Limits on the Mass of a Composite Higgs Boson: an Update

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We discuss the bound on the mass of the Higgs boson arising from precision electroweak measurements in the context of the triviality of the scalar Higgs model. We show that, including possible effects from the underlying nontrivial dynamics, a Higgs boson mass of up to 500 GeV is consistent with current data.

1. Introduction: Triviality of the Standard Higgs Model

Current results from the LEP Electroweak Working Group [1] favor a Higgs boson mass that is relatively light. The "best-fit" value [1] for the Higgs mass is 106 GeV, somewhat less than experimental lower bound [2] of 114.1 GeV (at 95% confidence level). The 95% CL upper bound from precision measurements, in the context of the standard model, is 222 GeV. It is possible that, as these data suggest, the Higgs boson lies around the corner and will be discovered at relatively low masses. On the other hand, it is important to consider alternatives and to understand what class of models can be consistent with precision electroweak tests. In this talk, we will show that even minor modifications to the standard electroweak theory allow for a substantially heavier Higgs boson[2].

This task is motivated by the fact that the standard one-doublet Higgs model *does not strictly exist* as a continuum field theory [4, 5, 6]. This is because the β-function for the Higgs-boson self-coupling is *positive*. For any finite low-energy coupling, the running coupling-constant has a Landau pole: it diverges at some finite energy. Conversely, defining the model in terms of a momentum-space cutoff Λ, the continuum limit is found by taking $\Lambda \to \infty$ while holding all low-energy properties fixed. In this limit, one finds that $\lambda \to 0$ — *i.e.* the only continuum limit is free or trivial.

The triviality of the scalar sector of the standard one-doublet Higgs model implies that this theory is only an effective low-energy theory valid below some finite cut-off scale Λ. Given a value of $m_H^2 = 2\lambda(m_H)v^2$, there is an *upper* bound on Λ. An *estimate* of this bound [7] can be obtained by integrating the one-loop β-function, which yields

$$\Lambda \lesssim m_H \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right) . \tag{1}$$

For a light Higgs, the bound above is at uninterestingly high scales and the effects of the underlying dynamics can be too small to be phenomenologically relevant. For a Higgs mass of order a few hundred GeV, however, effects from the underlying physics can become important. We will refer to these theories generically as "composite Higgs" models.

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[1]This value for the Higgs mass arises from using the value $\Delta\alpha_{had}^{(5)} = 0.02738 \pm 0.00020$ for the contribution to the running of α_{em} from hadrons.

[2]For a more complete discussion, see [3] and references therein.

2. T, S, and U in Composite Higgs Models

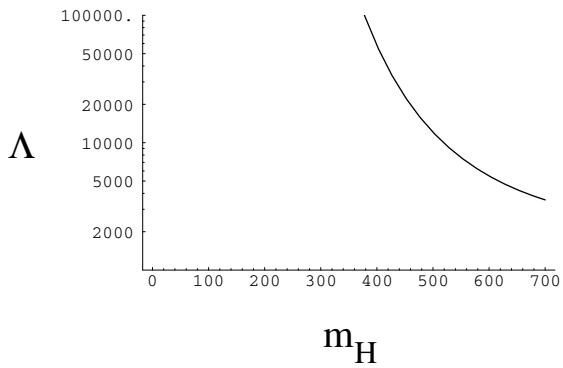


Figure 1: Upper bound on scale Λ as per eqn. (1).

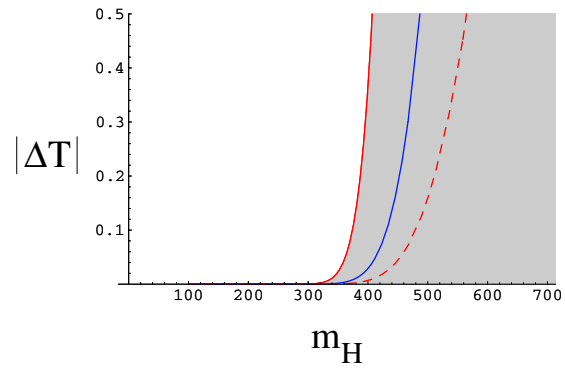
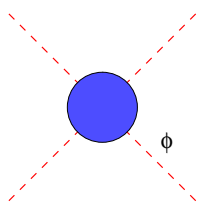


Figure 2: Lower bound on expected size of $|\Delta T|$ as per eqn. (3), for $|b|\kappa^2 = 16\pi^2$, 4π, and 3.

