

Proving Higgs Bosons are scalars at a Linear Collider

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The threshold dependence and angular correlations of Higgs-strahlung in e^+e^- collisions can be used to demonstrate the spinless nature of the Standard Model Higgs boson. This method, and its possible extension to heavy neutral MSSM Higgs bosons, is discussed.

1. The next generation of colliders will most likely discover the Higgs boson, or whatever other mechanism is responsible for Electroweak Symmetry Breaking. In order to distinguish whether any newly discovered particle is the expected Higgs boson or something else not yet predicted, it is important to experimentally determine its properties. One such task is the verification of the scalar nature of the Higgs boson.

The isotropic decay of a Higgs boson is a necessary consequence of the zero spin of the Higgs boson but is not sufficient to confirm its spinless nature, since other higher spin objects may also exhibit isotropic decays [1]. While such an observation is doubtless an interesting consistency check, it is constructive to consider other methods. In this talk I will describe the verification of the spinless nature of the Standard Model (SM) Higgs boson from the threshold behaviour and angular correlations of Higgs-strahlung in e^+e^- collisions, $e^+e^- \rightarrow ZH$. I then discuss the possible extension of this method to the heavy Higgs bosons of the Minimal Supersymmetric Standard Model (MSSM).

2. As a first step, it is useful to examine the Higgs-strahlung cross-section and angular distributions for arbitrary spin 'Higgs bosons' without specifying the model. (Although a boson of non-zero spin is undoubtedly *not* our traditional Higgs boson, I shall, in the interests of simplicity continue to refer to it as such, and will continue to use the term *Higgs-strahlung* for its radiation off a Z boson.) Using only the properties of the angular momentum operator, the helicity amplitude for the $Z^* \rightarrow ZH$ current can be written as [2],

$$\langle Z(\lambda_Z)H(\lambda_H)|Z^*(m)\rangle = \frac{g_W M_Z}{\cos \theta_W} d_{m, \lambda_Z - \lambda_H}^1(\theta) \Gamma_{\lambda_Z \lambda_H}, \quad (1)$$

where λ_Z , λ_H and m are the helicities of the real Z, Higgs and virtual Z bosons respectively, and $d_{mm'}^j(\theta)$ are the usual d-functions resulting from the projection of different angular momentum eigenstates on to one another. $\Gamma_{\lambda_Z \lambda_H}$ are *reduced helicity amplitudes* which are model dependent, and normalized to be dimensionless. If the Higgs sector is CP conserving then the parity relation between helicity amplitudes for a Higgs boson of spin J and parity \mathcal{P} can be exploited:

$$\Gamma_{\lambda_Z \lambda_H} = (-1)^J \mathcal{P} \Gamma_{-\lambda_Z - \lambda_H}. \quad (2)$$

In this way, the Higgs-strahlung cross-section is given by,

$$\sigma = \frac{G_F^2 M_Z^6 (v_e^2 + a_e^2)}{24\pi s^2 (1 - M_Z^2/s)^2} \beta \left[|\Gamma_{00}|^2 + 2 |\Gamma_{11}|^2 + 2 |\Gamma_{01}|^2 + 2 |\Gamma_{10}|^2 + 2 |\Gamma_{12}|^2 \right], \quad (3)$$

where v_e and a_e are the vector and axial-vector Z charges of the electron; M_Z is the Z-boson mass, \sqrt{s} the centre-of-mass energy, and the normalized Z/H velocity is $\beta = 2p/\sqrt{s}$ with p the Z/H three-momentum in the centre-of-mass frame. The Standard Model cross-section is recovered by inserting, $\Gamma_{00} = -E_Z/M_Z$, $\Gamma_{10} = -1$ and setting all other helicity amplitudes to zero, resulting in an excitation curve rising linearly with β near the threshold, $\sigma \sim \beta$. It is this distinctive property of the SM which can be exploited to verify the spinless nature of the Higgs boson.

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Similarly, the polar angle distribution of the Higgs and Z bosons in the final state can be written,

$$\frac{d\sigma}{d\cos\theta} \propto \sin^2\theta \left[|\Gamma_{00}|^2 + 2|\Gamma_{11}|^2 \right] + [1 + \cos^2\theta] \left[|\Gamma_{01}|^2 + |\Gamma_{10}|^2 + |\Gamma_{12}|^2 \right], \quad (4)$$

Inserting the SM reduced helicity amplitudes one sees that the SM distribution of the polar angle θ is isotropic near the threshold.

