A new study of the physics of the top quark threshold at a future electron-positron linear collider is presented. It is shown that using measurements of the total cross section, the forward-backward charge asymmetry and the position of the peak of the top momentum distribution, it is possible to determine simultaneously the top quark mass, $\alpha_s(M_Z)$ and the top quark width with experimental precisions of, respectively, 18 MeV, 0.0015 and 32 MeV, assuming an integrated luminosity of $300 \text{fb}^{-1}$. It is also found that, with the same assumptions, it will be difficult to determine the top Yukawa coupling with a precision much better than 30%.

1. Introduction

Earlier linear collider studies of the top threshold focused on the determination of the top quark mass (see, for instance, [6] and the proceedings of previous Linear Collider Workshops). A strong correlation between the top mass and the strong coupling constant, $\alpha_s(M_Z)$ was noticed, so that both quantities had to be measured at the same time, through a simultaneous two-parameter fit ([3] and references therein). The correlation limited the precision that could be achieved for the top mass to about 300 MeV. Furthermore, when next-to-next-to-leading order corrections to the $t\bar{t}$ cross section at threshold were computed (see [4] for a complete discussion), they were found to be large and to disturb the determination of the mass at the level of about 500 MeV. In 1999 there was a substantial breakthrough when two new definitions of top mass (“potential subtracted” and “1S” [4]) were proven to be much less sensitive to higher order corrections than the pole mass used previously. As a welcome side effect, correlations between $\alpha_s$ and these new masses were found to be much reduced [7], so that a determination of $m_t$ with less than 100 MeV experimental error and about 100–150 MeV theoretical uncertainty became feasible [7].

The study presented here continues the work of [7], extending it and completing it. It uses not only the $t\bar{t}$ cross section, but also the forward-backward charge asymmetry and the position of the peak of the top momentum distribution, as in [3]. The experimental study follows the one in [7] and is based on the following efficiencies, backgrounds and systematic uncertainties [1]:

- For the $t\bar{t}$ cross section: efficiency of 41%, systematic error of 3% and remaining background level of 8.5 fb, constant at all energy points.
- For the forward-backward asymmetry: efficiency of 11% and negligible systematic error.
- For the position of the peak of the momentum distribution: 41% efficiency and 4% systematic error.

An integrated luminosity of $300 \text{fb}^{-1}$ has been assumed, distributed equally on a nine-point scan around a center-of-mass energy equal to $2m_t$ and an additional energy point well below threshold in order to measure the background. The latest TESLA beam energy spectrum has been assumed [8]. The top quark mass has been taken as 175 GeV, $\alpha_s(M_Z)$ has been taken as 0.120, the Higgs mass as 120 GeV and the Standard Model values have been used for the top width and the top Yukawa coupling. The theoretical predictions of Kühn, Ježabek and Teubner et al., as implemented in the TOPPIK program ([5] and subsequent updates) have been used.
2. The Top Mass and the Strong Coupling Constant

To start with, a two-parameter fit with $m_t$ and $\alpha_s(M_Z)$ is performed, as in [7], but now using the larger integrated luminosity mentioned in the previous section and including also the information in the forward-backward charge asymmetry ($A_{FB}$) and position of the peak of the top momentum distribution. The uncertainties that come out of the fit, including only experimental errors, are the following:

$$\Delta m_t = 16 \text{ MeV} \quad \Delta \alpha_s = 0.0011 \quad \rho = 0.34,$$

where $\rho$ is the correlation coefficient between $m_t$ and $\alpha_s$. As in the study in [7], theoretical uncertainties, at around 100 MeV, will largely dominate the overall precision on the top mass, and they will most likely be also non-negligible in the $\alpha_s$ measurement, although this goes beyond the intended scope of this talk.

Once the possibility of measuring precisely the top mass and $\alpha_s$ with a $\bar{t}t$ threshold scan has been established one may turn the attention to measuring other quantities, like the top quark width and the top quark Yukawa coupling.

