Dispersive Techniques for α_s, $R_{\rm had}$ and Instability of the Perturbative Vacuum

Y. Srivastava
Departement di Fisica dell'Università and INFN, Perugia and Physics Department, Northeastern University, Boston
S. Pacetti
Dipartimento di Fisica dell'Università and INFN, Perugia
G. Pancheri
INFN Laboratori Nazionali di Frascati, Italy
A. Widom
Northeastern University, Boston

Recent dispersive techniques developed by us are applied to discuss three problems: 1. A long standing discrepancy between the measurements of $R(s)$ for $\sqrt{s} = (5 \div 7.5)$ GeV by Crystal Ball and MARK I has been analyzed and its consequences analyzed for the number of contributing quarks. 2. Noting that the perturbative α_s has the wrong analyticity, analytic models consistent with asymptotic freedom (AF) and confinement have been constructed and applied to discuss τ decay. 3. It is shown that AF leads to a wrong sign for Im $(\alpha(s))$ which signals an instability of the perturbative QCD vacuum.

1. THE DISPERSIVE METHOD

Recently, we have developed accurate numerical schemes to handle the "inverse" problem, that is how to retrieve the imaginary part of a physical quantity (for example, a cross section, or a form factor in the time-like region) using data for its real or the reactive part (usually in the space like region) [1, 2].

It has been reviewed at this conference by S. Pacetti (see his contribution T20), and its applications to the nucleon form factor have been discussed. Here these techniques have been applied to three following problems.

2. DETERMINATION OF $R(s)$ FOR $\sqrt{s} = (5 \div 7.5)$ GEV

The purpose of this analysis is to once again focus on a long standing discrepancy between data from Crystal Ball [3] and MARK I [4] in the energy region $\sqrt{s} = (5 \div 7.5)$ GeV. To arrive at a model independent answer to this problem we have applied the following procedure (for details we refer the reader to [5]). We write a dispersion relation for the derivative of the polarization tensor $\Pi(t)$ whose imaginary part is proportional to $R(s)$. As input, we use all available data for $R(s)$ from outside the disputed region and for its asymptotic part contributions from the five light quark flavors. We then solve the integral equation for $R(s)$ in the disputed region. As shown in Figure 1, very good agreement is found with the CB data. This implies that some additional contribution to the asymptotic behavior is mandatory to reproduce the MARK I data. As shown in Figure 2, we find that a low mass, spin zero quark of charge $(-1/3)$ is quite adequate for this purpose.

It is worthwhile to point out that it is not inconceivable for the CB data to be consistent with the MARK I data given their different selection criteria for what constituted $R(s)$. If one inquires into the selection criteria for $R(s)$ in the CB data, one is struck by the remarkable fact that in it only two jet $(q\bar{q})$ events were included. More precisely, all events with more than 20% imbalance in energy between forward-backward, left-right and up-down hemispheres were discarded. Were there any decays from beyond that of the simple $(q\bar{q})$ type, they were not counted. MARK I had imposed no such restriction.

Given the importance consequences for the standard model that it entails, we stress upon the necessity of an independent measurement of R in this region. One possibility is through the radiative technique at $B\bar{B}$ machines[5].

3. ANALYTIC MODELS FOR α_s WITH APPLICATION TO τ DECAY

Several authors have noted that the perturbative $\alpha_s(s)$ has the wrong analyticity[6, 7]. For example, the 1-loop AF formula

$$\alpha_{1-loop}(s) = \left(\frac{1}{b}\right)\frac{1}{\ln(-s/\Lambda^2)}, \qquad (1)$$

has a pole at space-like value $s = -\Lambda^2$. Higher loops suffer from the same disease. Of course, analyticity derived from unitarity forbids any singularity for space like s (< 0).

