Strongly Interacting $W$ Bosons at $e^-e^-$ Colliders

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If no Higgs boson exists, scattering amplitudes of massive vector bosons become strong at TeV energies. Below the threshold where new resonances appear, they are described by an effective chiral Lagrangian, which introduces a small number of new universal parameters at each order of a low-energy expansion. These parameters can be measured in (quasi-)elastic scattering processes of massive vector bosons in $e^-e^+$ collisions. Analyzing processes such as $e^-e^+ \rightarrow \nu_\ell \nu_\ell W^-W^-$, a sensitivity of the order $10^{-3}$ can be reached at a high-luminosity 1 TeV linear collider.

I. INTRODUCTION

Until an elementary Higgs boson has been found and established as the origin of electroweak symmetry breaking, the possibility must be taken into account that it does not exist, and mass generation is accomplished by other means. This is particularly true since the electroweak precision data are sensitive to Higgs effects essentially via two parameters ($S$ and $T$) only [1, 2]. In the absence of a light Higgs boson, their values could be generated by new strong interactions in the TeV range [3].

In a model-independent approach, one may adopt the minimal scenario that no new particles or more exotic effects exist up to the TeV range. However, since any model with massive fermions and vector bosons only is non-renormalizable as a quantum field theory, some scattering amplitudes rise without bound if calculated to lowest order in perturbation theory, eventually saturating the limit imposed by unitarity [4]. In quasi-elastic vector boson scattering, this bound is saturated already at $\sqrt{s} \approx 1.2$ TeV. Hence, one expects measurable effects of new interactions not far beyond 1 TeV.

II. VECTOR BOSON SCATTERING

If the Higgs boson is omitted from the Standard Model, its Lagrangian can nevertheless be expressed as a spontaneously broken gauge theory, with the gauge symmetry being non-linearly realized [5]. In this approach, the next-to-leading-order corrections to vector boson scattering amplitudes correspond to dimension-4 quartic interaction terms

$$L_4 = \alpha_4 \left[ \text{Tr}(V_\mu V^\mu) \right]^2,$$

(1)

$$L_5 = \alpha_5 \left[ \text{Tr}(V_\mu V^\mu) \right]^3,$$

(2)

$$L_6 = \alpha_6 \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV^\nu),$$

(3)

$$L_7 = \alpha_7 \text{Tr}(V_\mu V^\mu) \text{Tr}(TV_\nu) \text{Tr}(TV^\nu),$$

(4)

$$L_{10} = \alpha_{10} \frac{1}{2} \left[ \text{Tr}(TV_\mu) \text{Tr}(TV^\mu) \right]^2.$$

(5)

If a custodial $SU(2)$ symmetry is realized in bosonic interactions [6], only the first two coefficients are important.

The quartic interactions $L_4, 5, 6, 7, 10$ affect the cross sections, the energy behaviour, and the angular distributions of $2 \rightarrow 2$ vector boson scattering processes. In high-energy $e^-e^+$ collisions, such processes are initiated by $W$ and $Z$ bosons which are radiated off the incoming electrons/positrons:

$$e^+e^- \rightarrow \nu_\ell \nu_\ell W^+W^- : \quad W^+W^- \rightarrow W^+W^-$$

$$e^+e^- \rightarrow \bar{\nu}_\ell \nu_\ell ZZ : \quad W^+W^- \rightarrow ZZ$$

$$e^-e^- \rightarrow \nu_\ell \nu_\ell W^-W^- : \quad W^-W^- \rightarrow W^-W^-$$

$$e^-e^- \rightarrow \bar{\nu}_\ell e^-W^+Z : \quad W^+Z \rightarrow W^+Z$$

$$e^+e^- \rightarrow e^+e^-ZZ : \quad ZZ \rightarrow ZZ$$

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Since the $Z$ charge of leptons is suppressed compared to the $W\nu$ coupling, the event rate of the first three processes is enhanced compared to the last two ones. This fact leaves us with three channels, of which one requires an $e^-e^-$ collider.

In the kinematic range where the Mandelstam variables respect the relation $|\hat{s}| \sim |\hat{t}| \sim |\hat{u}| \gg m_W^2$, the longitudinal polarization states of $W$ and $Z$ bosons dominate the scattering amplitudes of the $2 \rightarrow 2$ subprocesses, eq. (6). Assuming $SU(2)$ custodial symmetry, these amplitudes can be expressed in terms of a single function $A(\hat{s}, \hat{t}, \hat{u})$:

\[
A(W^+W^- \rightarrow ZZ) = A(\hat{s}, \hat{t}, \hat{u}),
\]

\[
A(W^+W^- \rightarrow W^+W^-) = A(\hat{t}, \hat{u}, \hat{s}) + A(\hat{s}, \hat{t}, \hat{u}),
\]

\[
A(W^-W^- \rightarrow W^-W^-) = A(\hat{t}, \hat{u}, \hat{s}) + A(\hat{u}, \hat{t}, \hat{s}).
\]

The relations eq. (7) hold in the limit $|\hat{s}|, |\hat{t}|, |\hat{u}| \gg m_W^2$.

