowing large $A_\mu$, i.e., large left-right smuon mixing, significantly enlarges the range of possible $a_\mu$. The envelope contours scale linearly with $\tan \beta$. The 1σ and 2σ allowed $a_\mu^\text{SUSY}$ ranges are shown, and the discovery reaches of linear colliders with $\sqrt{s} = 500$ GeV and 1 TeV are given by the vertical blue lines.

Conclusions. Finally, these bounds provide fresh impetus for searches for lepton flavor violation, which is also mediated by sleptons and charginos/neutralinos.

We now turn to specific models, where $a_\mu$ is correlated to many other observables. We first consider the framework of minimal supergravity, where models are completely specified by the parameters $m_0$, $M_1/2$, $A_0$, $\tan \beta$, and $\text{sign}(\mu)$. The first three are the universal scalar, gaugino, and trilinear coupling masses at $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV. We determine the entire weak scale superparticle spectrum through two-loop renormalization group equations [31] with one-loop threshold corrections and superparticle masses [32].

In minimal supergravity, many potential low-energy effects are eliminated by scalar degeneracy. However, $a_\mu^\text{SUSY}$ is not suppressed in this way and may be large. In this framework, $\text{sign}(a_\mu^\text{SUSY}) = \text{sign}(\mu M_{1,2})$. As is well-known, however, the sign of $\mu$ also enters in the supersymmetric contributions to $B \to X_s \gamma$. Current constraints on $B \to X_s \gamma$ require $\mu M_3 > 0$ if $\tan \beta$ is large. Gaugino mass unification implies $M_{1,2} M_3 > 0$, and so a large discrepancy in $a_\mu$ is only possible for $a_\mu^\text{SUSY} > 0$, in accord with the new measurement.

In Fig. 2, the 2σ allowed region for $a_\mu^\text{SUSY}$ (hatched) in minimal supergravity, for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan \beta$. The dark red regions are excluded by the requirement of a neutral LSP and by the chargino mass limit of 103 GeV, and the medium blue (light yellow) region has LSP relic density $0.1 \leq \Omega h^2 \leq 0.3$ ($0.025 \leq \Omega h^2 \leq 1$). The area below the solid (dashed) contour is excluded by $B \to X_s \gamma$ (the Higgs boson mass), and the regions probed by the tri-lepton search at Tevatron Run II are below the dotted contours.
parameter space again consistent with all constraints and possessing the desired relic density [33]. The cosmologically preferred regions of minimal supergravity are probed by many pre-LHC dark matter experiments [34]. Note, however, that the sign of \( \mu \) preferred by \( a_\mu \) implies destructive interference in the leptonic decays of the second lightest neutralino, and so the Tevatron search for trileptons is ineffective for 200 GeV < \( m_0 < 400 \) GeV [35].

We close by considering anomaly-mediated supersymmetry breaking. One of the most striking predictions of this framework is that the gaugino masses are proportional to the corresponding beta function coefficients, and so \( M_1, M_3 < 0 \). Anomaly-mediation therefore most naturally predicts \( a_\mu \approx 0 \) [36, 37], in contrast to the observed deviation.

A more detailed quantitative analysis [3] in minimal anomaly-mediation finds that, even allowing a 1 \( \sigma \) deviation in \( a_\mu \), it is barely possible to obtain 2 \( \sigma \) consistency with the \( B \rightarrow X_\gamma \gamma \) constraint. Minimal anomaly mediation is therefore disfavored. The dependence of this argument on the characteristic gaugino mass relations of anomaly mediation suggests that similar conclusions will remain valid beyond the minimal model.

In conclusion, the recently reported deviation in \( a_\mu \) is easily accommodated in supersymmetric models. Its value provides model-independent upper bounds on masses of observable superpartners and already discriminates between well-motivated models.

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