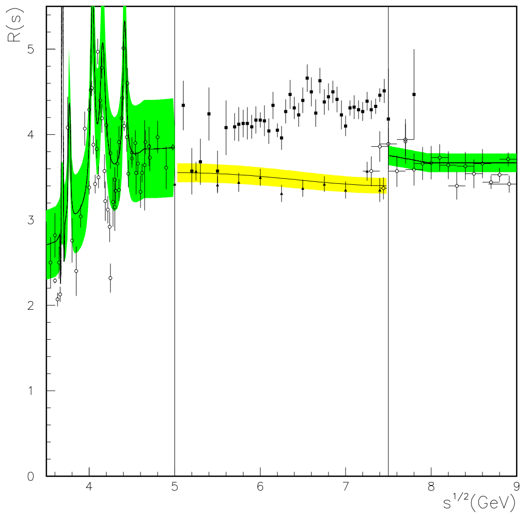

Figure 1: Values of $R(s)$ (light gray band) obtained from the integral equation with five spin 1/2 quarks in the interval [5 GeV, 7.5 GeV] of s.

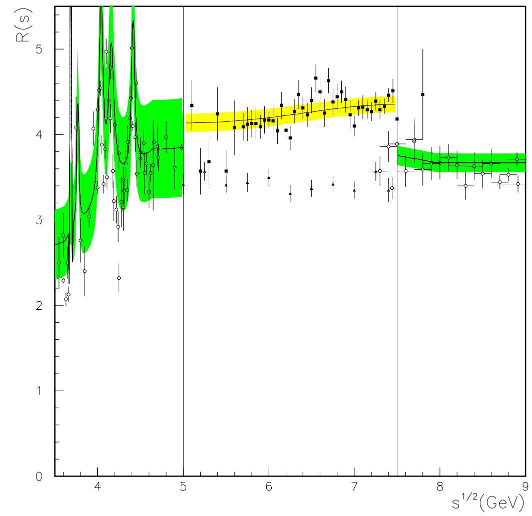

Figure 2: Values of $R(s)$ (light gray band) obtained from the integral equation with five spin 1/2 quarks plus one scalar quark.

Several cures for the above have been suggested. For example, in [6] the imaginary part for α_s computed from AF

$$\mathrm{Im}\big(\alpha_1(s)\big) = \left(\frac{1}{b}\right) \frac{\pi}{\ln(s/\Lambda^2)^2 + \pi^2}\vartheta(s), \tag{2}$$

is used in an unsubtracted dispersion relation (which converges thanks to the behavior $1/(\ln(s/\Lambda^2))^2$ in the asymptotic region) to compute the real part. Thus, one has

$$\alpha_1(s) = \frac{1}{b}\left[\frac{1}{\ln(-s/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 + s}\right]. \tag{3}$$

The second term cancels the unwanted pole. This procedure has been generalized up to 3 loops. An added curiosity: $\alpha_I(0) = (1/b)$, is finite and universal to all loops.

So why not stop here? The lacuna is that $\alpha_I(s)$ is too tame; has not enough "oomph" to produce all that QCD is advertised to possess. That is, to obtain, confinement of quarks and glue, infinite number of Regge trajectories, etc. etc., one would then have to add - in an ad hoc fashion - a confining potential to produce a reasonable hadronic spectrum. E.g., on the lattice, where the Wilson area law is imposed automatically, so there $\alpha(0) \to$ constant or even zero. Parenthetically, the embarrassment on the lattice is that the same prescription leads to a linear potential $V(r) \to r$ rather than $(1/r)$ for QED.

A different analytic model is due to Nesterenko[7], where the AF pole is eliminated multiplicatively:

$$\alpha_{II}(s) = \frac{1}{b}\left[\frac{(1+\frac{\Lambda^2}{s})}{\ln(-s/\Lambda^2)}\right]. \tag{4}$$

Here $\alpha_{II}(s)$ increases as s goes to zero.

