Beam Sizes in Collision and Flip-Flop States at KEKB

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Abstract
Evaluating a simplified linear model of the beam-beam interaction, self-consistent horizontal beta functions, emittances and beam sizes are computed for the two unequal colliding beams in KEKB. For head-on collisions only one equilibrium solution exists at the nominal tunes. However, if for off-center collisions the quadrupolar component of the beam-beam force becomes defocusing, we obtain two solutions, one of which describes a flip-flop state with increased size of the positron beam. This result may explain observations of sudden luminosity drops.

1 INTRODUCTION
During KEKB operation drops in the luminosity are observed, which are associated with step changes in the horizontal and/or vertical beam sizes at the interaction point (IP) [1]. Often the LER horizontal beam size increases. Changes in the beam size appear to be correlated with small orbit variations. In particular, a hysteresis is observed when the beam-beam separation at the collision point is varied. In this report, we study a simple linear model of the self-consistent horizontal optics and emittances for the two unequal colliding beams, evaluate their dependence on the betatron tune and the beam-beam tune shift, and demonstrate the existence of flip-flop solutions for off-center collisions.

2 SELF-CONSISTENT OPTICS AND BEAM SIZES
In collision, the beam emittance and beta functions are changed by the focusing force of the opposing beam. Neglecting the change in the other beam (weak-strong approximation), the horizontal dynamic beta function, $\beta_{x,1(2)}$, at the collision point is usually obtained as [2]

$$b_{x,1(2)} \equiv \frac{\beta_{x,0,1(2)}}{\beta_{x,1(2)}} = \frac{\sin 2\pi(Q_{1(2)} + \Delta Q_{1(2)})}{\sin 2\pi Q_{1(2)}}$$

(1)

where

$$Q_{1(2)} + \Delta Q_{1(2)} = \frac{1}{2\pi} \arccos \left( \cos 2\pi Q_{1(2)} - 2\pi \xi_{0,1(2)} \sin 2\pi Q_{1(2)} \right)$$

(2)

and $Q_{1(2)}$ denotes the unperturbed horizontal betatron tune of beam 1 (or beam 2). The subindices 1 and 2 refer to the electron (HER) and positron beam (LER), respectively; the subindex ‘0’ signifies the values of beta function and emittance without the focusing effect of the opposing beam.

Table 1: Parameters relevant to the flip-flop analysis.

<table>
<thead>
<tr>
<th>variable</th>
<th>HER</th>
<th>LER</th>
</tr>
</thead>
<tbody>
<tr>
<td>hor. beam-beam tune shift $\xi_x$</td>
<td>0.049</td>
<td>0.055</td>
</tr>
<tr>
<td>vert. beam-beam tune shift $\xi_y$</td>
<td>0.025</td>
<td>0.037</td>
</tr>
<tr>
<td>hor. tune</td>
<td>44.520</td>
<td>45.505</td>
</tr>
<tr>
<td>vert. tune</td>
<td>41.587</td>
<td>43.575</td>
</tr>
<tr>
<td>hor. beta function $\beta_{x,0}$</td>
<td>63 cm</td>
<td>59 cm</td>
</tr>
<tr>
<td>vert. beta function $\beta_{y,0}$</td>
<td>0.7 cm</td>
<td>0.7 cm</td>
</tr>
<tr>
<td>vert. beam-beam tune shift $\xi_y$</td>
<td>0.025</td>
<td>0.037</td>
</tr>
<tr>
<td>single-bunch current</td>
<td>0.48 mA</td>
<td>0.63 mA</td>
</tr>
</tbody>
</table>

The parameter $\xi_{0,1(2)}$ is the horizontal beam-beam tune shift, calculated from the unperturbed beta functions and emittances,

$$\xi_{0,1(2)} \equiv \frac{N_{2(1)r_0}}{2\pi e_{x,0,2(1)}} \beta_{x,0,1(2)} \beta_{y,0,2(1)}$$

(3)

where $r_0$ denotes the classical electron radius. For the parameter values of KEKB, summarized in Table 1, the (inverse) normalized dynamic beta functions evaluate to $b_{x,1} = 2.40$ and $b_{x,2} = 4.78$.

Since the actual beam-beam tune shift, $\xi_{1,2}$, depends on the dynamic beta function, Eq. (1) does not describe a self-consistent solution of the problem. Neither can it account for flip-flop phenomena or for the simultaneous existence of more than one equilibrium state. The flip-flop effect with linearized beam-beam force for round beams was recently analyzed by A. Otboyev and E. Perevedentsev [3], who computed self-consistent beta functions and equilibrium emittances. We here follow and extend their formalism, and then apply it to the KEKB case of flat beams with unequal parameters. For simplicity, we limit the discussion to the horizontal plane, in which flip-flop effects are frequently observed.