In an $SU(2)_W \times U(1)_Y$ invariant scalar theory of a single doublet, all interactions of dimension less than or equal to four also respect a larger "custodial" symmetry [8, 9] which insures the tree-level relation $\rho = M_W^2/M_Z^2 \cos^2\theta_W \equiv 1$. The leading custodial-symmetry violating operator is of dimension six [10, 11] and involves four Higgs doublet fields ϕ. In general, the underlying theory does not respect the larger custodial symmetry, and we expect the interaction



$$\Rightarrow \frac{b\kappa^2}{2!\,\Lambda^2}(\phi^\dagger \overleftrightarrow{D}^\mu \phi)^2 \,, \tag{2}$$

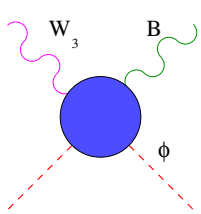
to appear in the low-energy effective theory. Here b is an unknown coefficient of $\mathcal{O}(1)$, and κ measures size of couplings of the composite Higgs field. In a strongly-interacting theory, κ is expected [12, 13] to be of $\mathcal{O}(4\pi)$.

Deviations in the low-energy theory from the standard model can be summarized in terms of the "oblique" parameters [14, 15, 16, 17, 18] S, T, and U. The operator in eqn. 2 will give rise to a deviation ($\Delta\rho = \varepsilon_1 = \alpha T$)

$$|\Delta T| = |b|\kappa^2 \frac{v^2}{\alpha(M_Z)\Lambda^2} \gtrsim \frac{|b|\kappa^2\, v^2}{\alpha(M_Z^2)\, m_H^2} \exp\left(-\frac{8\pi^2 v^2}{3m_H^2}\right) \,, \tag{3}$$

where $v \approx 246$ GeV and we have used eqn. 1 to obtain the final inequality. The consequences of eqns. (1) and (3) are summarized in Figures 1 and 2. The larger m_H, the lower Λ and the larger the expected value of ΔT. Current limits imply $|T| \lesssim 0.5$, and hence $\Lambda \gtrsim 4\,\text{TeV}\cdot\kappa$. (For $\kappa \simeq 4\pi$, $m_H \lesssim 450$ GeV.)

By contrast, the leading contribution to S arises from



$$\Rightarrow -\frac{a}{2!\,\Lambda^2}\,\{[D_\mu, D_\nu]\phi\}^\dagger\,[D^\mu, D^\nu]\phi \,. \tag{4}$$

This gives rise to ($\varepsilon_3 = \alpha S/4\sin^2\theta_W$)

$$\Delta S = \frac{4\pi a v^2}{\Lambda^2} \,. \tag{5}$$

It is important to note that the size of contributions to ΔT and ΔS are very different

$$\frac{\Delta S}{\Delta T} = \frac{a}{b}\left(\frac{4\pi\alpha}{\kappa^2}\right) = \mathcal{O}\left(\frac{10^{-1}}{\kappa^2}\right) . \tag{6}$$

Even for $\kappa \simeq 1$, $|\Delta S| \ll |\Delta T|$.

Finally, contributions to U ($\varepsilon_2 = -\frac{\alpha U}{4\sin^2\theta_W}$), arise from

$$\frac{c g^2 \kappa^2}{\Lambda^4}(\phi^\dagger W^{\mu\nu}\phi)^2 \tag{7}$$

and, being suppressed by Λ^4, are typically much smaller than ΔT.

