Accelerating Muons to 2400 GeV/c with Dogbones Followed by Interleaved Fast Ramping Iron and Fixed Superconducting Magnets

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The first acceleration stage for this muon collider scenario includes twenty passes through a single two GeV Linac. Teardrop shaped arcs of 1.8 Tesla fixed field magnets are used at each end of the Linac. This *dogbone* geometry minimizes muon decay losses because muons pass through shorter arcs when their gamma boost is low. Two 2200 m radius hybrid rings of fixed superconducting magnets and iron magnets ramping at 200 Hz and 330 Hz are used as part of the second stage of muon acceleration. Muons are given 25 GeV of RF energy per orbit. Acceleration is from 250 GeV/c to 2400 GeV/c and requires a total of 86 orbits in both rings; 82% of the muons survive. The total power consumption of the iron dipoles is 4 megawatts. Stranded copper conductors and thin magnetic laminations are used to reduce power losses.

1. Introduction

For a $\mu^+\mu^-$ collider [1], muons must be rapidly accelerated to high energies while minimizing the kilometers of radio frequency (RF) cavities and magnet bores. Cost must be moderate. Some muons may be lost to decay but not too many. In the first stage of acceleration, consider twenty passes through a two GeV Linac and see if enough muons survive decay. A single continuos Linac with teardrop shaped arcs of fixed field magnets at each end is adopted. Muon decay losses are minimized; muons pass through shorter arcs when their gamma boost is low. The overall geometry looks like a dogbone [2]. More time is available for the second stage of acceleration due to the gamma boost. Consider a ring of fixed superconducting magnets alternating with iron combined function magnets rapidly cycling between full negative and full positive field [3]. This interleaved geometry increases the average bending field achievable in a fast ramping synchrotron and thus reduces muon decay losses.

2. Dogbone Layout with 1.8 Tesla Fixed-Field Magnets

A neutrino factory as outlined in the recent Brookhaven study [4] provides 20 GeV muons which have enough energy to explore CP violation in the lepton sector. Further acceleration to 60 GeV may be enough to reach a low mass Higgs as suggested by theory and recent measurements at LEP.

Twenty passes through a 2 GeV Linac would accelerate muons from 20 to 60 GeV. Sets of teardrop shaped arcs as shown in Figure 1 are used at used at both end of the 2 GeV linac. To minimize magnet cost 45^0 turns are used with short straight sections to line up the arcs. For each teardrop, the length added to the curved sections by the two straight sections is $(4-2\sqrt{2})/2\pi =$ 18.6%. Take a muon lifetime of 2.2×10^{-6} seconds, 1.8 Telsa dipoles, a 70% dipole packing fraction, and a 133 meter long 2 GeV Linac with 15 MV/meter. The total magnet bore length required is 7000 meters, 11% longer that the Fermilab Tevatron. Muon survival after twenty passes through the 2 GeV Linac is 95.5%. Squaring this percentage the luminosity is 91.8% of what it would be in a Higgs factory if there had been no decay loss in accelerating the muons from 20 to 60 GeV. The magnet cell length may have to be short to provide good acceptance for the muons in the arcs. An alternating gradient design where the magnet lamination change shape within a magnet

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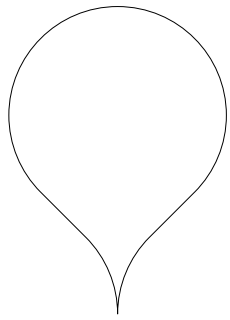

Figure 1: Sets of teardrop shaped magnet arcs each with 1.8 Tesla iron dipoles are used at each end of a 2 GeV Linac.

avoids magnet ends and makes it easier to consider superconducting wire rather than copper. The magnets do have to be at full field constantly, so power consumption is an issue.

Finally note that in a dogbone geometry, muons can orbit clockwise in one end and counter-clockwise in the other end, which may help to preserve polarization. If muons are 100% polarized, the $\mu^+\mu^- \to$ Higgs cross section doubles (versus the case of zero polarization).

3. Interleaved Fast Ramping Iron and Fixed Superconducting Magnets: Sagittas, Power Consumption, and Muon Survival

A lattice cell for a ring of interleaved fast ramping magnets and fixed field superconducting magnets is shown in Figure 2. The gradient dipoles buck the superconducting magnets at the start of a cycle and work in unison with the superconducting magnets at the end of a cycle. The magnetic field swings from full negative to full positive in the gradient dipoles.

The sagitta of a muon in a magnet increases linearly with increasing magnetic field, B. It decreases linearly with increasing momentum, p. And it increases as the square of the length of a magnet, ℓ. The size of the sagitta directly affects the size of magnet bores because the sagitta *changes* throughout a cycle. The sagitta is given by $R - \sqrt{R^2 - (\ell/2)^2}$, where $R = p/.3B$. At 250 GeV, the sagitta is 5mm for a 2 meter long 8 Telta magnet and 11 mm for a 6 meter long 2 Telsa magnet. As momentum increases, the sagitta in the 8 Tesla magnets decreases towards zero and the sagitta in the 2 Tesla magnets goes somewhat past zero.

