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The Photon–Pion Transition Form Factor for Virtual Photons

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We discuss the photon to meson transition form factor for virtual photons, which can be measured in e^+e^- collisions. We demonstrate that this form factor is independent of the shape of the meson distribution amplitude over a wide kinematical range. This leads to a parameter-free prediction of perturbative QCD to leading twist accuracy, which has a status comparable to the famous leading-twist prediction of the cross section ratio R.

1. INTRODUCTION

Exclusive reactions in QCD involving a large momentum scale are amenable to a perturbative treatment. A particular perturbative approach is the so-called hard scattering formalism [1], where the transition amplitude of a certain process is written in a factorized form as a convolution of a hard scattering amplitude specifying a partonic subprocess at a large scale and a universal, that is, process independent, hadronic distribution amplitude. While the hard scattering amplitude is perturbatively calculable, distribution amplitudes describe the soft transition from partons to hadrons and thus cannot be derived from QCD as yet. Therefore, in order to make reliable predictions for exclusive reactions, it is crucial to obtain informations about the shape of distribution amplitudes from other sources.

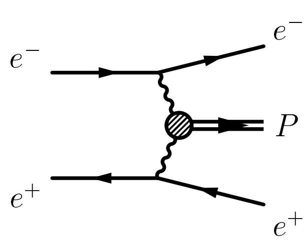

Figure 1: Sketch of the transition form factor as measured in $e^+e^- \to e^+e^-P$.

The simplest exclusive observable is the form factor for transitions from a real or virtual photon to a pseudoscalar meson P, measurable in electron-positron scattering, $e^+e^- \to e^+e^-P$, see sketch in Figure 1. The analysis of the CLEO data [2] for real photons have been used to constrain the distribution amplitudes for the cases of the pion and the eta, eta', see, for instance [3]–[8]. It has been infered that these distribution amplitudes are close to their asymptotic form.

Generically, the distribution amplitude Φ_P of a pseudoscalar meson can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$, the eigenfunctions of the leading-order evolution kernel:

$$\Phi_P(\xi, \mu_F) = \Phi_{AS}(\xi) \left[1 + \sum_{n=2,4,\dots}^{\infty} B_n^P(\mu_F)\, C_n^{3/2}(\xi) \right], \tag{1}$$

where Φ_{AS} denotes the asymptotic meson distribution amplitude,

$$\Phi_{AS}(\xi) = \frac{3}{2}(1-\xi^2). \tag{2}$$

ξ is related to the usual longitudinal momentum fraction x of the quark with respect to the meson by $\xi = 2x - 1$. The Gegenbauer coefficients B_n^P depend on a factorization scale μ_F in the following way:

$$B_n^P(\mu_F) = B_n^P(\mu_0) \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n}, \tag{3}$$

with μ_0 being a typical hadronic scale for which we choose a value of 1 GeV. Since the anomalous dimensions γ_n are positive fractional numbers any distribution amplitude evolves into Φ_{AS} at large scales. The Gegenbauer coefficients contain non-perturbative information and are therefore principally unknown.

The topic of this talk is an investigation of the photon-to-meson transition form factor for virtual photons. In particular, we address the question whether we can obtain additional informations for the Gegenbauer coefficients B_n^π of the distribution amplitude of the produced meson from the measurement of the form factor at current and planned e^+e^- colliders. We will limit ourselves to the case of a pion and only briefly comment on eta, eta' towards the end of the talk. A more detailed account of the analysis will be presented in [9].

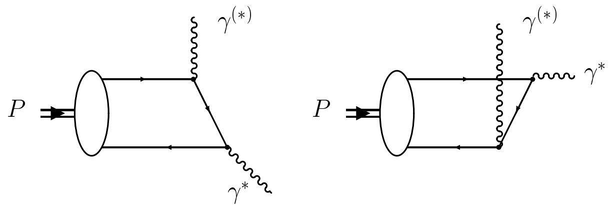

Figure 2: Born graphs contributing to the $P\gamma^{(*)}$ transition form factor.