The basic equations governing the evolution of the beta functions are [3]

$$b_1^2 = 1 + 2c_1x_1 \frac{b_2}{e_2} - x_1^2 \frac{b_2^2}{e_2^2}$$

(4)

$$b_2^2 = 1 + 2c_2x_2 \frac{b_1}{e_1} - x_2^2 \frac{b_1^2}{e_1^2}$$

(5)

were $c_{1(2)} \equiv \cot(2\pi Q_{1(2)})$, $b_{1(2)} \equiv \beta_{x,0,1(2)}/\beta_{x,1(2)}$, $x_{1(2)} \equiv 2\pi \xi_{0,1(2)}$, and $e_{1(2)} \equiv \xi_{x,1(2)}/\xi_{x,0,1(2)}$.

Figure 1 displays the graphical method [4] of solving Eqs. (4) and (5). Plotting the two curves $b_1(b_2)$ and $b_2(b_1)$,
solutions to (4) and (5) are given by their intersections. As can be seen, for the parameters considered and for constant emittances, \(e_1 = e_2 = 1\), there is only one intersection and, hence, no flip flop is expected. Figure 2 shows an equivalent picture obtained by neglecting the quadratic terms in (4) and (5). The difference to Fig. 1 is insignificant.

If the beams collide with a horizontal offset, the quadrupolar component of the beam-beam force may change sign. Figure 3 shows the graphical solution for an unperturbed beam-beam tune shift parameter \(\xi_0\) equal to \(-1/4\) times the nominal value. Still there is only one intersection.

Next we include the variation in emittance. Following Ref. [5], or ignoring the oscillatory term in the solution of Ref. [3], the emittance changes with the strength of the beam-beam focusing according to

\[
e_{1,2} = 1 + p_{1,2} \cot \frac{\pi Q_{x,1,2}}{\beta_{x,1,1}} \sqrt{1 + 2 p_{1,2} \cot \frac{\pi Q_{x,1,2}}{\beta_{x,1,1}} - p_{1,2}^2},
\]

where \(e_{1,2} \equiv \epsilon_x, 1(2)/\epsilon_{x,0,1}(2)\) and \(p_{1,2} \equiv x_{1,2}b_{2,1}/e_{2,1}/b_{1,2}\). This equation is illustrated in Fig. 4.

Figures 5 and 6 shows a more precise SAD computation of the dynamic emittances and beta functions as a function of the beam-beam tune shift, provided by H. Koiso, which accounts for the exact ring optics. The emittance variation in Fig. 5 agrees within 10% with the simplified estimate of Eq. (6) and Fig. 4.
Figure 5: Dynamic emittance in units of the unperturbed emittance, \(e_{1,2}\), as a function of beam-beam tune shift for the low and high-energy rings of KEKB, computed by SAD. (Courtesy H. Koiso)

Figure 6: Horizontal beta functions \(\beta_{1,2}/\beta_{0,1,2} \equiv 1/b_{1,2}\) as a function of beam-beam tune shift for the low and high-energy rings of KEKB, computed by SAD. (Courtesy H. Koiso)

Figure 7: Graphical solution of Eqs. (4)–(5) for emittances that vary linearly with the strength of the beam-beam force as in Eqs. (7) and (8). Plotted is the electron beta function, \(b_1 = \beta_{x0,e}/\beta_{x,e}\), as a function of the positron beta function, \(b_2 = \beta_{x0,p}/\beta_{x,p}\), for the nominal parameters of Table 1.

The situation changes dramatically, if we invert the sign of the beam-beam tune shift, in order to model a situation with off-center collisions. Figure 8 illustrates a typical example, where we consider an unperturbed tune shift equal to \(-0.25 \xi_{0,1(2)}^{\text{nom}}\). In this case there are two intersections, i.e., two solutions. This is quite different from the result for constant emittances in Fig. 3. One of the two solutions represents a large increase of the positron beta function (small value of \(b_2\)), possibly consistent with the observed flip-flop state.

The self-consistent beta functions and emittances depend on the tunes of both beams. Figures 9 and 10 illustrate the dependence of the normalized beam sizes \(\sigma_{x,1(2)}/\sigma_{x0,1(2)} = \sqrt{e_{1(2)}/b_{1(2)}}\) on the tunes in either ring, respectively, for the nominal beam-beam tune shift. In this calculation, we have approximated the variation of the coefficients \(k_1\) and \(k_2\) in Eqs. (7) and (8) with the tunes \(Q_{1,2}\).
Figure 8: Graphical solution of Eqs. (4)–(5) assuming a negative beam-beam tune shift \( \xi_{0,1(2)} = -0.25 \xi_{0,1(2)}^{\text{nom}} \), for emittances that vary linearly with the strength of the beam-beam force as in Eqs. (7) and (8). Plotted is the electron beta function, \( b_1 = \beta_{x,e} / \beta_{x,e} \), as a function of the positron beta function, \( b_2 = \beta_{x,p} / \beta_{x,p} \).

as

\[
\kappa_{1(2)} \approx k_{0,1(2)} \cot(2\pi Q_{1(2)}) / \cot(2\pi Q_{0,1(2)}). \quad (9)
\]

Figure 9: Self-consistent horizontal beam sizes \( \sigma_{x} / \sigma_{x0} \) as a function of the positron tune (right). The positron tune is set to 0.505.

