Energy–flow method for multi–jet effective mass reconstruction in the highly granulated TESLA calorimeter

V.L. Morgunov*
DESY, TESLA-FLC, Hamburg†

Energy-flow techniques start to gain interest for reconstruction of particles and jets in current and future tracker-calorimeter detector systems. For the TESLA detector with the TPC tracker and the high granular calorimeters proposed an improved energy-flow algorithm is developed and the actual status of jet reconstruction is described. Future improvements of the method are discussed. There is also an attempt to write the strict mathematical definition of the reconstruction problem.

1. Preface

The energy–flow algorithm for jet energy reconstruction and calibration was used for many years by different collaborations [7, 9], [8, 18], [2, 3, 10, 15], [4, 5, 6, 16]. Recently it was applied in the CDF collaboration [1, 17] in a new CDF jet algorithm using tracking, calorimetry and shower maximum detectors. The reconstruction procedures were concentrated on the finding of jet energy and its corrections, software compensation of e/π ratio and compensation for dead material inside the calorimeter volume.

It is possible to find all particles in the event the charged as well as the neutral ones with a perfect tracker and a highly granular calorimeter without dead material inside. Such a calorimeter can measure all properties of particles—energy, direction, momentum and particle type with reasonable accuracy (see [11]). We would like to separate jet finding from the particle finding as itself that will allow to make physical analysis at the interaction point independently from the calorimeter clustering and particle identification. The technique required is close to a real pattern recognition.

The detailed description of the TESLA calorimeter can be found in [19].

2. Energy–flow and reconstruction problem solution

The TPC measures the momentum of particle more precisely than the calorimeter measures its energy. The energy flow procedure is an attempt to replace calorimeter clusters that are created by charged tracks with TPC momentum of these particles to get better energy and effective mass resolution. An application of such a scheme in an environment with dense jets as at TESLA needs a finely segmented electromagnetic and hadronic calorimeters.

The formulae of the reconstruction problem definition are shown in Appendix.

The shower operator $\hat{S}$ (Eq. 3) creates the limited energy distributions in the calorimeter volume (separated showers) around $\vec{\Omega}_k$ direction. So, one term of the equation (Eqs. 4, 6) (one particle and one ID) creates the cluster—collection of the cells around $\vec{\Omega}_k$ direction that is the base for the clusterization method.

The proximity of the particles in the $\vec{\Omega}$ space is the biggest difficulty for the reconstruction procedure—that leads to the showers overlapping in the same calorimeter cell and/or in the same calorimeter volume.

The final set of the equations Eq. 6 is too complicated to solve it by the usual mathematical methods. The solution of such a kind of equation can be done by the pure none–analytical computation algorithm.

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*Vasiliy.Morgunov@desy.de; `www.desy.de/~morgunov`

†Permanent address: ITEP, B. Cheremushkinskaya, 25, Moscow 117218, Russia

The clustering can follow different strategies:

1. search and building of clusters independently from the available charged track information later followed by association to the tracks as usually applied (see [14]) or

2. the charged track information is used to start the cluster identification and collection of the hits.

The recent development of the TESLA detector reconstruction program shows then the second method is significantly better then the first one [12].

Now we briefly describe the new reconstruction procedure. It stars from the collection of calorimeter hits around the extrapolated helix at the distances about one cell size to create charged particle track core. The charged track core then checked for the muon hypothesis. At this step the cell amplitude information in both calorimeters significantly used for the recognition of the MIP's track.

The next step of the reconstruction is the iterative procedure that collects charged track cluster in the shells around the cluster core with larger and larger distances from the predicted helix (see [12] for the detail). The procedure finished when the collected energy is equal to the initial particle momentum with some accuracy. The accuracy is the tuning parameter depending on the calorimeter energy resolution. Charged track cluster is tested for the electromagnetic hypothesis during the shell's collection procedure using the energy density distribution and shower shape.

After such a procedure all charged part of the event in calorimeter is collected and assigned to the TPC measured tracks. The remaining of the hits are used for a more or less usual cluster finding algorithm which reconstructs the neutral part of event (see [13]). At this way the idea of energy–flow is carried out directly. This algorithm was realized in the reconstruction program developed by TESLA–FLC group DESY (http://www.desy.de/flc).

