

# Determination of the Muon Yukawa Coupling at High Energy $e^+e^-$ Linear Colliders

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The perspectives for the observation of the rare decay  $H^0 \rightarrow \mu^+\mu^-$  decay and the determination of the muon Yukawa coupling at a TeV-class and at a multi-TeV  $e^+e^-$  linear colliders are discussed. The signal for the decay can be obtained at  $\sqrt{s} = 0.8$  TeV and a first estimate of the coupling derived. A linear collider operating at 3 TeV, with high luminosity, is able to improve the accuracy on this couplings to 4% to 11% for  $120 \text{ GeV} < M_H < 150 \text{ GeV}$ .

## 1. Introduction

Understanding the mechanism of mass generation is one of the main quests for scientific research, today and in the coming decades. In the Standard Model (SM) the Higgs mechanism is held responsible for breaking the electro-weak symmetry and providing particles with their mass through their interactions with the Higgs field. Beyond the observation of a Higgs boson, the fundamental test that the Higgs couplings to particles scale as their masses needs to be verified experimentally, for gauge bosons and fermions separately. The potential of a TeV-class high luminosity  $e^+e^-$  linear collider (LC) has been already established for  $W$  and  $Z$  gauge bosons,  $t$ ,  $b$  and  $c$  quarks and for the  $\tau$  lepton [1]. We now discuss the possibility of measuring the muon Yukawa coupling by the determination of the branching fraction of the rare Higgs decay  $H^0 \rightarrow \mu^+\mu^-$ . This measurement would enable to complete the test of the coupling scaling for gauge bosons, quarks and leptons separately and thus ensuring that the observed Higgs boson is indeed responsible for the mass generation of all elementary particles. We present results for a TeV-class LC operating at a centre-of-mass energy  $\sqrt{s} = 0.8$  TeV and for a multi-TeV LC with  $\sqrt{s}=3.0$  TeV. We considered data sets of  $1 \text{ ab}^{-1}$ , corresponding to about two years of data taking at the nominal TESLA luminosity, and  $5 \text{ ab}^{-1}$ , corresponding to five years at the CLIC project luminosity, respectively.

In the SM, the branching fraction  $\text{BR}(H^0 \rightarrow \mu^+\mu^-)$  is of the order of only  $10^{-4}$ , for Higgs masses  $M_H$  below the  $WW$  threshold and even lower above it. But the signal can be efficiently reconstructed from the invariant mass of two oppositely charged muons both in the Higgs-strahlung,  $e^+e^- \rightarrow H^0 Z^0$ , and in the  $WW$  fusion,  $e^+e^- \rightarrow H^0 \nu_e \bar{\nu}_e$ , processes and isolated from the backgrounds. The signal reconstruction and background rejections have been studied using the SIMDET parametric detector simulation program [2], tuned to the expected responses for detectors at TeV and multi-TeV LCs. In particular, a muon momentum resolution  $\delta p/p^2 = 5 \times 10^{-5} \text{ GeV}^{-1}$  has been assumed. The signal and background cross sections have been obtained using the COM-PHEP program [3], while the SM  $\text{BR}(H^0 \rightarrow \mu^+\mu^-)$  has been computed with HDECAY [4]. The signal events have been generated using the PYTHIA-6 Monte Carlo [5] while parton level generation of the background processes has been performed with COMPHEP and the events have then been hadronised using PYTHIA [6].