The problem with this model is that it is "too singular," so that integrals (such as those needed for soft gluon summations [8])

$$\alpha_{av} = \frac{1}{s}\int_0^s ds'\alpha(s'), \tag{5}$$

are not convergent.

Taking our cue from the above, we have developed the following general strategy to develop a whole class of models for an analytic α - with 1 and 2 as special cases.

A simpler function to disperse is $(1/\alpha(s))$, which provides much physical insight. Let us recall that the vacuum Coulomb potential is modified in the following way for charged particles in a medium

$$\frac{\alpha_o}{r} \to \frac{\alpha_o}{\varepsilon r}, \tag{6}$$

where ε is the dielectric constant of the material. Hence, we shall consider

$$\varepsilon(s) = \frac{1}{\alpha(s)}. \tag{7}$$

For example, in the AF case, its imaginary part is much simpler, a constant

$$\mathrm{Im}\,(\varepsilon(s)) = -\pi b \vartheta(s). \tag{8}$$

Write an unsubtracted dispersion relation for ε

$$\varepsilon(s) = \varepsilon(-\Lambda^2) + \frac{(s+\Lambda^2)}{\pi}\int_0^\infty \frac{ds'\,\mathrm{Im}\big(\varepsilon(s')\big)}{(s'-s-i\delta)(s'+\Lambda^2)}. \tag{9}$$

In Eq. (9), confinement is easily imposed. Here, confinement means

$$\varepsilon(s=0) = 0. \tag{10}$$

Using Eq. (8) as the asymptotic limit from AF for $\mathrm{Im}(\varepsilon(s))$, we have a general class of models consistent with AF and confinement provided by (see Figure 3)

$$\mathrm{Im}\big(\varepsilon(s;\,p)\big) = -\pi b \frac{1}{1+(\Lambda^2/s)^p}; \qquad (0 < p \leq 1). \quad (11)$$

The model in [6] is obtained for $p = 1$.

We have considered two applications: 1. τ decay, Figure 4; 2. Hadronic transverse momentum distributions in W and Z decays. For $R_\tau^{\rm hadronic}$, AF analysis requires a large $\Lambda \approx 850\ MeV$. For values of $p \approx (0.5 \div 0.8)$, on the other hand, we obtain a much more reasonable value for $\Lambda \approx 300\ MeV$ to get agreement with the data.

Regarding the transverse momentum distribution broadening due to soft gluon summations, we reproduce previous phenomenological results [9] for values of p in the same range. Both these applications lead us to conclude that indeed the dispersive method is indeed capable of joining AF with confinement in a suitable way.

4. INSTABILITY OF THE PERTURBATIVE QCD VACUUM

In the previous section, we imposed confinement along with AF. Here we discuss further results of some importance regarding the nature of confinement itself through a vacuum instability induced by AF. The 1-loop result for $\mathrm{Im}\,(\varepsilon(s))$ may be decomposed into its quark and glue pieces as

$$\mathrm{Im}\,(\varepsilon(s)) = -\pi b_{\rm TOT}\vartheta(s), \quad (12)$$

with

$$b_{\rm TOT} = b_q + b_g > 0 \quad (13)$$

(for $N_F < 16$), $b_q < 0$ and $b_g > 0$.

In QED on the other hand, only charged particles contribute and the sign in Eq. (12) is reversed. (The "price" for it is the Landau ghost absent in QCD).

In QED, this positive sign is necessary for stability, since ε is related to the conductivity σ

$$\varepsilon(s) = \varepsilon_o + \frac{i\sigma(s)}{\sqrt{s}}. \quad (14)$$

Thus, for example, for Ohm's law to work, there must be dissipation, that is, $\mathrm{Re}\,(\sigma(s)) > 0$. For such "normal" systems, it is usual to define a noise temperature T_n which is positive.

