Signals for Noncommutative QED at High Energy e^+e^- Colliders

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We examine the signatures for noncommutative QED at e^+e^- colliders with center of mass energies in excess of 1 TeV such as CLIC. For integrated luminosities of 1 ab^{-1} or more, sensitivities to the associated mass scales greater than $\sqrt{s}$ are possible.

If the Planck scale is indeed of order a few TeV, some potential stringy effects may be observable at future colliders in addition to the existence of extra dimensions. One such possibility is that near the string scale space-time becomes noncommutative (NC), *i.e.*, the co-ordinates themselves no longer commute: $[x_\mu, x_\nu] = i\theta_{\mu\nu}$, where the $\theta_{\mu\nu}$ are a constant, frame-independent set of six dimensionful parameters[1]. This may occur in string theory in the presence of background fields. The $\theta_{\mu\nu}$ may be separated into two classes: (i) space-space noncommutivity with $\theta_{ij} = c_{ij}/\Lambda_B^2$, and (ii) space-time noncommutivity with $\theta_{0i} = c_{0i}/\Lambda_E^2$. The auxiliary quantities $(\hat{c}_E)_i = c_{0i}$ and $(\hat{c}_B)_k = \varepsilon_{ijk}c_{ij}$, with ijk cyclic, can be defined, where $\hat{c}_{E,B}$ are two, fixed, frame-independent unit vectors associated with the mass scales $\Lambda_{E,B}$. These NC scales are anticipated to lie above a TeV and NC effects will only become apparent as the these scales are approached. Since the commutator of the co-ordinates is *not* a tensor and is frame-independent, NC theories violate Lorentz invariance (but can be shown to conserve CPT), with the two vectors $\hat{c}_{E,B}$ being preferred directions in space, related to the directions of the background fields. Since momenta still commute in the usual way, energy and momentum remain conserved quantities. Since experimental probes of NC theories are sensitive to the directions of $\hat{c}_{E,B}$, experiments must employ astronomical coordinate systems and time-stamp their data so that, *e.g.*, the rotation of the Earth or the Earth's motion around the Sun does not wash out or dilute the effect through time-averaging.

It is possible to construct noncommutative analogs of conventional field theories following either the Weyl-Moyal (WM)[2] or Seiberg-Witten (SW)[3] approaches, both of which have their own advantages and disadvantages. In the SW approach, the field theory is expanded in a power series in θ which then produces an infinite tower of additional operators. At any fixed order in θ[4], the theory can be shown to be non-renormalizable. The SW construction can, however, be applied to any gauge theory with arbitrary matter representations. In the WM approach, only $U(N)$ gauge theories are found to be closed under the group algebra and the matter content is restricted to the (anti-)fundamental and adjoint representations. Further restrictions on matter representations apply when a product of group factors is present, such as in the SM[5]. These theories are at least one-loop renormalizable and appear to remain so even when spontaneously broken[6].

These distinctive properties of NC gauge theories render the construction of a satisfactory noncommutative version of the Standard Model (SM), the NCSM, quite difficult[7]. However, NCQED is a well-defined theory in the WM approach and we explore here its implications for very high energy e^+e^- colliders following this prescription. This version of NCQED differs from ordinary QED in several ways: (i) the $ee\gamma$ vertex picks up a Lorentz violating phase factor and is given by $e^{i\theta_{\mu\nu}p_i^\mu p_o^\nu/2}$ where $p_i i$ and p_o are the incoming and outgoing electron momenta; (ii) the NC theory predicts trilinear and quartic couplings between the photons that are, to leading order, linear and quadratic in the parameters $\theta_{\mu\nu}$, respectively, and are kinematics dependent; (iii) only the charges $Q = 0, \pm 1$ are allowed by gauge invariance in NCQED[8]. Thus quarks cannot be accommodated in the theory as it presently exists and an extension to a full NCSM is required. This implies that we can only examine the NC effects for the handful of processes which have external charged leptons or photons. Despite this limitation, NCQED provides a testing ground for the basic ideas behind NC quantum field theory and has had its phenomenological implications examined by a number of authors[9]. As we will see, the hallmark signal at colliders for NCQED

