# Progress toward a Muon Ring Cooler

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We describe progress toward the design and analysis of a storage ring for cooling a muon beam by the process of ionization cooling. Our primary strategy entails the design and optimization of the lattice using the code SYNCH, followed by the transfer of parameters to the code ICOOL which allows for the tracking of particles through rf cavities and absorbers of various dimensions. Our ultimate goal is to obtain longitudinal cooling and either transverse cooling or minimal transverse emittance heating.

#### 1. Introduction

We wish to construct a storage ring capable of providing longitudinal cooling of a muon beam through the ionization cooling[1] process. This technique consists of passing a momentum dispersed beam through an absorber material shaped so that the path length of the individual particles traversing the absorber increases in proportion to their momentum deviation, dp.

We adopt a strategy of utilizing the synchrotron design code SYNCH[2] to create a suitable ring lattice for the circulating beam. The lattice should have suitable space where the  $\beta_{\min}$  will be on the order of 25 cm to insure that multiple Coulomb scattering will not adversely heat the beam transversely. Also, the lattice should be as compact as possible in order to reduce decay losses of the muon beam. Suitable dispersion free drift regions should be provided to allow installation of rf cavities which yield an energy boost to the beam in order to compensate for the energy losses of beam as it traverses the absorbing material.

When an appropriate lattice is found, the lattice parameters are then transferred to an input file for the ICOOL code[3]. This code has been developed with the specific intent of tracking particles though absorbing materials and energy compensating rf cavities. We then analyze the single particle dynamics, the energy gain in the rf cavities, and the undesirable beam heating in the absorbers from Coulomb scattering and energy straggling which are superimposed on the beam cooling effects of ionization energy losses.

## 2. Lattice Design

The muon ring cooler is comprised of cells for bending and cooling and straight sections for injection and extraction . Each arc of the racetrack ring has four bending cells and each straight section has the length of two bending cells. Each bending cell has spaces for a wedge absorber and rf cavities. The system can cool both transversely and longitudinally, but the longitudinal cooling, which is proportional to the product of absorber wedge angle and dispersion, causes a reduction of the horizontal cooling. The cooling, together with heating from scattering and straggling in the absorber, gives theoretical equilibrium emittances[4]. Application of formulae for the transverse emittances leads to the choice of 25 cm for the absorber length and  $\beta_{min}$ .

The cell is symmetric with beam waists at the centers of the absorber and the rf cavities. The dipoles are located at the center of each set of four quadrupoles, which allows for dispersion in the absorber but not in the rf cavities. In order to maximize the momentum acceptance of the lattice, the cell phase advances are close to the tune  $\mu/2\pi = 3/4$ , midway between stopband values of 1/2 and 1. Finally, to maximize the transverse acceptances, the values of  $\beta_{max}$  are minimized.

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We have considered different types of dipoles, rectangular and sector, both with and without additional gradients. Most of the ICOOL simulations have used the cell of Figure 1, where the dipoles are sector type. For convenience, we simulate a combined function dipole with a 0-gradient dipole sandwiched between two thin quadrupoles.



Figure 1: Bending cell lattice

### 3. ICOOL implementation

SYNCH utilizes magnetic elements with hard edges, hence we also use similar elements in ICOOL. We first confirm that we observe the same beam dynamics of a muon bunch traversing an identical lattice, i.e. without absorbers and rf. After this has been achieved, we place wedge absorbers at the  $\beta_{min}$  point of the lattice. The width of the absorber along the central orbit of the beam is set to be 25 cm in order to match the  $\beta_{min}$  and thus insure that the beam has a low  $\beta$  throughout the absorber. In order to restore the energy lost from the ionization process in the absorber, we place a total of 2 m of rf in each bending cell. This is done at the initial and final 1 m of the cell where the beam dispersion is zero. We choose an rf cell frequency of 201.25 MHz and, for our initial results, a peak rf voltage of 10 MV/m is used within each cavity. We find that by setting the rf phase at 26° (0° phase corresponds to a net acceleration of zero) we can compensate for the 7.5 MeV energy loss in the absorbers.

## 3.1. Single particle dynamics

We confirm that the dynamics performance of the code ICOOL corresponds to that of the SYNCH code by following the paths of individual particles through several turns of a complete ring which is initially defined as eight successive bending cells without the straight cells. We find that all test particles (off-axis and off-momentum) give similar tracking results.

The absorber and rf components are then installed in ICOOL and we note the resulting beam dynamics, initially with the dE/dx energy loss implemented but without multiple Coulomb scattering and energy straggling. We see in Figure 2 the effect of varying the inclination of each wedge face  $(\theta_w)$  on the damping of dp for an off-momentum particle. The effects on the transverse phasespace of the absorbers with different wedge angles,  $\theta_w$ , can also be seen in Figure 2. For the 0° absorber case, the horizontal (dipole bend plane) x-px phase-space ellipse is collapsing inward showing the effects of ionization cooling whereas no damping of the longitudinal momentum is observed. As the angle of the wedge absorber increases, we see diminished transverse cooling.



Figure 2: The damping of longitudinal momentum and horizontal phase-space as a function of absorber wedge angle

No cooling for the 15° case while heating for the 20° case is evident. This is accompanied by an increase in the damping of the longitudinal momentum as the the wedge angle increases, thus clearly demonstrating the principle of emittance exchange when longitudinal cooling is achieved at the expense of emittance growth in the horizontal plane.

#### 3.2. Beam ensemble dynamics

To evaluate the performance of the lattice, we first determine the dynamic aperture of the lattice by generating a broad spectrum of particles in 6D phase-space and allowing these particle to transverse the ring for many turns, typically up to to 50 full turns. The surviving particles determine the admittance of the lattice. Subsequent runs are made utilizing the initial coordinates of these surviving particles and invoking the effects of the stochastic processes. We show, in Figure 3, results for the passage of particles through 16 full turns of a lattice which has been tuned for the transmission of a muon bunch with a central momentum  $p_0 = 500 \text{ MeV/c}$ . The initial invariant emittances for this example were  $\varepsilon_{nx} = 1.8 \times 10^{-3}$  m-rad,  $\varepsilon_{ny} = 4.4 \times 10^{-3}$  m-rad, and  $\varepsilon_{nz} = 18 \times 10^{-3}$  m-rad.

The ratio of surviving particles to initial for this example is 40%. By repeating this example, both with and without multiple Coulomb scattering and energy straggling, we confirm that this loss results from these stochastic processes. We define as a figure of merit the product of transmission ratio times the gain incurred from the reduction the phase-space volume, defined as  $\frac{\varepsilon_{x_i}}{\varepsilon_{x_f}} \frac{\varepsilon_{y_i}}{\varepsilon_{z_f}}$ . As shown in Figure 4 we achieve a factor of two gain in this figure of merit during the 16 full turns of this cooling ring.



Figure 3: The evolution of emittances in each phase-space plane



Figure 4: The transmission and total figure of merit as defined in the text

#### 4. Summary

We have transferred the beam transport parameters from SYNCH to ICOOL and find excellent agreement for the beam dynamics for cases without absorbers and rf. Using the code ICOOL, we observe that the our design results in cooling in both the longitudinal and vertical planes while showing modest emittance growth in the horizontal plane. This result represents a clear demonstration of the principle of emittance exchange. Future work will entail the addition of soft-edge quadrupole and dipole magnetic fields to the model and efforts to improve the transmission losses of the system.

## References

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