To next-to-leading order in the energy expansion, the amplitude $A(\hat{s}, \hat{t}, \hat{u})$ is given by

\[
A(\hat{s}, \hat{t}, \hat{u})_{\text{NLO}} = \frac{\hat{s}}{v^2} + \alpha_4 \frac{4(\hat{t}^2 + \hat{u}^2)}{v^4} + \alpha_5 \frac{8\hat{s}^2}{v^4}.
\]

### III. PHENOMENOLOGY

Qualitatively, the amplitude expressions eqs. (7), (8) are useful for predicting the dependence of the full processes eq. (6) on the parameters $\alpha_i$ in the energy range where the approximations are valid. One observes that the two processes accessible in $e^+e^-$ collision depend on the two parameters in a way which is quite different from the $e^-e^-$ process. In particular, the $e^+e^-$ amplitudes decrease in the lower left part of the $\alpha_4/\alpha_5$ plane, while the $e^-e^-$ amplitude increases. Obviously, the $e^-e^-$ channel should not be neglected as an important source of information in this measurement.

For a quantitative analysis in a realistic collider environment, one must consider the complete processes (6) where the initial vector bosons are represented by space-like virtual particles. Since in vector boson scattering processes large gauge cancellations are in effect which do not work for off-shell particles, together with the signal a large set of additional Feynman diagrams must be taken into account where the final-state vector bosons are radiated off the fermion lines, restoring the gauge cancellations for the full process. In $e^+e^-$ annihilation only, there is a second gauge-invariant class of diagrams contributing to the same final state which includes triple gauge boson production, where the final-state neutrinos come from on-shell $Z$ decay.

Apart from the processes listed in eq. (6), further vector boson scattering processes involving photons are possible in the Standard Model, which do not receive contributions from $\mathcal{L}_{4,5,6,7,10}$:

\[
e^\pm e^- \rightarrow e^\pm e^- W^\pm W^- : \gamma \gamma \rightarrow W^+W^- \\
e^\pm e^- \rightarrow e^\pm e^- W^- Z : \gamma W^- \rightarrow ZW^-.
\]

If no cuts are introduced, their total rates are enhanced by large logarithmic factors of the form $\ln \frac{m_Z^2}{m_e^2}$. Since final-state electrons/positrons may be lost in the beam-pipe, such photon-initiated processes constitute another major background to the signals listed above. However, by suitable cuts the background can be reduced to an acceptable level [7, 8].

In order to calculate the processes eq. (6), the chiral Lagrangian has been implemented in the CompHEP package [9], and the various signal and background processes has been evaluated in a complete tree-level calculation for typical values of the collider energy and luminosity [8]. This has been interfaced to the WHIZARD package [10] for multi-channel phase-space integration and event generation. As a result, Fig. 1 shows the distribution of the invariant mass of the signal process including the irreducible background with all cuts applied. Clearly, a non-zero value of $\alpha_4$ can be distinguished from $\alpha_4 = 0$. [This feature persists if the reducible background is added.]

### IV. RESULTS

In order to estimate the sensitivity of a Linear Collider to the parameters $\alpha_{4-10}$, the cross section prediction for a reference point in the parameter space has to be compared to the prediction for arbitrary values of those parameters. The discussion is simplified if conservation of the custodial symmetry is assumed in the scattering processes; this reduces the five-dimensional parameter space to a two-dimensional one. Adopting the
FIG. 1: (left) WW invariant mass distribution for the process $e^-e^- \to W^-W^-$ for a sample of unweighted Monte-Carlo events corresponding to an integrated luminosity of $\int L = 150 fb^{-1}$.

FIG. 2: (right) Exclusion contours for the hypothesis $\alpha_4, \alpha_5 = 0$, assuming $\sqrt{s} = 1$ TeV and an integrated $e^+e^-$ luminosity of $\int L = 1$ ab$^{-1}$ (50%/100% polarization). The 90% exclusion line has been obtained by combining the $W^+W^-$ and $ZZ$ channels (dark gray). The contour for the $W^-W^-$ channel (light gray) corresponds to an integrated $e^+e^-$ luminosity of $\int L = 100 fb^{-1}$ (100% polarization).

criterion that a point on the $(\alpha_4, \alpha_5)$ plane cannot be distinguished from the reference point at the center if the corresponding total event rate is within one standard deviation from the value at $\alpha_4 = \alpha_5 = 0$, the bands shown in Fig. 2 result for a $e^+e^-$ collider energy of 1 TeV, integrated luminosity of $\int L = 1 ab^{-1}$ and 100%/50% electron/positron polarization.

Due to the different parameter dependence of the cross section, the $e^-e^-$ channel adds independent information. If a luminosity comparable to the $e^+e^-$ option can be achieved, it serves not only as a cross-check for the two channels accessible in $e^+e^-$ collisions, but will further decrease the allowed area in the parameters $\alpha_4, \alpha_5$. If the custodial symmetry happens to be violated in the quartic vector-boson couplings, this channel is probably indispensable in order to disentangle contributions of the additional parameters $\alpha_6, \alpha_7, \alpha_{10}$.

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