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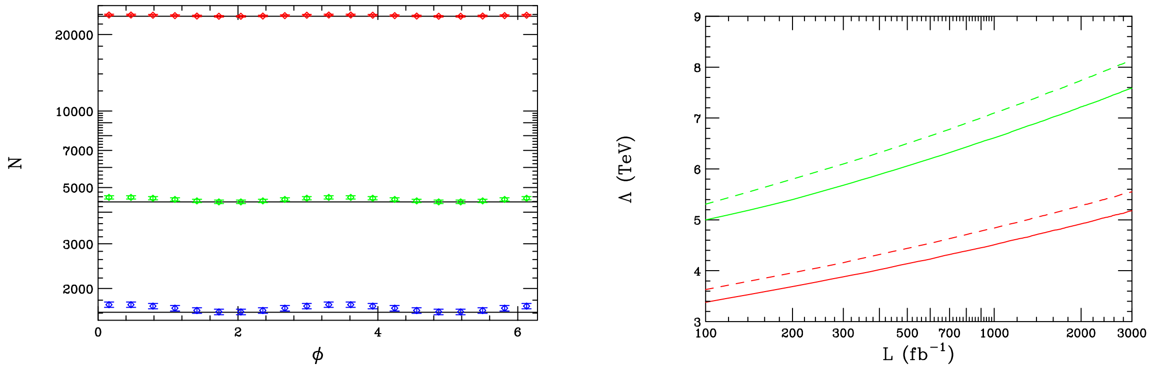

Figure 1: (Left) Binned ϕ distribution for Bhabha scattering at a 3 TeV CLIC assuming an integrated luminosity of 1 ab^{-1} with $|\cos\theta|$ cuts of 0.9, 0.7 and 0.5 (from top to bottom). The solid line is the SM prediction while the data assumes $c_{02} = 1$ and $\Lambda_E = 3$ TeV. (Right) Sensitivity to the scale Λ_E at a $\sqrt{s} = 3(5)$ TeV CLIC corresponding to the lower (upper) set of curves. The dashed (solid) curve is for the case $c_{01}(c_{02}) = 1$.

is the appearance of an azimuthally-dependent cross section in $2 \to 2$ processes; the azimuthal dependence arises from the existence of the two preferred directions discussed above. We note that in NC gauge theories, the leading NC contributions can be shown to take the form of new dimension-8 operators whose scale is set by $\Lambda_{E,B}$.

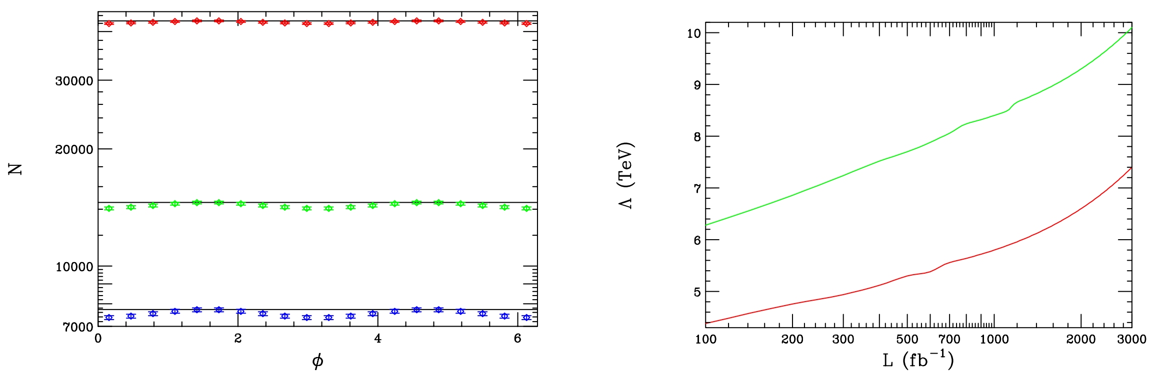

Figure 2: (Left) Same as the previous figure but now for Möller scattering assuming $c_{12} = 1$. (Right) Sensitivity to the scale Λ_B at a $\sqrt{s} = 3(5)$ TeV CLIC corresponding to the lower (upper) curves.

A caveat to our analysis below is the question of how/if the SM couplings of the Z boson to e^+e^- are modified in the NC case, as they contribute to Bhabha and Möller scattering. In truth, this lies outside the realm of the NCQED model and can only be addressed within a full NCSM. Here we will assume that these Z couplings get rescaled by kinematic-dependent exponents in a manner identical to photons. Within the SW approach we know that this is indeed what happens for on-shell electrons, to leading order in $\theta_{\mu\nu}$, and we might expect it to remain true in higher orders.

Very high energy e^+e^- colliders such as CLIC will allow us to probe values of $\Lambda_{E,B}$ up to several TeV provided sufficient luminosity, $\sim$ 1 ab^{-1}, is available. To demonstrate this claim we will examine three specific processes: Bhabha and Möller scattering, as well as pair annihilation. In all cases we assume the incoming e$^-$ direction to be along the $z-$axis.