2. THE $\gamma^* - \pi$ TRANSITION FORM FACTOR

The γ^*-π transition form factor $F_{\pi\gamma^*}$ is formally defined through the $\gamma^*\gamma^*\pi$ vertex:

$$\Gamma_{\mu\nu} = -ie^2\, F_{\pi\gamma^*}(Q^2, Q'^2)\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha q'^\beta\,, \tag{4}$$

where q and q' denote the photon momenta with respective spacelike virtualities $Q^2 = -q^2$, $Q'^2 = -q'^2$. For the following discussion, it is convenient to express $F_{\pi\gamma^*}$ in terms of the average photon virtuality $\overline{Q}^2$ and a dimensionless parameter ω:

$$\overline{Q}^2 = \frac{1}{2}(Q^2 + Q'^2), \qquad \omega = \frac{Q^2 - Q'^2}{Q^2 + Q'^2}, \tag{5}$$

with $-1 \le \omega \le 1$. The two photons cannot be distinguished such that the transition form factor is symmetrical under $\omega \leftrightarrow -\omega$.

Since we are interested in the leading twist behaviour of $F_{\pi\gamma^*}$ we employ the collinear approximation, that is, we neglect partonic transverse momenta. Power corrections arising from transverse momenta will be estimated later on. Thus, the leading twist expression to next-to-leading order (NLO) α_s reads [10]

$$\begin{aligned} F_{\pi\gamma^*}(\overline{Q}, \omega) &= \frac{1}{3\sqrt{2}}\frac{f_\pi}{\overline{Q}^2}\int_{-1}^{1} d\xi\, \frac{\Phi_\pi(\xi, \mu_F)}{1 - \xi^2\omega^2} \\ &\times \left[1 + \frac{\alpha_s(\mu_R)}{\pi}\, \mathcal{K}(\omega, \xi, \overline{Q}/\mu_F)\right]. \end{aligned} \tag{6}$$

The function $\mathcal{K}(\omega, \xi, \overline{Q}/\mu_F)$ parametrizes the $\mathcal{O}(\alpha_s)$ corrections, which have been calculated in [10, 11] within the $\overline{\mathrm{MS}}$ scheme. The factorization scale μ_F and the renormalization scale μ_R are both of the order $\overline{Q}$. In the above expression, we have taken into account the lowest, that is, valence Fock state only. $f_\pi \approx 131\,\mathrm{MeV}$ is the well-known pion decay constant. The Born graphs contributing to the transition form factor are shown in Figure 2

Using the expansion (1) and taking μ_R to be independent of ξ the transition form factor (6) can be rewritten in the following form:

$$\begin{aligned} F_{\pi\gamma^*}(\overline{Q}, \omega) &= \frac{f_\pi}{\sqrt{2}\,\overline{Q}^2}\Bigg[c_0(\omega, \mu_R) \\ &+ \sum_{n=2,4,\dots} c_n(\omega, \mu_R, \overline{Q}/\mu_F)\, B_n^\pi(\mu_F)\Bigg], \end{aligned} \tag{7}$$

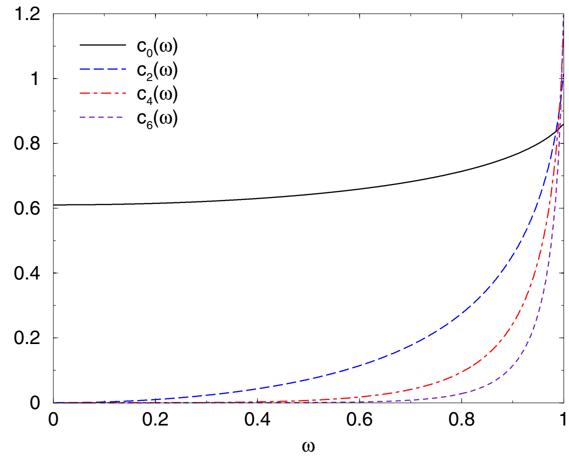

Figure 3: The coefficients $c_n(\omega)$ in the expansion (7) of the $\pi\gamma^*$ form factor. NLO corrections are included with $\mu_F = \overline{Q}$. Through α_s the coefficients depend mildly on μ_R, which is chosen as $\mu_R = 2\,\mathrm{GeV}$.

with analytically computable functions $c_n(\omega, \mu_R, \overline{Q}/\mu_F)$.

