# Measuring the Phases of $G_E$ and $G_M$ of the Nucleon

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The nucleon electromagnetic form factors $G_E$ and $G_M$ are complex quantities in the time-like region. The absolute values can be determined by measuring the angular distribution of the nucleons in $e^+e^- \rightarrow N\bar{N}$. The complex phase can only be determined by measuring one or more polarizations of the initial or final state. For PEP-N, we can use unpolarized $e^+$ and $e^-$ and measure the polarization of one of the outgoing nucleons.

## 1. INTRODUCTION

The electromagnetic form factors of the nucleons, $G_E(Q^2)$ and $G_M(Q^2)$, depend on $Q^2$.

In the space-like region they are relatively real. They have been measured using $e^- + N \rightarrow e^- + N$ elastic scattering where $Q^2 = |q^2|$ is the absolute value of the four-momentum transfer from the incoming lepton to the nucleon. The techniques used are:

1. Rosenbluth separation: Measurements made at least two angles and fixed $Q^2$.

$$d\sigma/d\Omega \propto \frac{G_E(Q^2)^2 + \tau G_M(Q^2)^2}{1+\tau} + 2\tau G_M(Q^2)^2 \sin^2(\theta/2)$$

2. Polarized beam and polarized target

   (a) Target polarization in scattering plane $\perp q$ vector:

   $$A_\perp = -P_e P_t \frac{\sqrt{2\tau(1-\varepsilon)} G_E G_M}{\varepsilon G_E^2 + \tau G_M^2}$$

   (b) Target polarization $\parallel$ to $q$ vector:

   $$A_\parallel = -P_e P_t \frac{\sqrt{1-\varepsilon^2} \tau G_M^2}{\varepsilon G_E^2 + \tau G_M^2}$$

3. Polarized beam and polarization of recoil nucleon:

   (a) Recoil polarization in scattering plane $\perp q$ vector:

   $$P_x = -P_e \frac{\sqrt{2\tau(1-\varepsilon)} G_E G_M}{\varepsilon G_E^2 + \tau G_M^2}$$

   (b) Recoil polarization $\parallel$ to $q$ vector:

   $$P_z = P_e \frac{\sqrt{1-\varepsilon^2} \tau G_M^2}{\varepsilon G_E^2 + \tau G_M^2}$$

The first method gives only the absolute value of the form factors. Combining measurements 2a and 2b gives the relative sign through: $G_E/G_M = \sqrt{\tau(1+\varepsilon)/2\varepsilon} \cdot A_\perp/A_\parallel$. Combining measurements 3a and 3b gives the relative sign through: $G_E/G_M = -\sqrt{\tau(1+\varepsilon)/2\varepsilon} \cdot P_x/P_z$. All three methods have been used in the space like region for both the proton and the neutron. [1]

In the time-like region $q^2$ is positive and the form factors are complex, so it is necessary to measure $|G_E|$, $|G_M|$ and the Phase Difference. These can be determined using the process $e^+e^- \rightarrow N\bar{N}$. The momentum transfer $q^2 = s$ where $s$ is the square of the center of mass energy. The $N\bar{N}$ are in an L=0 ($G_s$) of L=1 ($G_d$) state with $G_M = G_s - G_d$ and $G_E = \sqrt{s}/(2M) \cdot G_s + 2G_d$. At threshold $G_d = 0$ and thus $G_M(4M^2) = G_E(4M^2)$ and $\text{Im}[G_E G_M^*] = 0$. The VMD model of Dubnickova, Dubnicka, and Strizenec [2] gives predictions of the magnitude and phases of the form factor and predict significant non zero phase difference in the PEP-N energy region.

The following experiments are possible.

1. Unpolarized beam particles.

   (a) Rosenbluth separation: Measurements made at at least two angles and fixed $Q^2$.

   $$d\sigma/d\Omega = \frac{\alpha^2 \sqrt{1 - 4M^2/x}}{4s}$$

   $$[|G_E(s)|^2 \sin^2(\theta)/\tau + |G_M(s)|^2(1 + \cos^2\theta)]$$

   (b) Recoil polarization $\perp$ to scattering plane:

   $$P_y = -\frac{\sin(2\theta)\,\text{Im}[G_E G_M^*]/\sqrt{\tau}}{|G_E|^2 \sin^2(\theta)/\tau + |G_M|^2(1 + \cos^2\theta)}$$

2. One longitudinally polarized beam particle

   (a) Recoil polarization $\parallel$ to baryon:

   $$P_z = -P_e \frac{2\cos(\theta)|G_M|^2}{|G_E|^2 \sin^2(\theta)/\tau + |G_M|^2(1 + \cos^2\theta)}$$

(b) Recoil polarization in scattering plane $\perp$ to baryon:

$$P_x = -P_e \frac{2\sin(\theta)\,\mathrm{Re}[G_E G_M^*]/\sqrt{\tau}}{|G_E|^2 \sin^2(\theta)/\tau + |G_M|^2(1+\cos^2\theta)}$$

Method 1b allows us to measure the phase difference once the absolute values have been determined using the Rosenbluth separation (1a). The measurement can be carried out at all scattering angles simultaneously.

## 2.  MEASURING POLARIZATION

The polarization of the recoil nucleon can be measured in a polarimeter similar to the one used recently at JLAB [4]. The method is to precisely measure the incoming trajectory, re-scatter the recoil nucleon on a carbon target, and then measure the outgoing angle to get the angular dependence of the re-scatter. The number of counts $N(\Phi')$ is given by:

$$N(\theta', \phi') = N_o(1 + P \times A_{\mathrm{eff}} \sin\Phi') \qquad (1)$$

where $P$ is the polarization to be measured, $A_{\mathrm{eff}}$ is the effective analyzing power, and $\Phi'$ is the second scattering angle. This may require a separate detector unless we can embed a precise position measuring system within the multi-layered shower counter. The $\Lambda$ form factors could be measured by using the self analyzing power of the decay. Corrections must be made for the precession of the baryon spin by the field of the vertex magnet.

### 2.1.  Rates and Errors

We estimate that there will be about 200 $N\bar{N}$/day. I assume that we have an analyzing power similar to that at JLAB which is 0.5 at 230 MeV and 0.1 at a few GeV. The probability of scattering $P_s$ in the polarimeter is 0.01 to 0.1 giving the figure of merit $A_{\mathrm{eff}}\sqrt{P_s} \sim 0.07$. The error on the nucleon polarization is given by $\delta P_s \sim 1./(A\sqrt{P_s N})$ where $N$ is the number of nucleons going into the polarimeter. With these parameters, a 100 day run yields $\delta P \sim 0.1$.

## 3.  CONCLUSION

$G_E(s)$ and $G_M(s)$ and their relative phase can be measured at PEP-N using unpolarized beams and measuring the recoil polarization of the Nucleon.

## REFERENCES

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