Finally, independent information can be obtained by considering the final-state fermion distributions in the decay $Z \rightarrow f\bar{f}$. Denoting the fermion polar angle in the Z rest frame with respect to the Z flight direction in the laboratory frame by θ_* , the double differential distribution in θ and θ_* is given by.

$$\begin{aligned} \frac{d\sigma}{d\cos\theta d\cos\theta_*} \propto & \sin^2\theta \sin^2\theta_* |\Gamma_{00}|^2 + \frac{1}{2}[1 + \cos^2\theta][1 + \cos^2\theta_*] \left[|\Gamma_{10}|^2 + |\Gamma_{12}|^2 \right] \\ & + [1 + \cos^2\theta] \sin^2\theta_* |\Gamma_{01}|^2 + \sin^2\theta [1 + \cos^2\theta_*] |\Gamma_{11}|^2 \\ & + \frac{2v_e a_e}{(v_e^2 + a_e^2)} \frac{2v_f a_f}{(v_f^2 + a_f^2)} 2\cos\theta \cos\theta_* \left[|\Gamma_{10}|^2 - |\Gamma_{12}|^2 \right]. \end{aligned} \quad (5)$$

Note that in the SM the reduced helicity amplitudes Γ_{01} and Γ_{11} are zero and consequently the $[1 + \cos^2\theta] \sin^2\theta_*$ and $\sin^2\theta [1 + \cos^2\theta_*]$ correlations are absent.

3. Of course, these formulae are not useful unless one has expressions for the reduced helicity amplitudes in a particular model. To find these, consider the most general current describing the Z^*ZH vertex,

$$J_\mu = \frac{g_W M_Z}{\cos\theta_W} T_{\mu\alpha\beta_1\ldots\beta_J} \varepsilon^*(Z)^\alpha \varepsilon^*(H)^{\beta_1\ldots\beta_J}, \quad (6)$$

where ε^α is the usual spin-1 polarization vector and $\varepsilon^{\beta_1\ldots\beta_J}$ is the spin- J polarization tensor which is symmetric, traceless and orthogonal to the 4-momentum of the Higgs boson $p_H^{\beta_i}$. The tensor $T_{\mu\alpha\beta_1\ldots\beta_J}$ is effectively transverse due to the conservation of the lepton current. By choosing the most general tensor for the appropriate spin, and comparing with Eq.(1), expressions for the reduced helicity amplitudes can be found.

For example, the most general tensor for a $J^P = 0^+$ Higgs coupling to the Z boson is given by,

$$T^{\mu\alpha} = a_1 g_\perp^{\mu\alpha} + a_2 k_\perp^\mu q^\alpha, \quad (7)$$

where $q = p_Z + p_H$, $k = p_Z - p_H$ and \perp denotes orthogonality to q . Projecting with the polarization vector, and comparing this with Eq.(1), one obtains $\Gamma_{00} = (-a_1 E_Z - \frac{1}{2} a_2 s^{3/2} \beta^2)/M_Z$, $\Gamma_{10} = -a_1$ and all other reduced helicity amplitudes vanish. Clearly, the SM is restored with the choice $a_1 = 1$ and $a_2 = 0$.

In this way, it is straightforward to write down general reduced helicity amplitudes for theories with higher spin Higgs bosons. In general, the coefficients of the tensors in $T^{\mu\alpha\beta_1\ldots\beta_J}$ (a_1 and a_2 in the 0^+ example above) may be functions of β , so the full behaviour of the cross section is still unknown. However, near threshold, where β is small, one may make a power series expansion of these functions, and predict the steepest possible β dependence of the cross-section in the threshold region. This prediction can be examined experimentally allowing one to rule out certain spin states.

A full analysis of all possible J^P states [3] reveals that every spin-parity combination yields a faster than β rise of the cross-section at threshold, except for the 0^+ , 1^+ and 2^+ states. In particular, states of $J < 3$ and odd parity present at least a β^3 rise of the cross-section near threshold, and for $J \geq 3$ the behaviour is $\sim \beta^{2J-3}$ and $\sim \beta^{2J-1}$ for $(-1)^{JP} = \pm 1$ respectively. It is therefore a simple matter to rule out the majority of J^P states by simply measuring the cross-section rise at threshold [4].

4. The exceptional cases must be ruled out using extra observations. Generally, the requirement that the 1^+ or 2^+ states mimic the linear β rise of the 0^+ state places restrictions on the reduced helicity amplitudes. For a 2^+ Higgs boson, the term in the Z^*ZH coupling which provides a

linear rise ($g^{\alpha\beta_1} g_{\perp}^{\mu\beta_2} + g^{\alpha\beta_2} g_{\perp}^{\mu\beta_1}$) also contributes to all possible reduced helicity amplitudes. Consequently all of the reduced helicity amplitudes in this model are non-zero and would provide angular correlations in the decay of the Z boson, Eq.(5), in direct contradiction of the SM. For example, if one then observes no $[1 + \cos^2 \theta] \sin^2 \theta_*$ or no $\sin^2 \theta [1 + \cos^2 \theta_*]$ correlations then a 2^+ state is ruled out.