3. Limits on a Composite Higgs Boson

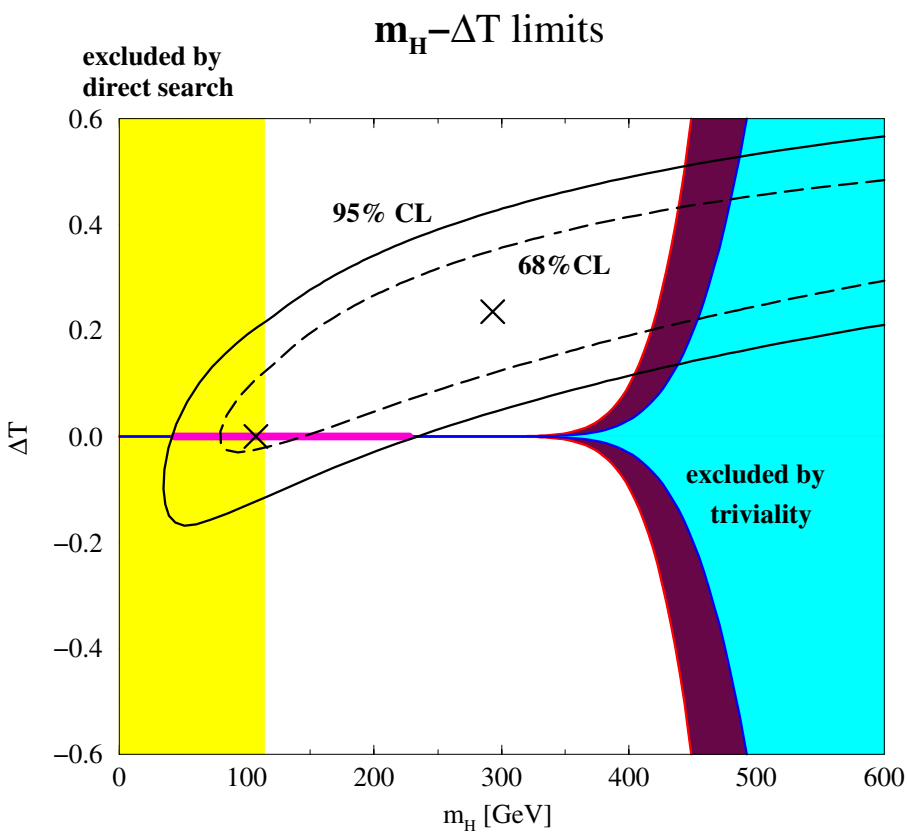


Figure 3: 68% and 95% CL regions allowed [3] in $(m_H, \Delta T)$ plane by precision electroweak data [1]. Fit allows for m_t, α_s, and α_{em} to vary consistent with current limits [3]. Also shown by the the thick line on the $\Delta T = 0$ axis is the usual one-dimensional 95% CL limit quoted on the Higgs boson mass in the standard model, and the corresponding best fit. The triviality bound curves are for $|b|\kappa^2 = 4\pi$ and $4\pi^2$, corresponding to representative models [3]

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From triviality, we see that the Higgs model can only be an effective theory valid below some high-energy scale Λ. As the Higgs becomes heavier, the scale Λ *decreases*. Hence, the expected size of contributions to T *grow*, and are larger than the expected contribution to S or U. The limits from precision electroweak data in $(m_H, \Delta T)$ plane shown in Figure 3. We see that, for positive ΔT at 95% CL, the allowed values of Higgs mass extend to well beyond 800 GeV. On the other hand, not all values can be realized consistent with the bound given in eqn. (1). As shown

in figure 3, values of Higgs mass beyond approximately 500 GeV would likely require values of ΔT much larger than allowed by current measurements.

We should emphasize that these estimates are based on dimensional arguments, and we are not arguing that it is *impossible* to construct a composite Higgs model consistent with precision electroweak tests with m_H greater than 500 GeV. Rather, barring accidental cancellations in a theory without a custodial symmetry, contributions to ΔT consistent with eqn. 1 are generally to be expected.

These results may also be understood by considering limits in the (S, T) plane for *fixed* (m_H, m_t). In Figure 4, changes from the nominal standard model best fit (m_H = 84 GeV) value of the Higgs mass are displayed as contributions to $\Delta S(m_H)$ and $\Delta T(m_H)$. Also shown are the 68% and 95% CL bounds on ΔS and ΔT consistent with current data. We see that, for m_H greater than $\mathcal{O}$(200 GeV), a positive contribution to T can bring the model within the allowed region.