Some preliminary results of multi-jet mass reconstruction are shown at Figure **??**, **??**. *No further kinematic fit was applied in reconstruction.*

Figure **??** shows the mass resolution for Z bosons, the top histogram is for a cms energy of the Z^0 pole. The resolution is at the predicted limit for ideal energy–flow procedure (see [13]). The bottom histogram shows the effective mass resolution of two highly boosted jets for the specific process $e^+e^- \to \gamma + Z^0 \to \gamma q\bar{q}$ at 500 GeV cms energy. Particles overlapping inside the individual jets deteriorates the predicted ideal resolution.

Figure **??**–top shows the di–jet mass resolution for the $e^+e^- \to Z^0 + H^0 \to \nu\bar{\nu} + W^+W^-$ at 340 GeV (H^0 mass = 170 GeV). The bottom histogram is the di–jet mass resolution for the $e^+e^- \to W^+W^-$ at 500 GeV. Again the resolution is worse due to the showers overlapping in boosted jets.

For the comparison of the multi–jet mass reconstruction by the conventional method one can see [14].

3. Conclusion

The actual version of the reconstruction program for the TESLA detector allows reconstruction of Z and W in boosted jets within 2.7–4.5 GeV sigma of di–jet effective mass.

Further improvement is expected when the tracking information used at the start of the clustering procedure.

The optimal granularity of the Tile–HCAL structure will be the subject of further studies.

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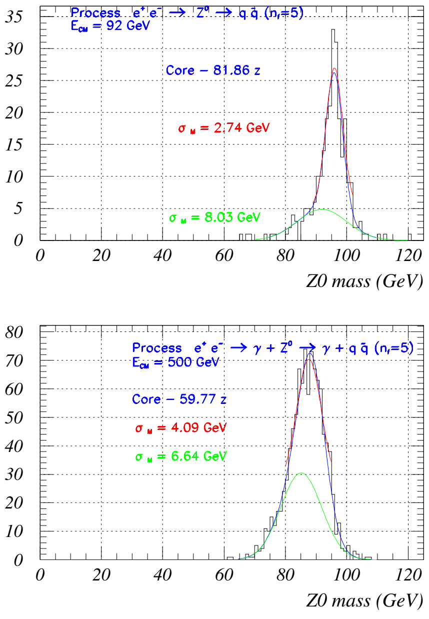

Figure 1: Reconstructed Z^0 mass from 2 jets for different energies. Single gaussian fit get a bad χ^2 for these curves. The core of di–jet mass reconstruction are gaussians (in red); it shows the limit of performance. Sometimes reconstruction program make mistakes, such cases are shown as the second gaussian (in green).

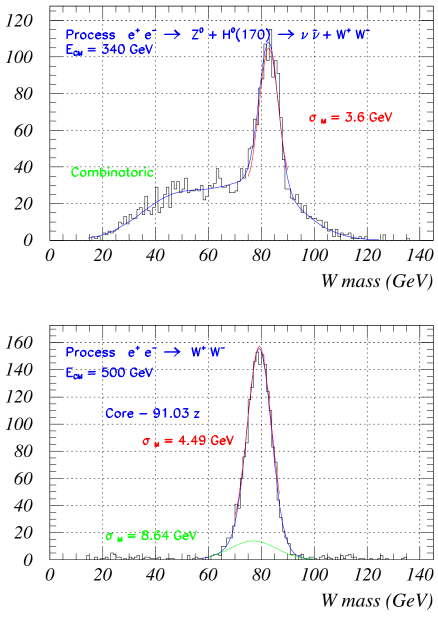

Figure 2: Reconstructed $W^{\pm}$ mass with combinatoric background (upper histogram). Reconstructed $W^{\pm}$ mass from 2 jets (lower histogram), jets cut: $Cos < 0.85$

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Appendix

Reconstruction problem definition

The energy flux of stable particles at the interaction point (IP) can be presented as a "hedgehog" of vectors in the direction space $\vec{\Omega}$:

$$E_{IP}(\vec{\Omega}) = \sum_{i}^{N_{IP}} \delta(\vec{\Omega}_i)\sqrt{\vec{p}_i^{\,2} + m_i^2} = \sum_{i}^{N_{IP}} \delta(\vec{\Omega}_i)E_i \tag{1}$$

where: N_{IP} is number of particles appearing at the IP with $\vec{p}_i$, m_i, E_i—the momentum, mass and energy of each particle.