Consider the feasibility of an iron dominated magnet which cycles from -2 to $+2$ Tesla [3]. First calculate the energy, W, stored in a 2 Tesla field in a volume 6 m long, .03 m high, and .08 m wide. The permeability constant, μ_0, is $4\pi \times 10^{-7}$. $W = B^2/2\mu_0[\text{Volume}] = 23\,000$ Joules. Next given 6 turns, an LC circuit capacitor, and a 250 Hz frequency; estimate current, voltage, inductance, and capacitance. The height, h, of the aperture is .03 m. The top and bottom coils may be connected as two separate circuits to halve the switching voltage.

V SC Quad	$\mp$2 Tesla Gradient Dipole	8 Tesla Supercon-ducting Dipole	$\mp$2 Tesla Gradient Dipole	H SC Quad

Figure 2: Lattice cell for a ring to accelerate muons. The gradient dipole magnetic field starts at -2 Telsa and ends at $+2$ Tesla. At the start of an acceleration cycle, the gradient dipoles oppose the bending and focusing of the superconducting quadrapoles and dipole. At the end of an acceleration cycle, the gradient dipoles bend and focus in the unison with the superconducting quadrapoles and dipole. H signifies a horizontal quadrapole and V signifies a vertical quadrapole.

$$B = \frac{\mu_0 N I}{h} \longrightarrow I = \frac{Bh}{\mu_0 N} = 8000 \text{ Amps}; \qquad W = .5\, L\, I^2 \longrightarrow L = 2\, W/I^2 = 720\, \mu\text{H}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \longrightarrow C = \frac{1}{L\,(2\pi f)^2} = 560\, \mu\text{F}; \qquad W = .5\, C\, V^2 \longrightarrow V = \sqrt{2W/C} = 9000 \text{ Volts}$$

Now calculate the resistive energy loss, which over time is equal to one-half the loss at the maximum current of 8000 Amps. The one-half comes from the integral of cosine squared. Table I gives the resistivity of copper. A six-turn copper conductor 3 cm thick, 10 cm high, and 7800 cm long has a power dissipation of 15 kilowatts.

$$R = \frac{7800\ (1.8\, \mu\Omega\text{-cm})}{(3)\,(10)} = 470\, \mu\Omega$$

$$P = I^2 R \int_0^{2\pi} \cos^2(\theta)\, d\theta = 15\,000 \text{ watts/magnet}$$

Table I Resistivity, magnetic saturation, and coercivity of conductors, cooling tubes, and soft magnetic materials [9].

Material	Composition	ρ ($\mu\Omega$-cm)	B Max (Tesla)	H_c (Oersteds)
Copper	Cu	1.8	—	—
Stainless 316L	Fe 70, Cr 18, Ni 10, Mo 2, C .03	74	—	—
Hastelloy B	Ni 66, Mo 28, Fe 5	135	—	—
Titanium 6Al–4V	Ti 90, Al 6, V 4	171	—	—
Titanium 8Al–1Mo–1V	Ti 90, Al 8, Mo 1, V 1	199	—	—
Pure Iron [10]	Fe 99.95, C .005	10	2.16	.05
1008 Steel	Fe 99, C .08	12	2.09	0.8
Grain–Oriented	Si 3, Fe 97	47	1.95	.1
NKK Super E-Core	Si 6.5, Fe 93.5	82	1.8	
Metglas 2605SA1 [11]	Fe 81, B 14, Si 3, C 2	135	1.6	.03
Supermendur	V 2, Fe 49, Co 49	26	2.4	.2

Calculate the dissipation due to eddy currents in this conductor, which will consist of transposed strands to reduce this loss [5–8]. To get an idea, take the maximum B-field during a cycle to be that generated by a 0.05m radius conductor carrying 24000 amps. This ignores fringe fields from the gap which will make the real answer higher. The eddy current loss in a rectangular conductor made of transposed square wires 1/2 mm wide (sometimes called Litz wire) with a perpendicular magnetic field is as follows. The width of the wire is w.

$$B = \frac{\mu_0 I}{2\pi r} = 0.096 \text{ Tesla};$$

$$P = [\text{Volume}]\frac{(2\pi\, f\, B\, w)^2}{24\rho} = [.03\ .10\ 78]\ \frac{(2\pi\ 250\ .096\ .0005)^2}{(24)\ 1.8 \times 10^{-8}} = 3000 \text{ watts}$$