The first process we consider is Bhabha scattering which proceeds through both $s-$ and $t-$channel gauge boson exchanges. There are no new amplitudes to consider in this case, but each vertex picks up the kinematic dependent phase discussed above. As demonstrated[9] in our earlier work, the NC modifications to the SM result only appear in the interference term in the squared matrix element and are sensitive only to finite Λ_E. This NC effect appears through the cosine of the relative phase: $\Delta_{Bhabha} = \phi_s - \phi_t = \frac{-1}{\Lambda_E^2}[c_{01}t + \sqrt{ut}(c_{02}c_\phi + c_{03}s_\phi)]$, where t and u are the usual Mandelstam variables. Independently of the particular values of c_{0i}, the result-

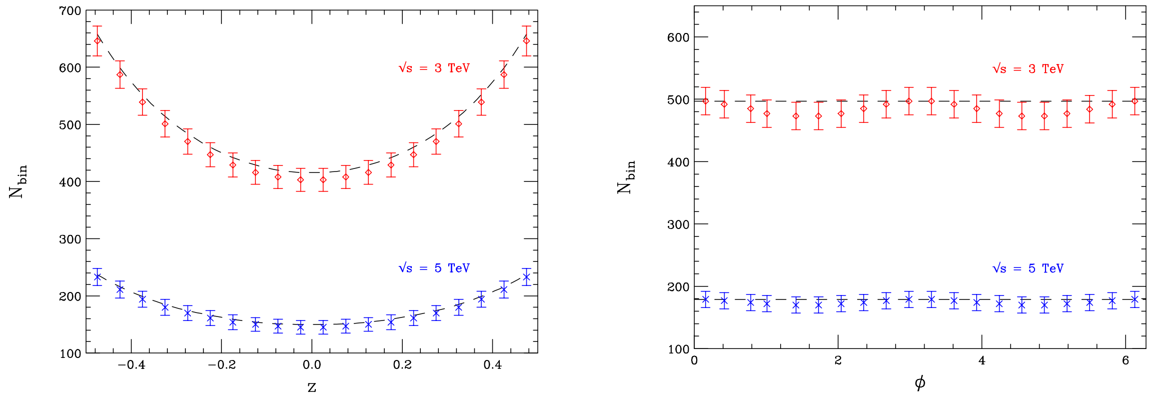

Figure 3: Shifts in the $z = \cos\theta$ and ϕ distributions for the process $e^+e^- \to \gamma\gamma$ at a 3 or 5 TeV CLIC assuming an integrated luminosity of 1 ab^{-1}. The dashed curves show the SM expectations while the 'data' assumes $c_{02} = 1$ and $\Lambda_E = \sqrt{s}$. A cut of $|z| < 0.5$ has been applied in the ϕ distribution.

ing $\cos\theta$ distribution is modified; however, for c_{02} and/or c_{03} nonvanishing, the cross section also picks up a periodic azimuthal (*i.e.*, ϕ) dependence as is shown in Fig. 1. While other new physics may lead to modifications of the $\cos\theta$ distribution, only Lorentz violation can induce a $\phi-$dependence in the cross section. Note that the ϕ dependence becomes more pronounced as stiffer cuts on the scattering angle, which selects more central events where both u and t are large, are applied. We find that beam polarization is not particularly useful for NC searches in Bhabha scattering; the Left-Right Polarization asymmetry, A_{LR}, is found to be rather insensitive to NC effects showing, in particular, little azimuthal dependence. The reach for Λ_{NC} in this case is also presented in Figure 1 where we see that it is of order $\sim 1.5\sqrt{s}$.

The next reaction we examine is Möller scattering. As in the case of Bhabha scattering, the NC effects appear only in the cosine of the phase of the interference term between the two $t-$ and $u-$channel amplitudes : $\Delta_{Moller} = \phi_u - \phi_t = \frac{-\sqrt{ut}}{\Lambda_B^2}[c_{12}c_\phi - c_{31}s_\phi]$. Note that, unlike Bhabha scattering, Möller scattering is sensitive to a finite Λ_B. If either c_{12} or c_{31} is nonzero, an azimuthal dependence is seen in the cross section as displayed in Fig. 2. (Note that there is no sensitivity to a nonvanishing c_{23}.) The oscillatory behavior is somewhat more pronounced here than in Bhabha scattering. Again, no additional sensitivity arises from the azimuthal dependence of A_{LR}. The reach for Λ_{NC} from Möller scattering is seen from Fig. 3 to exceed that for Bhabha scattering.

