

# Precision Determination of $V_{ub}$ at an $e^+e^-$ B Factory

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Current methods of determining  $V_{ub}$  are dominated by theoretical uncertainties. We present Monte Carlo simulations of three promising methods of determining  $V_{ub}$  with small theoretical and experimental errors. We find that with data samples of order  $1,000 \text{ fb}^{-1}$  the B factories will attain combined experimental errors of a few % on  $V_{ub}$ , much smaller than the theoretical errors associated with new inclusive methods. Lattice QCD offers the promise of rate calculations of exclusive semileptonic decays with errors of a few %. A data sample of order  $10,000 \text{ fb}^{-1}$ , beyond the capabilities of the current B factories, may be required to achieve an experimental error on the exclusive rate comparable to the theoretical error.

## 1. Current Methods

CLEO was the first experiment to measure the ratio  $|V_{ub}|/|V_{cb}|$  by detection of inclusive leptons from semileptonic B meson decay beyond the kinematic endpoint for final states containing charm mesons [1]. Experimentally this is a clean measurement but only 5–20% of the leptons from  $b \rightarrow u\ell\nu$ , depending on the model, populate the endpoint region and so a large extrapolation, and hence a large theoretical uncertainty ( $\sim 20\%$ ) severely limits the precision of the measurement. However this method is still useful as a reality check of more precise methods.

More recently CLEO was able to reconstruct exclusive transitions using several analysis techniques that exploit the hermiticity of the CLEO detector to reconstruct the neutrino four vector [2, 3]. Due to large backgrounds it has so far been found necessary to continue to work mostly in the lepton endpoint region.  $|V_{ub}|$  is extracted from the measured branching fraction, and the rate which is computed using a variety of theoretical methods. The theoretical error is large ( $\sim 20\%$ ) and dominates the experimental errors as the form factors governing the transitions are not derived from first principles, and the transitions are measured in only part of the phase space.

## 2. Future Methods

There are two main theoretical approaches to improve the determination of  $|V_{ub}|$ . The first of these is the operator product expansion (OPE) which is able to predict the inclusive  $b \rightarrow u\ell\nu$  rate to 5–10% within experimentally tractable regions of phase space (either the region of  $m_{\text{had}} < m_D$  or the endpoint of the  $q^2$  spectrum). The second theoretical approach is lattice QCD (LQCD). There has been a great deal of progress in LQCD in the last few years and within a few years unquenched calculations of the rate for exclusive semileptonic decays will be available. The LQCD calculations may have an error of a few % or better. Further discussion of the theoretical issues in the extraction of a precision value of  $|V_{ub}|$  can be found in [4, 5, 6].

The simulations reported here are for a symmetric  $e^+e^-$  B Factory. We use a fast (parametric) Monte Carlo of the CLEO III detector called TRKSIM with the CLEO  $Y(4S) \rightarrow B\bar{B}$  event generator. Efficiencies and  $S/B$  for asymmetric B factories will differ slightly but the general conclusions are likely to be unchanged. Powerful suppression of the  $b \rightarrow c\ell\nu$  background can be achieved by full reconstruction of the companion  $B$  decaying to  $B \rightarrow D^{(*)}(n\pi)$ . As  $B$  tagging has a relatively low efficiency ( $2.85 \times 10^{-3}$ ) [7], the technique was impractical for most analyses with pre