What if the sign is reversed? Then the system is not dissipative instead is an "amplifier" for which, the noise temperature $T_n < 0$. The notion of a noise temperature may be appreciated by considering a system with two energy levels $E_1 > E_0$. The probability ratio for finding the state with these energies is given by

$$\frac{P_1}{P_0} = e^{-\frac{(E_1-E_0)}{k_B T_n}}, \quad (15)$$

Thus, we see that the "normal" situation is for $T_n > 0$ whereas the "amplifier" case has $T_n < 0$. An artificially pumped system such as a MASER or a spin system has $T_n < 0$. But such a system is **unstable**. We have shown elsewhere that such systems exhibit a Klein paradox for photons [10].

Turning to QCD then, since the perturbative ground state has $\mathrm{Im}\,(\varepsilon(s)) < 0$, we conclude that such a system is unstable. This is a pleasing physical result since it implies that the perturbative ground state containing free quarks and glue is an excited (higher energy) state whereas the states of lower energy containing the hadrons must be the true ground state of QCD.

5. ACKNOWLEDGMENTS

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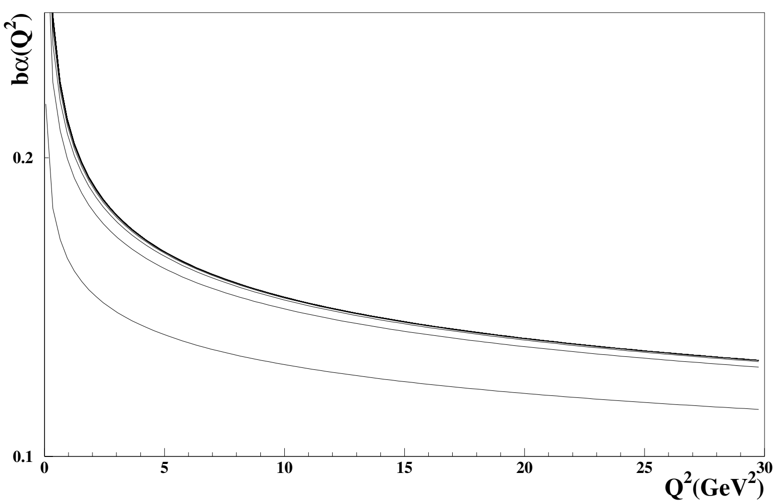

Figure 3: Values of $\alpha(Q^2)$ space-like obtained by integrating the dispersion relation (9) with the imaginary part (11) for $p = 1/5$ (lower curve), $2/5, \ldots, 4$ (higher curve) ($\Lambda = 100\ MeV$).

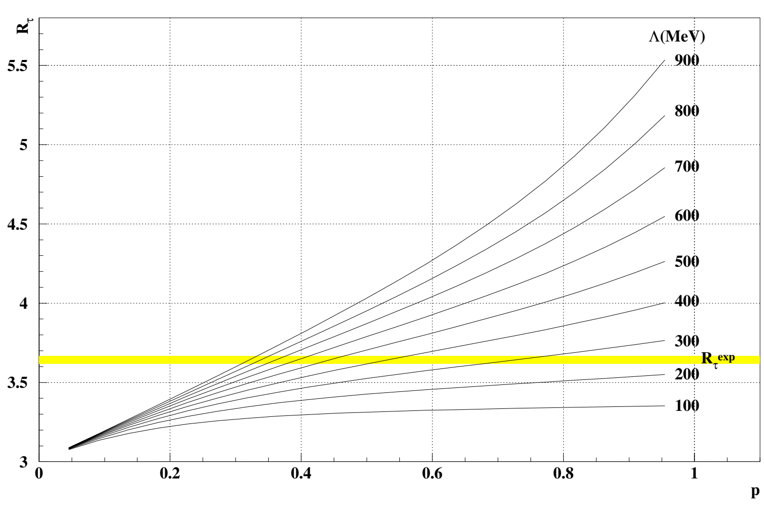

Figure 4: R_τ, obtained with different values of p and Λ^2, compared with its experimental value (gray band).