The last case we consider is pair annihilation. In addition to the new phases that enter the $t-$ and $u-$ channel amplitudes, there is now an additional $s-$channel photon exchange graph involving the NC-generated three photon vertex discussed above. This new amplitude is proportional to the sine of the phase: $\Delta_{PA} = \frac{-s}{2\Lambda_E^2}[c_{01}c_\theta + c_{02}s_\theta c_\phi + c_{03}s_\theta s_\phi]$. As in Bhabha scattering, this modification to the SM result appears to lowest order as a dimension-8 operator that probes finite Λ_E. As before, the $\cos\theta$ distribution is modified for all values of the c_{0i}, but a nonvanishing c_{02} and/or c_{03} is required to produce an azimuthal dependence. The resulting angular distributions are shown in Fig. 3 for the case $c_{02} = 1$. Writing $c_{01} = \cos\alpha$ and $c_{02} = \sin\alpha\cos\beta$, and $c_{03} = \sin\alpha\sin\beta$, Fig. 4 displays the reach for Λ_E for CLIC energies for several values of α.

In summary, we have examined the effects of NCQED in several $2 \to 2$ scattering processes at high energy e^+e^- colliders. We find that these effects produce an azimuthal dependence in the cross sections, providing a unique signature of the Lorentz violation inherent in these theories. The search reaches for the NC scale in a variety of processes are summarized in Table I for both $\sqrt{s} = 500$ GeV and CLIC energies. We see that these machines have a reasonable sensitivity to NC effects and provide a good probe of such theories.

References

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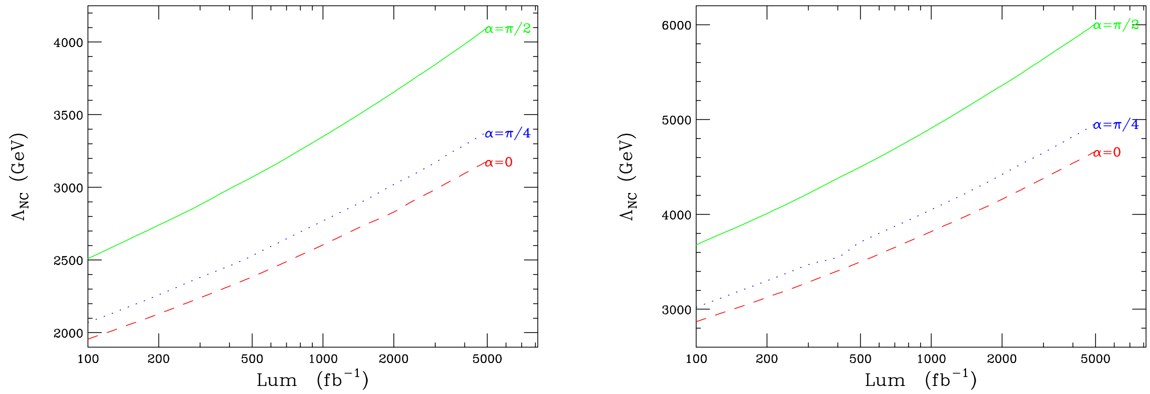

Figure 4: Reach for Λ_E at a (left) 3 TeV or a (right) 5 TeV CLIC as a function of the integrated luminosity for the process $e^+e^- \to \gamma\gamma$ following the notation in the text.

Process	Structure Probed	$\sqrt{s}$=500 GeV	$\sqrt{s}$=3 TeV	$\sqrt{s}$=5 TeV
$e^+e^- \to \gamma\gamma$	Space-Time	740 – 840 GeV	2.5 – 3.5 TeV	3.8 – 5.0 TeV
Moller Scattering	Space-Space	1700 GeV	5.9 TeV	8.5 TeV
Bhabha Scattering	Space-Time	1050 GeV	4.5 – 5.0 TeV	6.6 – 7.2 TeV
$\gamma\gamma \to \gamma\gamma$	Space-Time	700 – 800 GeV		
	Space-Space	500 GeV		
$\gamma\gamma \to e^+e^-$	Space-Time	220 – 260 GeV	1.1 – 1.3 TeV	1.8 – 2.1 TeV
$\gamma e \to \gamma e$	Space-Time	540 – 600 GeV	3.1 – 3.4 TeV	4.8 – 5.8 TeV
	Space-Space	700 – 720 GeV	4.0 – 4.2 TeV	6.3 – 6.5 TeV

Table I Summary of the 95% CL search limits on the NC scale Λ_{NC} from the various processes considered above at a 500 GeV e^+e^- linear collider with an integrated luminosity of 500 fb^{-1}or at a 3 or 5 TeV CLIC with an integrated luminosity of 1 ab^{-1}. The sensitivities are from the first two papers in [9]. The $\gamma\gamma \to e^+e^-$ and $\gamma e \to \gamma e$ analyses of Godfrey and Doncheski include an overall 2% systematic error not included by Hewett, Petriello and Rizzo.

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