The charged tracks change their initial direction due to the bending in the strong magnetic field that leads to jet energy flux distortion at the calorimeter inner surface.

The energy flux at the calorimeter surface $E_{\text{face}}(\vec{\Omega}, \vec{r})$ can be written as:

$$E_{face}(\vec{\Omega}, \vec{r}) = \hat{\mathcal{T}} E_{IP}(\vec{\Omega}) = \sum_{k}^{N_{\text{face}}} \delta(\vec{\Omega}_k)E_k(\vec{r}_k) \tag{2}$$

where: $\hat{\mathcal{T}}$ is the transport operator through the relatively small amount of matter (VTX + TPC); N_{face} is the number of particles that reached the calorimeter surface; $\vec{r}_k$ is the particle impact point at the calorimeter surface and $\vec{\Omega}_k$ is the particle direction at the calorimeter surface. The operator $\hat{\mathcal{T}}$ includes: small multiple scattering collisions and small energy losses, particle decay, and the effect of motion in the magnetic field for charged particles.

The energy deposition distribution in the calorimeter volume $E_{\text{dep.}}(\vec{r})$ can be written as:

$$E_{\text{dep.}}(\vec{r}) = \hat{S} E_{\text{face}}(\vec{\Omega}, \vec{r}) \tag{3}$$

where: $\hat{S}$ is the shower transformation operator. It produces the spatial energy distribution in calorimeter volume from the initial particle energy flux at the calorimeter surface.

The operator $\hat{S}$ can be formally represented as a sum of operators of separated particle types (ID) with different behaviour in the calorimeter.

$$\hat{S} = \hat{S}_e + \hat{S}_\gamma + \hat{S}_\mu + \hat{S}_{\text{charg.had.}} + \hat{S}_{\text{neut.had.}} = \sum_{\text{ID}} \hat{S}_{\text{ID}}\delta(\text{ID}) \tag{4}$$

The amplitude (A_{ic}) distribution in the calorimeter electronic cells can be written by the simplest operator $\hat{\mathcal{J}}$:

$$A_{ic}(\vec{r}_{ic}) = \hat{\mathcal{J}} E_{\text{dep.}}(\vec{r}) = \int_{V_{ic}} C_s\, E_{\text{dep.}}(\vec{r})\, dV \tag{5}$$

where: C_s is the conversion coefficient that depends on the calorimeter sampling structure and particle ID; V_{ic}—electronic cell volume with $\vec{r}_{ic}$ as a center.

After substitutions we have the set of equations for $A_{ic}(\vec{r}_{ic})$ that are measurable amplitudes at the center $\vec{r}_{ic}$ of electronic cell volume V_{ic}:

$$A_{ic}(\vec{r}_{ic}) = \sum_{\text{ID}} \sum_{k}^{N_{\text{face}}} \int_{V_{ic}} C_s\, \delta(\text{ID})\, \delta(\vec{\Omega}_k)\, \hat{S}_{\text{ID}} \sqrt{\vec{p}_k^{\,2}(\vec{r}_k) + m_k^2}\, dV \tag{6}$$

The number of such equations is equal to the number of calorimeter cells for event with non–zero amplitude in it.

The backward problem for this system of equations is the energy–flow reconstruction problem.

It was solved in some way for many different HEP detectors.

Analogs of such a kind of problem (successfully solved!) are the adjoint problem for the design of nuclear power reactors and the image reconstruction in the modern gamma–chamber in nuclear medicine.

Taking into account the definition of operator $\hat{S}$ one can recognize well known calorimeter energy–flow formula in this equations, that is a sum of energy over all calorimeter cells: $\mathcal{E} = \mathcal{E}_e + \mathcal{E}_\mu + \mathcal{E}_{\text{charg.had.}} + \mathcal{E}_\gamma + \mathcal{E}_{\text{neut.had.}}$.

In reality any calorimeter cell can carry the signals from many initial particles!

A few more remarks on this problem:

This formula is the strict mathematical definition of the bootstrap method for the fast simulation programs.

The equations can be rewritten as a sum of charged and neutral particles separately with the parametrization of shower operators $\hat{S}$ to make an analytical investigation of its properties.

If one reduces the cell volume up to zero the overlapping still would take place for the shower volume.