4. The Top Quark Seesaw Model

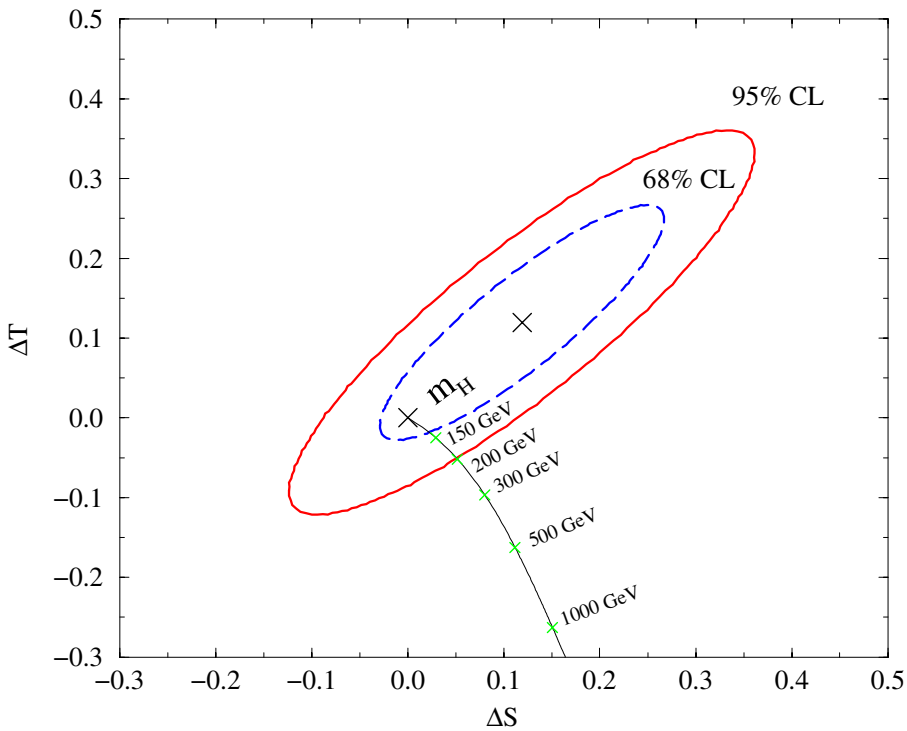

Figure 4: 68% and 95% CL regions allowed in $(\Delta S, \Delta T)$ plane by precision electroweak data [1]. Fit allows for m_t, α_s, and α_{em} to vary consistent with current limits [3]. Standard model prediction for varying Higgs boson mass shown as parametric curve, with m_H varying from 84 to 1000 GeV.

The top-quark seesaw theory of electroweak symmetry breaking [20, 21, 22, 23] provides a simple example of a model with a potentially heavy composite Higgs boson consistent with electroweak data. In this case, electroweak symmetry breaking is due to the condensation, driven by a strong topcolor [24] gauge interaction, of the left-handed top-quark with a new right-handed singlet fermion χ. Such an interaction gives rise to a composite Higgs field at low energies, and the mass of the top-color gauge boson sets the scale of the Landau pole Λ [25]. The weak singlet χ_L and t_R fields are introduced so that the 2×2 mass matrix,

$$\begin{pmatrix} 0 & m_{t\chi} \\ m_{\chi t} & m_{\chi\chi} \end{pmatrix} \tag{8}$$

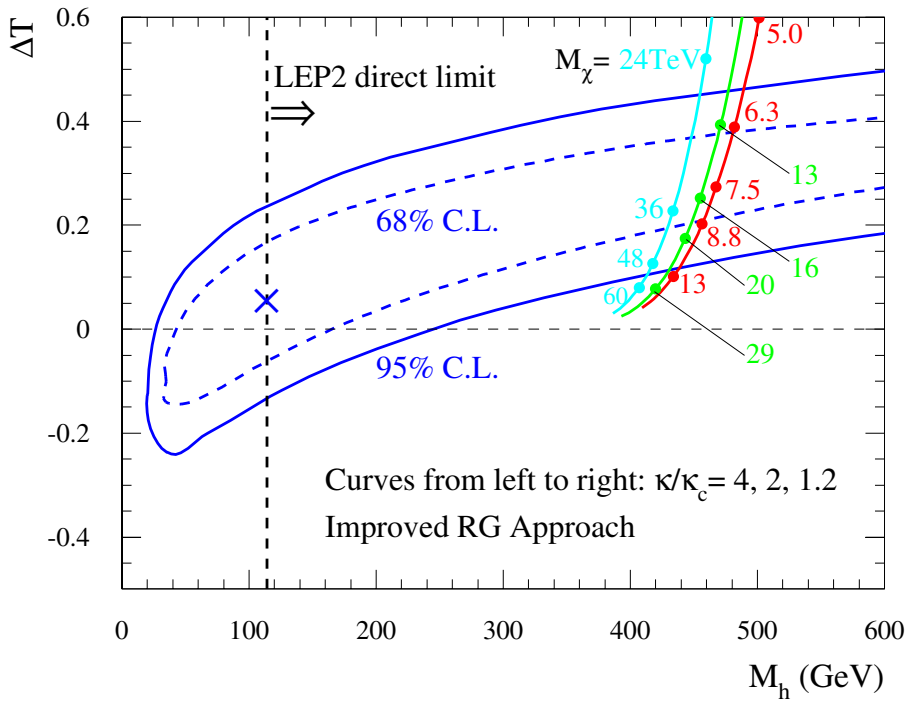

Figure 5: ΔT vs. m_H for the top-quark seesaw model plotted for various values of the mass of the heavy singlet quark, m_χ, and various values of the (strong) topcolor-coupling, $\kappa \propto g_{tc}^2$, superimposed on fit to summer 2000 electroweak precision data. Courtesy of Hong-Jian He, [19].

is of seesaw form ($m_{\chi\chi} \gg m_{t\chi}$, $m_{\chi t}$) and has a light eigenvalue corresponding to the observed top quark. The value of $m_{t\chi}$ is related to the weak scale, and its value is estimated to be 600 GeV [20].