Cooling water will be needed, so calculate the eddy current losses for cooling tubes made from type 316L stainless steel. More exotic metals with higher resistivities are also available as shown in Table III. Choose 2 tubes per 3 cm × 10 cm stranded copper conductor for a total length of 78 × 2 = 156 m. Take a 12 mm OD and a 10 mm ID. Subtract the losses in the inner *missing* round conductor. The combined eddy current loss in the copper plus the stainless steel is 4200 watts (3000 + 2400 - 1200).

$$P(12\,\text{mm}) = [\text{Volume}]\,\frac{(2\pi\, f\, B\, d)^2}{32\,\rho} = [\pi\ .006^2\ 156]\,\frac{(2\pi\, 250\ .096\ .012)^2}{(32)\ 74\times 10^{-8}} = 2400 \text{ watts}$$

$$P(10\,\text{mm}) = [\text{Volume}]\,\frac{(2\pi\, f\, B\, d)^2}{32\,\rho} = [\pi\ .005^2\ 156]\,\frac{(2\pi\, 250\ .096\ .010)^2}{(32)\ 74\times 10^{-8}} = 1200 \text{ watts}$$

Eddy currents must be reduced in the iron not only to decrease power consumption and cooling, but also because they introduce multipole moments which destabilize beams. If the laminations are longitudinal, it is hard to force the magnetic field to be parallel to the laminations near the gap. This leads to additional eddy current gap losses [12]. So consider a magnet with transverse laminations as sketched in Figure 1 and calculate the eddy current losses. The yoke is either 0.28 mm thick 3% grain oriented silicon steel [13–15] or 0.025 mm thick Metglas 2605SA1 [11]. The pole tips are 0.1 mm thick Supermendur to raise the field in the gap [16].

$$\begin{aligned}
\text{P(3\% Si–Fe)} &= [\text{Volume}]\frac{(2\pi\, f\, B\, t)^2}{24\rho} \\
&= [6\,((.42\ .35) - (.20\ .23))]\,\frac{(2\pi\ 250\ 1.6\ .00028)^2}{(24)\,47\times 10^{-8}} = 27\,000 \text{ watts} \\
\text{P(Metglas)} &= [\text{Volume}]\frac{(2\pi\, f\, B\, t)^2}{24\rho} \\
&= [6\,((.42\ .35) - (.20\ .23))]\,\frac{(2\pi\ 250\ 1.6\ .000025)^2}{(24)\,135\times 10^{-8}} = 75 \text{ watts} \\
\text{P(Supermendur)} &= [\text{Volume}]\frac{(2\pi\, f\, B\, t)^2}{24\rho} \\
&= [6\ .09\ .02]\,\frac{(2\pi\ 250\ 2.2\ .0001)^2}{(24)\,26\times 10^{-8}} = 210 \text{ watts}
\end{aligned}$$

Eddy currents are not the only losses in the iron. Hysteresis losses, $\int \mathbf{H}\cdot d\,\mathbf{B}$, scale with the coercive force, H_c, and increase linearly with frequency. Anomalous loss [10] which is difficult to calculate theoretically must be included. Thus I now use functions fitted to experimental measurements of 0.28 mm thick 3% grain oriented silicon steel [17], 0.025 mm thick Metglas 2605SA1 [11], and 0.1 mm thick Supermendur [17].

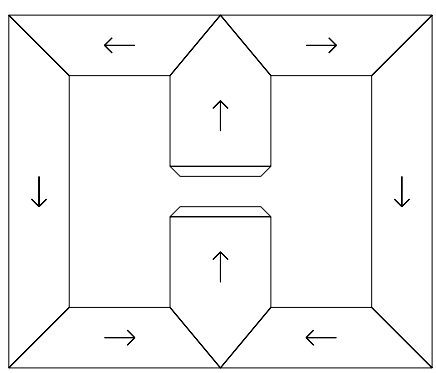

Figure 3: A two dimensional picture of an H frame magnet lamination with grain oriented 3% Si–Fe steel. The arrows show both the magnetic field direction and the grain direction of the steel. Multiple pieces are used to exploit the high permeability and low hysteresis in the grain direction [13–15]. If Metglas 2605SA1 is used for the yoke, multiple pieces are not needed, except for the poles. The pole tips are an iron–cobalt alloy for flux concentration exceeding 2 Tesla.

$$\begin{aligned}
\text{P(3\% Si–Fe)} &= 4.38 \times 10^{-4}\, f^{1.67}\, B^{1.87} \\
&= 4.38 \times 10^{-4}\, 250^{1.67}\, 1.6^{1.87} = 10.7 \text{ w/kg} = 49\,000 \text{ watts/magnet} \\
\text{P(Metglas)} &= 1.9 \times 10^{-4}\, f^{1.51}\, B^{1.74} \\
&= 1.9 \times 10^{-4}\, 250^{1.51}\, 1.6^{1.74} = 1.8 \text{ w/kg} = 7900 \text{ watts/magnet} \\
\text{P(Supermendur)} &= 5.64 \times 10^{-3}\, f^{1.27}\, B^{1.36} \\
&= 5.64 \times 10^{-3}\, 250^{1.27}\, 2.2^{1.36} = 18 \text{ w/kg} = 1600 \text{ watts/magnet}
\end{aligned}$$

Table II Magnet core materials.