An offset between the two beams at the collision point distorts the closed orbit, introduces a change in the linear focusing, and excites additional higher-order resonances. As indicated earlier in this paper, we only consider the variation in the quadrupolar focusing force, and approximate the change in the focusing due to a varying beam-beam separation by a common multiplication factor \( M_\xi \) for the two beam-beam tune shift parameters. This is based on the assumption that a small beam-beam offset reduces the strength of linear focusing experienced at the collision point by a similar factor for either beam, provided the sizes of the two beams are equal (note that they will not remain equal once a flip-flop state is established). For larger offsets, the beam-beam focusing force changes sign, which we model by a negative value for \( M_\xi \).

Figures 11–13 illustrate the dynamic variation of beta function, emittances and beam sizes as a function of a positive multiplication factor \( M_\xi \). The beta functions decrease more strongly than the emittances increase as a function of the beam-beam tune shift, such that the IP beam sizes shrink for higher current. Equivalent results for a negative multiplication factor \( M_\xi \) are shown in Figs. 14–16. Consistent with Fig. 3, in the latter case two solutions coexist. The additional solution appears to be of the flip-flop type. It is characterized by a large increase in the LER IP beta function (Fig. 14), a decrease in the emittance (Fig. 15) and a resulting net growth of the IP beam size (Fig. 16).

A tentative explanation of the observed hysteresis may then be the following. For a sufficiently large beam-beam offset of about \( 2\sigma_x \), the ‘quadrupolar’ component of the horizontal beam-beam force changes sign, i.e., the force becomes defocusing instead of focusing, and there emerges a new equilibrium, which represents a flip-flop state. Therefore, repeated changes in the sign of \( \xi \) — due to varying beam-beam separation —, might induce transitions between the different solutions that exist for \( \xi < 0 \).

3 CONCLUSIONS

Calculations of horizontal equilibrium sizes for head-on colliding beams at KEKB suggest the existence of a unique equilibrium solution. If the beams are horizontally separated sufficiently far that the ‘quadrupolar’ component of the beam-beam force is defocusing, two self-consistent solutions coexist, one of which describes a flip-flop state, in which the positron beam is blown up. This appears consistent with some of the observations.

Our analysis was based on a simplified model, which considers only the horizontal plane, a linearized beam-beam force, a linear dependence of the emittance on the beam-beam tune shift, and a common scale factor for both beam-beam parameters representing the effect of a trans-
Figure 11: Self-consistent dynamic beta functions $\beta_{1,2}/\beta_{0,1,2} \equiv 1/b_{1,2}$, as a function of a common positive multiplication factor $M_\xi$ for both tune shift parameters. This multiplication factor is intended to model a change in linear focusing arising from a beam-beam offset. The tunes are set to 0.520 (HER, $e^-$) and 0.505 (LER, $e^+$), respectively.

Figure 12: Self-consistent dynamic emittances $\epsilon_{1,2} \equiv \epsilon_{1,2}/\epsilon_{0,1,2}$, as a function of a common positive multiplication factor $M_\xi$ for both tune shift parameters. This multiplication factor is intended to model a change in linear focusing arising from a beam-beam offset. The tunes are set to 0.520 (HER, $e^-$) and 0.505 (LER, $e^+$), respectively.

verse offset. All of these approximations could be improved. Future extensions might also include the vertical plane, bunch length and crossing angle, as well as the non-linear components of the force including an arbitrary beam-beam separation.

4 ACKNOWLEDGEMENTS

I thank K. Oide for suggesting this study, H. Fukuma, Y. Funakoshi, H. Koiso, K. Ohmi, K. Oide, and J. Urakawa for the hospitality at KEK and for helpful discussions, W. Herr and F. Ruggiero for their support, and the organizers and participants of the FNAL 2001 beam-beam workshop, especially T. Sen and M. Xiao, for their interest in this work.

5 REFERENCES

Figure 15: Self-consistent dynamic emittances $e_{1,2} \equiv \epsilon_{1,2}/\epsilon_{0,1,2}$, as a function of a common negative multiplication factor $M_\xi$ for both tune shift parameters. This multiplication factor is intended to model a change in linear focusing arising from a beam-beam offset. The tunes are set to 0.520 (HER, $e^-$) and 0.505 (LER, $e^+$), respectively.

Figure 16: Self-consistent dynamic beam sizes $\sigma_{1,2}/\sigma_{0,1,2} \equiv \sqrt{\epsilon_{1,2}/b_{1,2}}$, as a function of a common negative multiplication factor $M_\xi$ for both tune shift parameters. This multiplication factor is intended to model a change in linear focusing arising from a beam-beam offset. The tunes are set to 0.520 (HER, $e^-$) and 0.505 (LER, $e^+$), respectively.