## 2. $H^0 \rightarrow \mu^+\mu^-$ at a TeV-class LC

While most of the profile of a light Higgs boson is best studied with lower energy operation of a LC,  $300 \text{ GeV} < \sqrt{s} < 500 \text{ GeV}$ , there are advantages in performing some measurements at energies around 1 TeV. The total Higgs production cross section is within a factor of two from that

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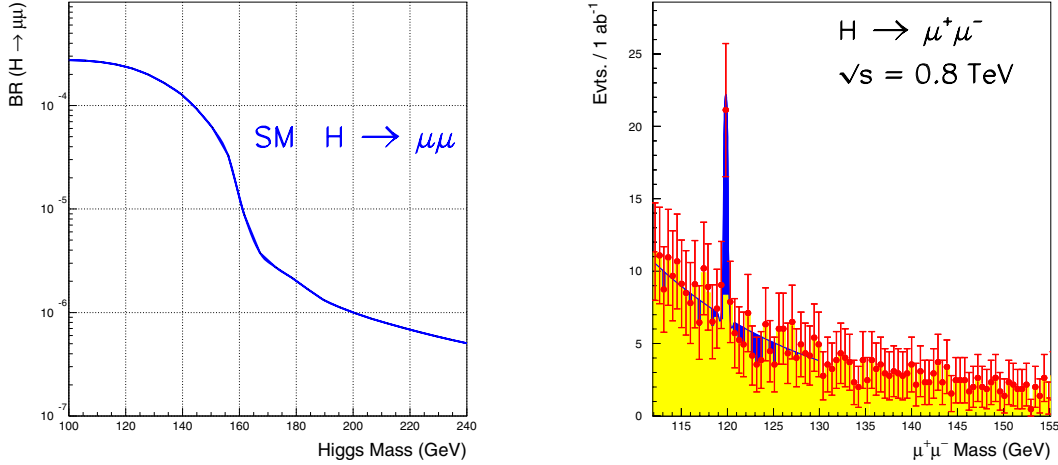


Figure 1: Left: SM prediction for  $BR(H \rightarrow \mu^+\mu^-)$  as a function of the Higgs boson mass. Right: distribution of the di-muon invariant mass  $M_{\mu\mu}$  for  $M_H=120 \text{ GeV}$ . The points with error bars represent  $1 \text{ ab}^{-1}$  of data at  $\sqrt{s}=0.8 \text{ TeV}$  and the continuous line the result of the fit used to extract the number of signal events.

at the peak for the Higgs-strahlung process and this is more than compensated by the increase in the achievable luminosity at these larger energies. Further the  $ZZ^*$  background is significantly decreased. We have considered  $\sqrt{s} = 0.8 \text{ TeV}$  with an integrated luminosity  $\mathcal{L} = 1 \text{ ab}^{-1}$ , corresponding to  $1.75 \times 10^5$  Higgs bosons produced in the  $e^+e^- \rightarrow H\nu\bar{\nu}$  channel and about 40  $H \rightarrow \mu^+\mu^-$  decays, for  $M_H = 120 \text{ GeV}$ .

The analysis starts from events with two oppositely charged, identified muons and significant missing energy and assumes an accurate knowledge of the Higgs mass, obtained at the LHC and lower energy LC operation. The main background from  $WW \rightarrow \mu\nu\mu\nu$  is reduced by cuts on the di-muon recoil mass  $M_{\mu\mu}^{\text{recoil}}$  and energy  $E_{\mu\mu}$ . The total background has been estimated including also the  $ZZ\nu\bar{\nu}$ ,  $WW\nu\bar{\nu}$  and the inclusive  $\mu\mu\nu\bar{\nu}$  processes, evaluated without the Higgs contribution. The resulting di-muon invariant mass is shown in Figure 1. The  $H \rightarrow \mu\mu$  signal can be extracted from the underlying background with more than  $5\sigma$  significance, for  $M_H \approx 120 \text{ GeV}$ . From the event rate, the product of production cross section and  $\mu\mu$  decay branching fraction  $\sigma(e^+e^- \rightarrow H\nu\bar{\nu}) \times BR(H \rightarrow \mu\mu)$  is measured with  $\simeq 30 \%$  accuracy.