The 1^+ state is even easier to experimentally disprove. Firstly, the observation of $H \rightarrow \gamma\gamma$ decays or the formation of Higgs bosons in photon collisions, $\gamma\gamma \rightarrow H$, rules out all spin-1 assignments as a result of the Landau-Yang theorem. In addition, the spin-parity relation among the reduced helicity amplitudes, Eq.(2), implies that Γ_{00} must vanish (indeed, this is also true for any J^P assignment where $(-1)^J \mathcal{P} = -1$, i.e. $2^-, 3^+, 4^-$ etc.). Therefore the observation of a $\sin^2 \theta \sin^2 \theta_*$ correlation in Eq.(5), as predicted by the Standard Model, eliminates this possibility. Finally, as with the 2^+ Higgs state, a linear β rise of the cross-section also requires that Γ_{01} and Γ_{11} be non-zero, so non-observation of $[1 + \cos^2 \theta] \sin^2 \theta_*$ or $\sin^2 \theta [1 + \cos^2 \theta_*]$ correlations also rules out a 1^+ Higgs boson.

Non-zero spin Higgs bosons of non-definite parity (i.e. when the Z^*ZH vertex is parity violating) are equally straightforward to disprove. In this case one may no longer use Eq.(2) to obtain the simple form of the (differential) cross sections seen in Eqs.(3-5). In particular, the polar angle distribution, Eq.(4), is modified to include a linear term proportional to $\cos \theta$, indicative of CP violation. The analysis, however, proceeds as in the fixed normality case, since the most general tensor vertex will be the sum of the even and odd parity tensors (with appropriate phase factors). One finds that, for every spin, only one of the even or odd parity tensors dominates at threshold, so that the procedure outlined above will also eliminate all mixed parity states with non-zero spins.

Also observe that this measurement can very easily rule out an odd parity Higgs boson, which has at best a $\sim \beta^3$ rise of the cross-section at threshold. It is, however, unable to distinguish between the SM 0^+ Higgs boson and a scalar Higgs boson of indefinite parity, since their threshold behaviour will be indistinguishable.

5. The MSSM contains two Higgs doublets, and consequently five physical Higgs states: two CP even (h and H), one CP odd (A), and two charged Higgs bosons (H^\pm). Clearly the lightest CP even Higgs boson spin can be determined via Higgs-strahlung, $e^+e^- \rightarrow Zh$, exactly as for the SM Higgs boson, as described above. The heavier Higgs bosons, however, present more of a challenge. Here I consider only the heavy neutral Higgs bosons.

At first sight our method seems hopeless for determining the spin of the heavy Higgs bosons. Heavy Higgs-strahlung, $e^+e^- \rightarrow ZH$, and production of the pseudoscalar in association with the light scalar, $e^+e^- \rightarrow Ah$, are, in this case, not useful since their cross-sections are suppressed by the square of $\cos(\alpha - \beta)$ making them prohibitively small. The only process of use for this method is the production of the pseudoscalar together with the heavy scalar, $e^+e^- \rightarrow AH$. However, this process, presents two major difficulties. Firstly, we have two unknown spins in the process (assuming that neither spin has been experimentally determined elsewhere), giving many more different spin combinations which must be ruled out. Secondly, and more significantly, in the MSSM the decay products of the spin one virtual Z boson are scalar particles, and consequently form a P-wave, with an expected threshold dependence $\sim \beta^3$. This slowly rising signal makes the threshold measurement much more difficult for experiment. Furthermore, any higher spin combination which one would naturally expect to form an S-wave, may mimic such a P-wave by having all their non-zero helicity amplitudes $\sim \beta$ at threshold.

However, one possible exploitable feature is that, since both H and A are scalars, the MSSM process has only one reduced helicity amplitude Γ_{00} . Consequently the differential cross-section with respect to the polar angle of the outgoing pair is proportional to $\sin^2 \theta$ (see Eq.(4)). By contrast, when one considers the possible tensor structures for the Z^*HA vertex which produce similar β^3 threshold cross-section dependences, one finds that most also contribute other non-zero reduced helicity amplitudes resulting in $[1 + \cos^2 \theta]$ correlations in the differential cross-section.

The exceptions to this rule occur when the H and A spins are identical and non-zero. In order to discount them, one must again resort to the further decay into fermion pairs. Note that this is much more complicated than for the SM Higgs boson since we must now consider the decay of a boson whose spin is unknown, in contrast to the decay of the well studied Z boson. This is beyond the scope of this contribution and will be reported elsewhere [5].

6. In summary, the J^P quantum numbers for a SM Higgs boson can be unambiguously determined by measuring the threshold dependence of the Higgs-strahlung cross-section, $e^+e^- \rightarrow ZH$, and the angular correlations $[1 + \cos^2 \theta] \sin^2 \theta_*$ and $[1 + \cos^2 \theta_*] \sin^2 \theta$. The observation of a linear dependence on β at threshold eliminates all spin-parity assignments different from 0^+ except 1^+ and 2^+ , and these last two assignments may then be ruled out by the absence of the above correlations. The method can be extended to the MSSM, where the processes $e^+e^- \rightarrow Zh$ and $e^+e^- \rightarrow HA$ can be used to experimentally verify the spinless nature of all neutral Higgs bosons of the theory.

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