Material	Thickness (mm)	Density (kg/m^3)	Volume (m^3)	Mass (kg)
3% Si–Fe	0.28	7650	0.6	4600
Metglas	0.025	7320	0.6	4400
Supermendur	0.1	8150	0.01	90

Table III 250 Hz dipole power consumption. Eddy current components of total core losses are 27 210 and 285 watts for 3% Si–Fe and Metglas.

Material	3% Si–Fe	Metglas
Coil Resistive Loss	15 000 watts	15 000 watts
Coil Eddy Current Loss	4200 watts	4200 watts
Total Core Loss	50 600 watts	9500 watts
Total Loss	69 800 watts	28 700 watts

In summary, a 250 Hz dipole magnet close to 2 Tesla looks possible as long as the field volume is limited and one is willing to deal with stranded copper and thin, low hysteresis laminations. Total losses can be held to twice the I^2R loss in the copper alone, using Metglas.

Now with a rough design for a fast ramping magnet in hand, work out the details of ring radii, RF requirements, and the fraction of muons that survive decay. The fraction of the circumference packed with dipoles is set at $P_F = 70\%$. As an example, consider two rings in a 2200 m radius tunnel with an injection momentum of 250 GeV/c. The first has 25% 8T magnets and 75% ±2T magnets and ramps from 0.5T to 3.5T. The second has 55% 8T magnets and 45% ±2T magnets and ramps from 3.5T to 5.3T.

$$B = \frac{250\,\text{GeV/c}}{.3\, P_F\, R} = \frac{250}{(.3)\,(.7)\,(2200)} = 0.54\,\text{Tesla}$$

$$p = (3.5\,\text{Tesla})\,(.3)\,(P_F)\,(R) = (3.5)\,(.3)\,(.7)\,(2200) = 1600\,\text{GeV/c}$$

$$p = (5.3\,\text{Tesla})\,(.3)\,(P_F)\,(R) = (5.3)\,(.3)\,(.7)\,(2200) = 2400\,\text{GeV/c}$$

Provide 25 GeV of RF. The first ring accelerates muons from 250 GeV/c to 1600 GeV/c in 54 orbits. and the second from 1600 GeV/c to 2400 GeV/c in 32 orbits. At what frequency do the two rings have to ramp?

$$\text{Time}\,(0.5T \to 3.5T) = \frac{(54)\,(2\pi)\,(2.2)}{300\,000} = 2.5\,\text{ms} \qquad \to 200\,\text{Hz}$$

$$\text{Time}\,(3.5T \to 5.3T) = \frac{(32)\,(2\pi)\,(2.2)}{300\,000} = 1.5\,\text{ms} \qquad \to 330\,\text{Hz}$$

How many muons survive during the 86 orbits from 250 GeV/c to 2400 GeV/c? N is the orbit number, $\tau = 2.2 \times 10^{-6}$ is the muon lifetime, and $m = .106$ GeV/c^2 is the muon mass.

$$\text{SURVIVAL} = \prod_{N=1}^{86} \exp\left[\frac{-2\pi R\, m}{[250 + (25\,N)]\, c\tau}\right] = 82\% \tag{1}$$

Only 1/6 of the 18% loss occurs in the second ring, so it is not crucial to run it as fast as 330 Hz. The 250 → 1600 GeV/c ring has 1200 6 m long dipoles ramping at 200 Hz. The 1600 → 2400 GeV/c ring has 725 6 m long dipoles ramping at 330 Hz. The weighted average rate is 250 Hz. If running continuously, the 1925 magnets would consume a weighted average of 29 kilowatts each for a total of 56 megawatts. But given a 15 Hz refresh rate for the final muon storage ring [1], the average duty cycle for the 250 → 2400 GeV/c acceleration rings is 6%. So the power falls to 4 megawatts, which is small. Finally note that one can do a bit better than 82% on the muon survival during final acceleration if the first ring is smaller, say 1000 m, rather than 2200 m.

Acknowledgments

I would like to thank S. Berg, K. Bourkland, S. Bracker, R. Fernow, J. Gallardo, C. Johnstone, H. Kirk, N. Marks, D. Neuffer, A. Otter, R. Palmer, A. Tollestrup, K. Tuohy, D. Walz, R. Weggel, E. Willen, and D. Winn for their help and suggestions.

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