### 3. $H^0 \rightarrow \mu^+\mu^-$ at a multi-TeV LC

Multi-TeV  $e^+e^-$  collisions provide a large sample of Higgs bosons, produced in the  $WW$  fusion process, becoming the dominant Higgs production mode since  $\sigma_{H\nu\bar{\nu}} \propto \log \frac{s}{M_H^2}$ . However, beyond  $\sim 3 \text{ TeV}$  the further gain in production cross section is largely lost due to the reduced acceptance, due to the forward Higgs boson production. A data set of  $5 \text{ ab}^{-1}$  at  $\sqrt{s}=3 \text{ TeV}$  corresponds to a sample of  $2.7 \times 10^6$  Higgs bosons and 650  $H \rightarrow \mu\mu$  decays, for  $M_H = 120 \text{ GeV}$ .

The analysis is again based on events with two identified muons and the background is suppressed in part by selection cuts on the di-muon recoil mass and energy. The  $H \rightarrow \mu\mu$  signal is clearly visible in the di-muon invariant mass distribution above the background. A signal significance of more than  $5\sigma$  is obtained up to  $M_H \approx 155 \text{ GeV}$ . The number of signal events is extracted from a fit to the di-muon invariant mass where the signal is modelled by a Gaussian distribution peaked at the nominal Higgs mass and the background by a polynomial curve fitted on the peak side bands (see Figure 2). The accuracies on the product of production cross section and  $\mu\mu$  decay branching fraction derived from the fitted number of signal events, are summarised in Table I for different values of  $M_H$ .

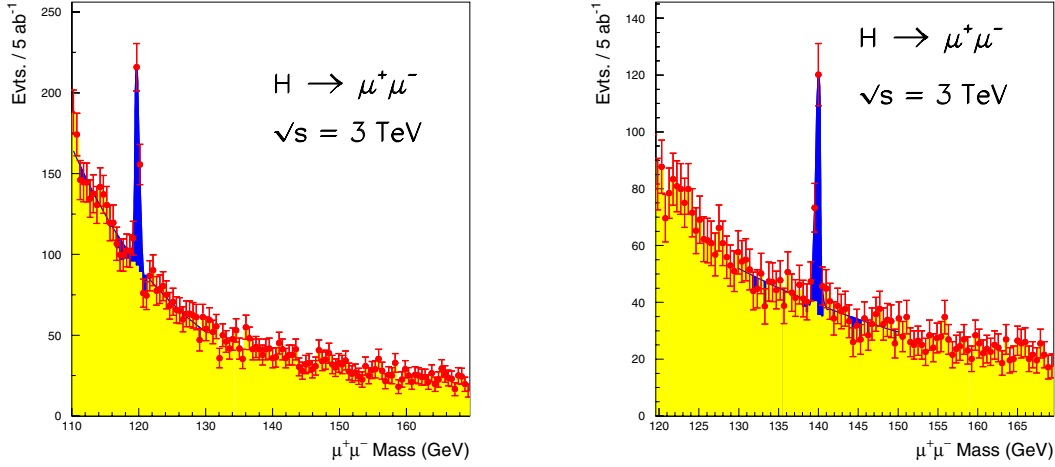


Figure 2: Distribution of the di-muon invariant mass  $M_{\mu\mu}$  for two data sets with  $M_H = 120$  GeV (left) and  $M_H = 140$  GeV (right). The points with error bars represent  $5 \text{ ab}^{-1}$  of data at  $\sqrt{s} = 3 \text{ TeV}$  and the continuous lines the results of the fits used to extract the numbers of signal events.

Table I Statistical accuracy on the product  $\sigma(e^+e^- \rightarrow H\nu\bar{\nu}) \times \text{BR}(H \rightarrow \mu\mu)$  and the  $g_{H\mu\mu}$  coupling expected with an integrated luminosity of  $5 \text{ ab}^{-1}$  at  $\sqrt{s} = 3 \text{ TeV}$ , for different values of the Higgs boson mass  $M_H$ .