The first four coefficients c_n are shown in Figure 3. The NLO corrections are evaluated using the two-loop expression of α_s for $n_f = 4$ flavours and $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 305\,\mathrm{MeV}$. We choose a factorization scale $\mu_F = \overline{Q}$, which is the virtuality of the quark propagators in Figure 2 at $\xi = 0$. We see a very rapid decrease of the coefficients as soon as one goes away from the real-photon limit $\omega \to 1$, where all coefficients behave as $c_n(\omega = 1) = 1 + \mathcal{O}(\alpha_s)$. This means that the transition form factor is sensitive to the Gegenbauer coefficients only for $\omega \to 1$. In this limit, however, the transition form factor measures the $(1+\xi)^{-1}$-moment of the pion distribution amplitude, which, apart from $\mathcal{O}(\alpha_s)$ corrections, is given by the sum over all Gegenbauer coefficients,

$$\langle(1+\xi)^{-1}\rangle = \frac{3}{2}\Big[1 + \sum_n B_n^\pi(\mu_F)\Big]. \tag{8}$$

The phenomenological analysis of the CLEO data [2] led to the constraint $\langle(1+\xi)^{-1}\rangle = 1.37$ at $Q^2 = 8\,\mathrm{GeV}^2$ [5]. Assuming that $B_n^\pi = 0$ for $n \ge 4$ this constraint translates into $B_2^\pi(\mu_0) = -0.15$, which implies the distribution amplitude being close to its asymptotic form, as already mentioned in the introduction.

Before we proceed to a discussion of the region away from the limit $\omega \to 1$, we have to comment on possible power corrections in the large ω region, where the transition form factor becomes sensitive to the end-point regions $\xi \to \pm 1$. This corresponds to the situation of the quark or antiquark in the pion having small momentum fraction and the internal quark between the photon vertices going on-shell. Large power corrections arising from, e.g., transverse momentum or meson mass effects, soft overlap contributions or the non-perturbative behaviour of α_s in the infrared region, may spoil the accuracy of the data analysis.

The most important of these corrections in the region where the CLEO data are available are the partonic transverse momentum effects. In order to estimate these effects we will

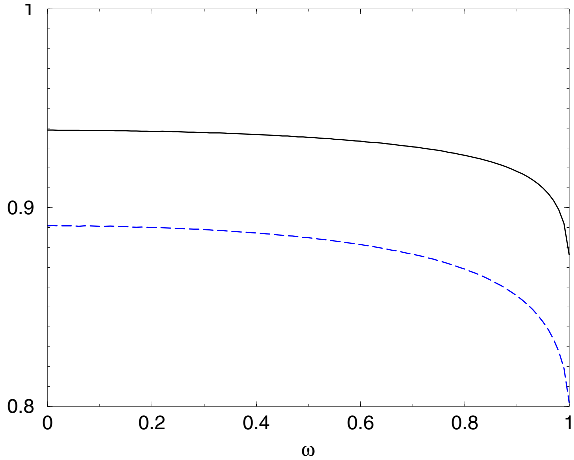

Figure 4: Ratio of $F_{\pi\gamma^*}(\overline{Q}, \omega)$ in the modified perturbative approach and in the LO leading-twist approximation at $\overline{Q}^2 = 4\,\text{GeV}^2$ (solid line) and at $\overline{Q}^2 = 2\,\text{GeV}^2$ (dashed line). Here we have used the wave function (10) and the asymptotic pion distribution amplitude Φ_{AS}.

employ the modified hard scattering approach [12], where the the expression (6) is replaced by

$$F_{\pi\gamma^*}(\overline{Q}, \omega) = \frac{1}{4\sqrt{3}\pi^2} \int d\xi\, d^2\mathbf{b}\, \hat{\Psi}^*_\pi(\xi, -\mathbf{b}, \mu_F) \times K_0(\sqrt{1+\xi\omega}\,\overline{Q}\,b)\, \exp\left[-S\left(\xi, b, \overline{Q}, \mu_R\right)\right]. \quad (9)$$