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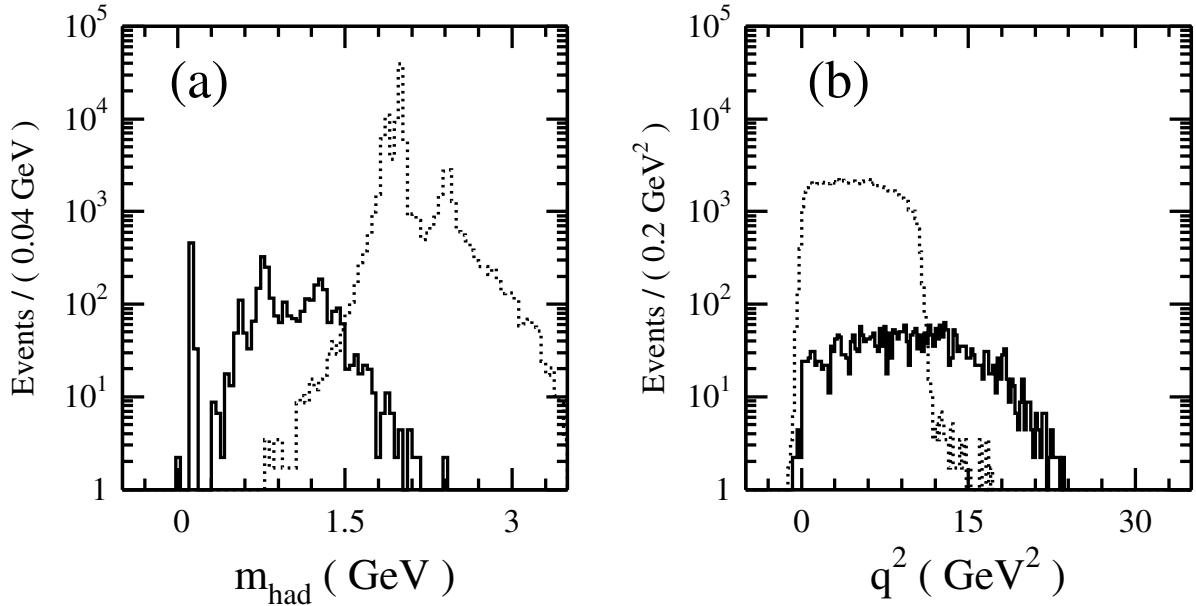


Figure 1: (a) Hadronic mass ( $m_{\text{had}}$ ) distribution for  $1000 \text{ fb}^{-1}$  data found with CLEO III fast MC. The solid histogram is the  $m_{\text{had}}$  distribution of  $b \rightarrow u\ell\nu$ , and the dashed histogram is the  $m_{\text{had}}$  distribution of  $b \rightarrow c\ell\nu$ . (b)  $q^2$  distribution for  $1000 \text{ fb}^{-1}$  data found with CLEO III fast MC. The solid histogram is the  $q^2$  distribution of  $b \rightarrow u\ell\nu$ , and the dashed histogram is the  $q^2$  distribution of  $b \rightarrow c\ell\nu$ .

B-factory samples, but will be used extensively in future. We assume 1% systematic error in lepton identification, and 2% systematic error in tracking in this paper.

### 2.1. Inclusive hadronic mass spectrum

The companion  $B$  is reconstructed and the remainder of the event is then required to have only one lepton, with momentum greater than  $1.4 \text{ GeV}$ , and missing mass consistent with a neutrino. We select  $b \rightarrow u\ell\nu$  events with  $m_{\text{had}} < m_D$ . Figure 1(a) shows the simulated  $m_{\text{had}}$  distribution for an integrated luminosity of  $1000 \text{ fb}^{-1}$ . Table I shows the estimate of the statistical and systematic error on  $V_{ub}$  measured with this method.

Table I Estimate of statistical and systematic errors on  $V_{ub}$  measured with  $m_{\text{had}}$  and  $q^2$  spectra, where  $S$  ( $B$ ) is the number of  $b \rightarrow u$  ( $c$ )  $\ell\nu$  events. The  $m_{\text{had}}$  method assumes a 10% systematic error in  $B$  for  $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$ . It further assumes that this systematic error can be reduced to 5% for  $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$  by vertexing, a better knowledge of  $D$  branching ratios, and a better knowledge of the form factors in  $B \rightarrow D/D^*/D^{**}\ell\nu$ . The  $q^2$  method assumes a 100% uncertainty in  $B$  for  $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$ , and a 20% uncertainty in  $B$  for  $\mathcal{L}_{\text{int}} \geq 500 \text{ fb}^{-1}$ . Note that errors in  $V_{ub}$  are experimental, not theoretical.

year	$\mathcal{L}_{\text{int}}(\text{fb}^{-1})$	$m_{\text{had}}$						$q^2$					
		$S$	$B$	$\delta V_{ub}^{\text{expt.}} (\%)$			$S$	$B$	$\delta V_{ub}^{\text{expt.}} (\%)$			$S$	$B$
				stat.	sys.	tot.			stat.	sys.	tot.		
2002	100	335	127	3.2	2.2	3.9	127	7	4.6	3.0	5.5		
2005	500	1675	635	1.5	1.5	2.1	635	36	2.0	1.2	2.3		
2010	2000	6700	2540	0.7	1.5	1.7	2538	144	1.0	1.2	1.6		

Table II Estimate of statistical and systematic errors on  $V_{ub}$  measured with the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decay. We assume  $S/B = 10/1$ , and a 10% uncertainty in  $B$  for this estimate. The numbers in the parentheses are for  $0.4 < p_\pi < 0.8 \text{ GeV}/c^2$ . Note that errors in  $V_{ub}$  are experimental, not theoretical.

