ODYSSEUS: AN ADAPTIVE 3D STRONG-STRONG BEAM-BEAM SIMULATION CODE

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Abstract

ODYSSEUS is a 3D simulation code for the beam-beam interaction in storage ring colliders. It is a true strong-strong simulation in which no constraints are placed on the distribution of particles in the beams. The program achieves its speed by adaptively choosing between alternative electromagnetic field calculation methods. Linear tracking through the ring and wake fields are included. We plan to include nonlinear tracking through the ring in the next version of the code.

1 INTRODUCTION

In storage ring colliders the beam-beam force can cause a blowup of the beam emittance, particle loss, or instability. Because the beam-beam force is strongly nonlinear, particle-tracking methods are useful for determining the dynamics of the beams. Strong-strong simulations, in which the force exerted by each beam on the opposing beam is calculated, are capable of modeling coherent instabilities of the beams. These simulations are very time-intensive because of the need to repeatedly calculate the electromagnetic field of each beam. To include longitudinal as well as transverse degrees of freedom, an unconstrained strong-strong code must use special techniques to improve its speed.

ODYSSEUS is an unconstrained 3D strong-strong beam-beam simulation that includes broadband wake fields [1, 2]. It is capable of rapidly calculating the electromagnetic field of a beam divided into many longitudinal slices because it adaptively chooses from a variety of different field computation methods. Different algorithms are used for the core, transverse tails, and longitudinal tails of the beam. The parameters of the program can be changed to model flat or round beams. Inclusion of the longitudinal degree of freedom and wake fields allows the investigation of previously inaccessible physics.

A flowchart for ODYSSEUS is shown in Figure 1. The individual calculations are described in detail in the following sections.

2 PARTICLE TRACKING

2.1 Storage Ring

On each simulated turn through the storage ring, each macroparticle is propagated through the linear optics of the storage ring, including chromaticity, synchrotron radiation excitation and damping, RF phase focusing, and wake field deflections. Macroparticles which have migrated past a transverse aperture are no longer considered in the simulation. Longitudinal and transverse short-range wake fields are modeled as a sum of broadband resonators.

2.2 Collisions

During its passage through the opposing bunch, the transverse position of each macroparticle may change appreciably because the vertical beta function at the interaction point in many colliders is comparable to the bunch length. ODYSSEUS handles the longitudinal variation of the electromagnetic field of the beam by dividing the beam into slices. The simulation collides each pair of slices sequentially, updating the transverse momenta and positions of each macroparticle after each pair-wise collision of slices. For each slice collision the slice electromagnetic field is calculated by one of the methods described below. Macroparticles undergo longitudinal oscillations and migrate from slice to slice, so on each turn the macroparticles are sorted according to their longitudinal position and are reassigned to slices.

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3 ELECTROMAGNETIC FIELD CALCULATION

3.1 Beam Core

If the number of macroparticles, \( N \), within a slice is very small, the integrated field at a probe beam macroparticle is calculated from radius vector from each opposing source beam macroparticle. Because it involves a sum over source-probe particle pairs, the number of calculations goes as \( N^2 \), making this method efficient only for \( N < 50 \).

For larger values of \( N \), the electromagnetic field is calculated on a rectangular grid (Particle-In-Cell method) using pre-calculated Green’s functions for charges on the grid points. The macroparticle charge is assigned to the grid points using one of the two area-weighted techniques described below. If the number of grid points \( N_g \) is less than about 200, the convolution of the charge density and Green’s function is done as a summation in coordinate space and the number of calculations required goes as \( N_g^2 \).

For larger values of \( N_g \), the convolution of the Green’s function and charge density is done as a multiplication in wavenumber space. The speed of this method is limited by the speed of the necessary Fourier transform to wavenumber space and the inverse transform back to coordinate space. Thus the number of calculations goes as \( N_g \log N_g \). To suppress edge effect problems in the Fourier transforms the size of the wavenumber space can be doubled in both directions and padded with zeros [3], although this is both to be unnecessary for the typical, Gaussian-like, beam charge distribution.

The grid itself is adaptive. The number of cells in each transverse direction and their aspect ratio change as needed to cover the beam, with a constraint on the aspect ratio, described below in Section 4.2.

3.2 Beam Tails

The tails of the beam, typically taken to be particles with a displacement of more than \((10/3) \sigma\) in any direction, are treated differently than the core particles. The tail particles have very little influence on the beam-beam force. They do respond to the beam-beam force, so a weak-strong calculation is used for them.