The modified Bessel function K_0 appears as the Fourier transform of the hard scattering kernel in leading order α_s. The transverse quark-antiquark separation $\mathbf{b}$ is Fourier conjugated to the partonic transverse momentum $\mathbf{k}_\perp$ and $\hat{\Psi}^*_\pi$ is the Fourier transform of the outgoing pion's wave function. The exponential is the Sudakov form factor which describes gluonic radiative corrections at scales intermediate between the confinement region and the hard region; for details see [12]. The most important feature of the Sudakov form factor is its damping of large quark-antiquark separations. Asymptotically, only configurations with vanishing transverse separations survive. Since b acts as an infrared cut-off, the factorization scale μ_F is to be taken as $1/b$. The renormalization scale is chosen according to the max-prescription [12] as $\mu_R = \max\{1/b, \sqrt{1+\xi}\,\overline{Q}, \sqrt{1-\xi}\,\overline{Q}\}$. Following [5, 13] we assume for the light-cone wave function in b-space the Gaussian ansatz

$$\tilde{\Psi}^\pi(\xi, \mathbf{b}) = \frac{2\pi f_\pi}{\sqrt{6}}\, \Phi_\pi(\xi) \exp\left[-\frac{(1-\xi^2)\, b^2}{16\, a_\pi^2}\right] \quad (10)$$

with $a_\pi^{-2} = 8\pi^2 f_\pi^2 (1 + B_2^\pi + B_4^\pi + \ldots)$ being the transverse size parameter. The prediction of the $\gamma \to \pi$ form factor in the modified perturbative approach using this wave function with $\Phi_\pi = \Phi_{\text{AS}}$ is in very good agreement with the CLEO data [5].

In order to demonstrate in which kinematical region the transverse momentum corrections are less important, in Figure 4 we show the ratio between the form factor evaluated in the modified hard scattering approach, Eq. (9), and the leading-twist approximation in LO α_s, i.e., neglecting the contributions from $\mathcal{K}(\omega, \xi, \overline{Q}/\mu_F)$ in Eq. (6). In both schemes we use the asymptotic pion distribution amplitude Φ_{AS}. It is interesting to note that while the dominant effects stem from $k_\perp$-corrections to the hard scattering amplitude, the Sudakov corrections amount to only less than about 1.5% in the kinematics considered here. We see that the transverse momentum corrections are negligible for $\overline{Q} \gtrsim 2\,\text{GeV}$ and $\omega \lesssim 0.9$, where they already provide less than 10% corrections. However, as can be seen in Figure 3, in this region the sensitivity to the Gegenbauer coefficients decreases very fast. While it appears difficult to pin down the individual values for the coefficients B_n^π, one at least should be able to discriminate between the wide range of theoretical results for the lowest B_n^π, ranging from a QCD sum rule analysis [14], which predicted $B_2^\pi(1\text{ GeV}) = 0.44$ and $B_4^\pi(1\text{ GeV}) = 0.25$, and a preliminary result from lattice QCD [15] providing $B_2^\pi = -0.41 \pm 0.06$ at a low scale, to name only a few.

We now turn to a discussion of the kinematical region away from the real-photon limit $\omega \to 1$. In particular, we investigate the limit $\omega \to 0$, where the two photons approximately have the same virtualities, $Q^2 \sim Q'^2$. The fast decrease of the functions c_n appearing in Eq. (7) can be understood by expanding the hard scattering kernel in Eq. (6) in powers of ω. Using the properties of the Gegenbauer polynomials, one finds

$$\begin{aligned} F_{\pi\gamma^*}(\overline{Q}, \omega) = \\ &\frac{\sqrt{2} f_\pi}{3\,\overline{Q}^2}\left[1 - \frac{\alpha_s(\overline{Q})}{\pi} + \frac{1}{5}\,\omega^2\left(1 - \frac{5}{3}\frac{\alpha_s(\overline{Q})}{\pi}\right)\right. \\ &\left. + \frac{12}{35}\,\omega^2 B_2^\pi(\mu_F)\left(1 + \frac{5}{12}\frac{\alpha_s(\overline{Q})}{\pi}\left[1 - \frac{10}{3}\ln\frac{\overline{Q}^2}{\mu_F^2}\right]\right)\right] \\ &+ \mathcal{O}(\omega^4, \alpha_s^2)\,, \end{aligned} \quad (11)$$

where we have chosen $\mu_R = \overline{Q}$. While the above result clearly demonstrates the insensitivity of the transition form factor to the Gegenbauer coefficients B_n^π as soon as ω departs from the limit $\omega \to 1$, it provides us with a parameter-free prediction from QCD to leading-twist accuracy in the small-ω region:

$$F_{\pi\gamma^*}(\overline{Q}, \omega) = \frac{\sqrt{2} f_\pi}{3\,\overline{Q}^2}\left[1 - \frac{\alpha_s(\overline{Q})}{\pi}\right] + \mathcal{O}(\omega^2, \alpha_s^2)\,. \quad (12)$$