3. The Top Quark Width

Earlier attempts have been made of determining the top quark width ($\Gamma_t$) from the $\bar{t}t$ threshold scan [2, 3], with results in less than perfect agreement with each other, at least apparently. Figure 1 shows the sensitivity of the threshold cross section to the top width. As it can be seen from the figure, there is a sizable sensitivity both at the peak of the cross section and at lower center-of-mass energies. The other two observables ($A_{FB}$ and peak of momentum distribution) have substantially reduced sensitivities. A three-parameter fit with $m_t$, $\alpha_s$ and $\Gamma_t$ leads to the following experimental uncertainties:

$$\Delta m_t = 18 \text{ MeV} \quad \Delta \alpha_s = 0.0015 \quad \Delta \Gamma_t = 32 \text{ MeV},$$

with all correlations between the three parameters being below 50%. The 32 MeV uncertainty on the top width corresponds to about a 2% measurement. This result is found to be in agreement with the results quoted in [3] once differences in the assumed luminosities and efficiencies are
taken into account, and the larger sensitivity to $\Gamma_t$ in the region of center-of-mass energies below $2m_t$ that is found with the new top mass definitions is factored in.

Being the top quark so heavy, a 2% determination of its width can be very useful in constraining models of new physics which would predict new particles that could be produced in top quark decays. The precise determination of the top width from the threshold scan allows putting constraints which are independent of the final state particles produced.

4. The Top Yukawa Coupling

Measuring the top Yukawa coupling could provide an important test of the Higgs mechanism for generating fermion masses. The exchange of a Higgs boson between the top and anti-top produced at threshold has been taken into account in the theory prediction by adding a Yukawa potential to the QCD $\bar{t}t$ potential [5]. The modification of the potential can have measurable effects in the observables studied here. However, Figure 2 shows that the sensitivity of the total cross section to the Yukawa coupling is not very large.

As an unrealistic starting point, a one-parameter fit is performed, fixing all parameters except for the top Yukawa coupling, $\lambda_t$. The fit returns an asymmetric uncertainty:

$$\frac{\Delta \lambda_t}{\lambda_t} = +0.17 -0.24.$$  

Given the lack of sensitivity, one can try to see whether there could be any gain obtained by relaxing somewhat the assumptions concerning systematic errors. In particular, the systematic error in the cross section determination has been lowered from 3% (taken from [1]) to 1%, which seems to be a reasonable educated guess, given the level of understanding achieved at electron-positron machines like LEP, where selection systematics routinely achieved the few per mille level. Assuming the 1% error, the uncertainty in the one-parameter fit decreases to:

$$\frac{\Delta \lambda_t}{\lambda_t} = +0.12 -0.17.$$  

From now on, this lower systematic error in the selection will be assumed in all fits.

The next step consists on leaving the top mass and $\alpha_s$ free in the fit while fixing the top width to its Standard Model value and including an external constraint on $\alpha_s(M_Z)$ of $\pm 0.001$. The constraint should come from a different determination of $\alpha_s$, like the one available, for instance, at Giga-Z. Under these conditions, a three-parameter fit leads to the following precisions:

$$\Delta m_t = 25 \text{ MeV} \quad \Delta \alpha_s = 0.001 \text{ (constraint)} \quad \frac{\Delta \lambda_t}{\lambda_t} = +0.31 -0.49,$$  

with correlations large, up to 81%, particularly among $m_t$ and $\lambda_t$.

Finally, one could also try to leave the top width free in the fit, and perform a four-parameter fit with an external constraint on $\alpha_s(M_Z)$. The results are:

$$\Delta m_t = 30 \text{ MeV} \quad \Delta \alpha_s = 0.001 \text{ (constraint)} \quad \Delta \Gamma_t = 35 \text{ MeV} \quad \frac{\Delta \lambda_t}{\lambda_t} = +0.33 -0.57.$$  

The simultaneous determination of the four parameters is possible without a large increase in the resulting uncertainties. Correlations increase slightly, to a maximum of 85%, again among $m_t$ and $\lambda_t$.

As it can be seen, a realistic determination of the top Yukawa coupling, which has to be done simultaneously with that of the top mass, is challenging, although not impossible.

In summary, while a simultaneous determination of the top quark mass and width and $\alpha_s(M_Z)$ with experimental precisions of 18 MeV, 32 MeV and 0.0015, respectively, seems feasible, it appears to be difficult to determine the top Yukawa coupling with a precision much better than around 30% under realistic conditions.
References