Longitudinal tail particles are subject to forces from the core of the opposing beam. A full calculation of the field from the opposing (strong) beam slice is performed, but the tails are assumed to have no effect on the strong beam (see Figure 2). Alternatively, the user may choose a strong-strong calculation for all slices.

The transverse tail particles are subject to a beam-beam force of similar magnitude to that experienced by the core particles. However, the high-wavenumber component of the charge distribution of the core has little influence on the field in the transverse tails, so the field used is that of a Gaussian charge distribution with the same charge and first- and second-order moments as the slice. The field from this Gaussian charge distribution is calculated from the rational approximation of Talman and Okamoto [4].

Figure 2: The beam is divided into longitudinal slices and the collision of each pair of slices is simulated in sequence. Slices are marked as either “weak” (white) or “strong” (gray) and the appropriate technique is applied when they interact. No two weak slices ever interact.

3.3 Interpolation Techniques

Whenever a grid-based method is used, it is necessary to interpolate from the fields on a grid to arbitrary probe macroparticle locations and to deposit charge on the grid from arbitrary source macroparticle locations. ODYSSEUS gives the user a choice between three interpolation methods: Nearest-Grid-Point (NGP); four-point Cloud-In-Cell (CIC); and a nine-point extension of the Triangular-Shaped-Cloud (TSC) technique [3]. Lower order interpolation methods tend to introduce noise into the simulation. Higher order interpolation methods reduce this noise but tend to broaden small features in the charge distribution. In ODYSSEUS a “sharpening function” is used during the convolution of the charge density and Green’s function in wavenumber space to compensate this broadening.

4 SPEED AND ACCURACY CONSIDERATIONS

4.1 Speed

ODYSSEUS is being run on a farm of 500 MHz Alpha based processors that operate under Linux. The fast Fourier transforms typically take about 90% of the computation time. The code is not parallelized, because one typically wishes to run the code with a large set of...
varying parameters (e.g., for tune scans). The speed of ODYSSEUS may be compared with that of a non-adaptive Particle-In-Cell (PIC) calculation by setting its options to include only the PIC method. The calculation is found to run 5 times faster with the adaptive options.

4.2 Field Error Due to Transverse Grid

For round beams a field calculation grid can be constructed out of square cells with equal numbers of cells in each dimension. This construction allows for accurate field calculation with a low number of cells. In contrast, the extreme aspect ratios of flat beams force the use of either large numbers of cells or individual cells with large aspect ratios. The Green’s function technique fails to provide a good approximation to the electromagnetic field when the cell aspect ratio is far from unity, especially when a scalar Green’s function is used. ODYSSEUS uses a two-component Green’s function and limits the cell aspect ratio. This limit is typically set to 1.4.

4.3 Field Error Due to Longitudinal Slicing

The electromagnetic field error introduced by dividing the beam into longitudinal slices is a source of noise that tends to increase the emittance of the simulated beam. This is a problem when simulating flat beams with a low natural emittance and large vertical beam-beam parameter $\xi_y$ when the number of slices is small. The noise produces a maximum possible value of $\xi_y$ in the simulation. In a particle-tracking simulation using longitudinal slices of uniform length $\Delta z$, we find that the maximum $\xi_y$ is

$$\xi_{y,\text{max}} = \frac{4\sqrt{3}}{\pi} \frac{\beta_y}{\Delta z} \sqrt{\delta}$$  \hspace{1cm} (1)

where $\beta_y$ is the vertical beta function at the interaction point and $\delta$ is the transverse damping decrement. Other physical or numerical effects may further reduce $\xi_y$. Figure 3 shows the vertical beam-beam parameter as a function of the number of slices. This behavior is consistent with the limit of Eq. (1).

5 CONCLUSIONS

The adaptive nature of ODYSSEUS allows it to run several times faster than a code using PIC methods alone. Simulations of several tens of thousands of turns can be completed in a reasonable amount of time, varying from hours to several days, depending on the parameters of the calculation.

ODYSSEUS has been applied to tune-plane scans with CESR parameters to determine the operating point with the best luminosity; to tune-plane scans at lower energy for the proposed CESR-c; to quantifying the effect of damping time on the maximum beam-beam parameter; and to determining the mechanism of the beam-beam limit in CESR. The luminosity tune scans agree very well with observations in CESR [5]. In the next version of ODYSSEUS we plan to include calls to BMAD [6], a particle-tracking code, to realistically model the remainder of the ring.

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