To leading-order α_s, this result has been derived a long time ago by the authors of [16]. The α_s-corrections can be found in Ref. [10] and have been rederived in [8] for the real-photon case on the basis of the conformal operator product expansion. In Fig. 5 we compare the approximations (11) and (12) with the full result (6). As we can see, the leading expression (12) provides a very good approximation not only for $\omega \to 0$, but in fact over a wide range of ω, up to about $\omega \simeq 0.5$, where α_s-corrections start to become important. Any clear deviation from the leading-twist prediction would signal large power corrections and therefore, this prediction well deserves experimental verification. It has a status comparable to the famous leading-twist expression of the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

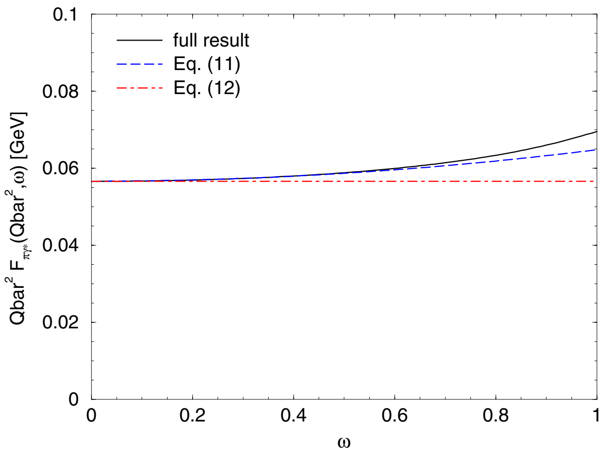

Figure 5: NLO leading-twist prediction for the scaled form factor $\overline{Q}^2 F_{\pi\gamma^*}(\overline{Q},\omega)$ at $\overline{Q}^2 = 4\,\mathrm{GeV}^2$ with $B_2^\pi(\mu_0) = -0.15$ and $B_n^\pi = 0$ for $n \geq 4$. For comparison we also show the approximations (11) and (12).

A completely analogous discussion with essentially the same conclusions can be pursued for $\gamma^* \to \eta,\ \eta'$ transitions. The analysis is, however, complicated through the mixing of η and η' and contributions from the gluon distribution amplitude at $\mathcal{O}(\alpha_s)$. The gluon contributions are negligible and again, we find that the transition form factors for the η and η' are hardly sensitive to the Gegenbauer coefficients over a wide range of kinematics.

3. CONCLUSIONS

We have investigated the possibility to exploit the $\gamma^* \to \pi$ transition form factor in order to determine the Gegenbauer coefficients B_n^π of the pion distribution amplitude. Performing an expansion in terms of the dimensionless kinematical parameter ω which is defined as the ratio of the difference and the sum of the two photon virtualities, we have been able to demonstrate that the form factor is independent of the shape of the pion distribution amplitude over a wide range of ω. As a consequence, one has a parameter-free prediction from QCD to leading-twist accuracy, which is valid in a large kinematical region, and which deserves experimental verification. Any observable deviation from this prediction is to be seen as a signal for power corrections.

While the data for the real-photon case, $\gamma \to \pi$, where $|\omega| = 1$, fixes the sum of the Gegenbauer coefficients, data for values of $|\omega|$ around 0.9, say, will allow for a discrimination of the wide range of theoretical predictions for the lowest B_n^π. Similar conclusions hold for $\gamma^* \to \eta,\ \eta'$ transitions.

Concerning the accessibility of the transition form factor at the running B-factories BarBar, Belle and CLEO, our studies have revealed that it seems possible, although challenging, to measure the form factor for $\overline{Q}^2 \lesssim 3\,\mathrm{GeV}^2$ both in regions of moderate ω and where $|\omega| \lesssim 1$. The planned asymmetrical e^+e^- collider at SLAC appears to be suitable for studies of the form factor after an upgrade to larger center of mass energies and luminosities.

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