The coupling of the top-quark to χ violates custodial symmetry in the same way that the top-quark mass does in the standard model. The leading contribution to T from the underlying top seesaw physics arises from contributions to W and Z vacuum polarization diagrams involving the χ. This contribution is positive and is calculated to be [20, 22, 23]

$$\Delta T = \frac{N_c}{16\pi^2 \alpha_{em}(M_Z^2)} \frac{m_{t\chi}^4}{m_{\chi\chi}^2 v^2} \approx \frac{0.7}{\alpha_{em}} \left(\frac{\Lambda^2}{m_{\chi\chi}^2}\right) \left(\frac{v^2}{\Lambda^2}\right) , \tag{9}$$

which is of the form of eqn. 2 with $b\kappa^2 \propto (\Lambda/m_{\chi\chi})^2$. Note that $\Lambda/m_{\chi\chi}$ *cannot* be small since top-color gauge interactions must drive $t\chi$ chiral symmetry breaking.

A recent detailed analysis of precision electroweak constraints [19, 23], taking into account the running of the Higgs self-coupling below the compositeness scale, yields the results shown in Figure 5. The results show that the top quark seesaw model essentially saturates the bounds implied by the triviality curves plotted in Figure 3.

5. Conclusions

In conclusion, the triviality of the Standard Higgs model implies that it is at best a low-energy effective theory valid below a scale Λ characteristic of nontrivial underlying dynamics. As the Higgs mass increases, the upper bound on the scale Λ decreases. If the underlying dynamics does not respect a custodial symmetry, it will give rise to corrections to T of order $\kappa^2 v^2/\alpha\Lambda^2$, while the contributions to S and U are likely to be much smaller. For this reason, it is necessary to consider limits on a Higgs boson in the $(m_H, \Delta T)$ plane. In doing so, we see that a Higgs mass larger than 200 GeV is consistent with precision electroweak tests if there is a positive ΔT. Absent

a custodial symmetry, however, Higgs masses larger than $\simeq$ 500 GeV are unlikely: the scale of underlying physics is so low that ΔT is likely to be too large.

6. Acknowledgments

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References

[1] S. Mele (LEP Electroweak Working Group) (2001), contributed to International Europhysics Conference on High- Energy Physics (HEP 2001), Budapest, Hungary, 12-18 Jul 2001.
[2] (2001), cERN-EP/2001-055.
[3] R. S. Chivukula, C. Holbling, and N. Evans, Phys. Rev. Lett. **85**, 511 (2000), arXiv:hep-ph/0002022.
[4] K. G. Wilson, Phys. Rev. **B4**, 3174 (1971).
[5] K. G. Wilson, Phys. Rev. **B4**, 3184 (1971).
[6] K. G. Wilson and J. B. Kogut, Phys. Rept. **12**, 75 (1974).
[7] R. Dashen and H. Neuberger, Phys. Rev. Lett. **50**, 1897 (1983).
[8] M. Weinstein, Phys. Rev. **D8**, 2511 (1973).
[9] P. Sikivie, L. Susskind, M. B. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).
[10] W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).
[11] B. Grinstein and M. B. Wise, Phys. Lett. **B265**, 326 (1991).
[12] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[13] H. Georgi, Phys. Lett. **B298**, 187 (1993), arXiv:hep-ph/9207278.
[14] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).
[15] M. E. Peskin and T. Takeuchi, Phys. Rev. **D46**, 381 (1992).
[16] M. Golden and L. J. Randall, Nucl. Phys. **B361**, 3 (1991).
[17] B. Holdom and J. Terning, Phys. Lett. **B247**, 88 (1990).
[18] A. Dobado, D. Espriu, and M. J. Herrero, Phys. Lett. **B255**, 405 (1991).
[19] H.-J. He (2001), private communication.
[20] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. **81**, 2634 (1998), hep-ph/9712319.
[21] R. S. Chivukula, B. A. Dobrescu, H. Georgi, and C. T. Hill, Phys. Rev. **D59**, 075003 (1999), hep-ph/9809470.
[22] H. Collins, A. Grant, and H. Georgi, Phys. Rev. **D61**, 055002 (2000), hep-ph/9908330.
[23] H.-J. He, C. T. Hill, and T. M. P. Tait (2001), arXiv:hep-ph/0108041.
[24] C. T. Hill, Phys. Lett. **B266**, 419 (1991).
[25] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. **D41**, 1647 (1990).