year	$\mathcal{L}_{\text{int}}(fb^{-1})$	$S$	$B$	$\delta V_{ub}^{\text{expt.}} (\%)$		
				stat.	sys.	tot.
2008	1000	590(29)	59(3)	4.3(9.8)	1.2	4.5(9.9)
?	10000	5900(290)	590(30)	0.7(3.1)	1.2	1.4(3.3)

## 2.2. Inclusive $q^2$ spectrum

Using the inclusive  $q^2$  endpoint results in a loss of statistics, but a gain in theoretical certainty compared to the low  $m_{\text{had}}$  region. The experimental advantage of this method compared to the  $m_{\text{had}}$  method is that  $S/B$  is more favorable, therefore this method will be more attractive with very large data samples.  $B$  tagging and event selection are the same as in the previous method. We select  $b \rightarrow u \ell \nu$  events with  $q^2 > 11.6 \text{ GeV}^2$ . Figure 1(b) shows the simulated  $q^2$  distribution for an integrated luminosity of  $1000 fb^{-1}$ . Table I also shows the estimate of the statistical and systematic experimental errors on  $V_{ub}$  obtained with this method.

## 2.3. Exclusive decays with Lattice prediction

LQCD aims to predict the decay rate of semileptonic decays such as  $D \rightarrow \pi \ell \nu$  and  $B \rightarrow \pi \ell \nu$  to  $\sim$  few %, which corresponds to  $\delta V_{ub}^{\text{stat.}} \sim 1\text{-}2\%$  [6]. However many consistency checks will be required to demonstrate that the estimated lattice precision has been achieved. Here we outline one possible method.

From an unquenched LQCD calculation of  $f_D$  and a measurement of  $D^+ \rightarrow \mu^+ \nu$  at a charm factory (for example CLEO-C [8]) we obtain a precision direct measurement of  $V_{cd}$ . Using this value of  $V_{cd}$ , with an unquenched LQCD calculation of the rate and form factor shape of  $D \rightarrow \pi \ell \nu$ , we can make a direct test of the lattice with a measurement of  $d\Gamma/dq^2(D \rightarrow \pi \ell \nu)$ . This measurement is also best done at a charm factory where the kinematics at threshold cleanly separates signal from background.

If the lattice passes the above test, the second step is to compare the lattice prediction of the shape of  $d\Gamma/dq^2(B \rightarrow \pi \ell \nu)$  to that of data at a B factory. If the shapes agree the third step is to measure  $\Gamma(B \rightarrow \pi \ell \nu)$  with data. To cleanly isolate the signal we fully reconstruct the companion  $B$  meson, and select, for example,  $B \rightarrow \pi \ell \nu$  events with the neutrino reconstruction or consistency methods. Combining the measurement with the lattice prediction of  $\Gamma(B \rightarrow \pi \ell \nu)$ , we extract  $V_{ub}$ . The theoretical error on  $|V_{ub}|$  may be as small as 1-2%. Table II shows the results of our simulation. To have a comparably small experimental error would require  $\sim 10,000 fb^{-1}$ . Such large data samples are beyond the reach of existing B factories where  $\sim 2000 fb^{-1}$  are expected by the year 2010.

## 3. Conclusion

All possible theoretically clean measurements in the B sector are very important even if they are redundant within the standard model. It is essential to pursue both CP violating and CP conserving measurements (i.e.,  $V_{ub}$ ) to test the Standard Model and look for new physics. Inclusive methods will achieve  $\delta V_{ub} \sim \text{few\%}(experiment) \sim 5\text{-}10\%(\text{theory})$ . The  $q^2$  endpoint is the method of choice. The first test of CKM at the 10% level will come from inclusive methods of determining  $V_{ub}$  and  $V_{cb}$ , and the measurements of  $\sin 2\beta$ , and  $V_{td}/V_{ts}$ . If the lattice can reach the predicted accuracy (1-2%), it will become the method of choice for measurements of  $V_{ub}$  (and  $V_{cb}$ ) in the second decade of the 21st Century. However the lattice must first be calibrated. A charm factory can provide unique and crucial tests of lattice predictions. A  $\sim 10,000\text{-}20,000 fb^{-1}$  data sample

is required to attain a total experimental error of 1-2% on  $V_{ub}$  commensurate with the lattice error.

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