$M_H$	120 GeV	140 GeV	150 GeV
$\delta(\sigma_{H\nu\bar{\nu}} \times \text{BR}) / (\sigma_{H\nu\bar{\nu}} \times \text{BR})$	0.072	0.120	0.210
$\delta g_{H\mu\mu} / g_{H\mu\mu}$	0.040	0.062	0.106

#### 4. Discussion and Conclusions

The  $g_{H\mu\mu}$  coupling can be extracted from the measured branching fractions with an independent measurement of the production cross section, obtained using the dominant  $b\bar{b}$  and/or  $WW^*$  Higgs boson decay modes. An uncertainty of 2% is assumed for the production cross section, extrapolating results from lower energies. The resulting accuracies on the muon Yukawa coupling are given in Table I.

A muon collider (FMC), running at energies around the  $H$  mass, can determine the product  $\sigma(\mu\mu \rightarrow H) \times \text{BR}(H \rightarrow b\bar{b})$  to a statistical accuracy of 3.5% to 0.2% for  $M_H \simeq 120$  GeV, depending on the assumed luminosity and beam energy spread [7, 8]. By taking the  $\text{BR}(H \rightarrow b\bar{b})$  measured at the LC to 2.5%, the combination of the FMC+NLC data allows to extract the  $g_{H\mu\mu}$  coupling with an accuracy of  $\simeq 2.2 - 1.3\%$ . To this accuracy, the  $b\bar{b}$  branching fraction contributes with comparable or dominant uncertainty depending on the FMC luminosity assumption.

The decay  $H \rightarrow \mu^+\mu^-$  may also be investigated at hadron colliders. However, a first study [9] has shown that the LHC could get a  $5\sigma$  signal of this decay only with an integrated luminosity in excess of  $2 \text{ ab}^{-1}$ . Therefore, it may become relevant only for a high luminosity upgrade. At higher energies, a VLHC would be able to get a  $5\sigma$  evidence with about  $0.25 \text{ ab}^{-1}$  and a  $g_{H\mu\mu}$  coupling determination with 10-14% accuracy for  $120 \text{ GeV} < M_H < 140 \text{ GeV}$  at  $\sqrt{s} = 200 \text{ TeV}$ .

The measurements of the muon Yukawa coupling allow to extend the test of the Higgs mechanism of mass generation to the lepton sector by verifying that

$$\frac{g_{H\mu\mu}}{g_{H\tau\tau}} = \frac{M_\mu}{M_\tau}.$$

The  $g_{H\tau\tau}$  coupling can be determined at the LC with an accuracy of 3% to 5% for  $120 \text{ GeV} < M_H < 140 \text{ GeV}$ , already at  $\sqrt{s} = 300 \text{ GeV} - 500 \text{ GeV}$  [1]. The accuracies of the determination of the two couplings being comparable, the test of the mass scaling can be obtained with a precision of 5.0%

to 8.0% for  $120 \text{ GeV} < M_H < 140 \text{ GeV}$  for the case of a LC and of 3.3% to 3.7% for  $M_H = 120 \text{ GeV}$  at a FMC, depending on the luminosity.

In summary, we discussed the measurement of the muon Yukawa coupling by the determination of the branching fraction of the rare Higgs decay  $H^0 \rightarrow \mu^+ \mu^-$  at  $\sqrt{s} = 0.8 \text{ TeV}$  and  $3.0 \text{ TeV}$ . The  $H \rightarrow \mu^+ \mu^-$  signal can be observed at the lower energy for  $M_H = 120 \text{ GeV}$  and the  $g_{H\mu\mu}$  coupling measured to  $\approx 15\%$ . This accuracy can be pushed to 4-11% for the wider Higgs mass range  $120 \text{ GeV} < M_H < 150 \text{ GeV}$  with  $e^+e^-$  collisions at  $\sqrt{s} = 3 \text{ TeV}$ . The anticipated accuracy of a multi-TeV LC in the determination of  $g_{H\mu\mu}$  is comparable to that achievable at a muon collider operating at the Higgs mass peak and more than two times better than at a 200 TeV VLHC hadron collider.

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