Status of CVC Tests from $e^+e^- \to$ Hadrons and τ-lepton Decays

S.I. Eidelman
Budker Institute of Nuclear Physics, Russia

New results on $e^+e^- \to$ hadrons coming from two detectors at the e^+e^- collider VEPP-2M are presented. Conserved vector current (CVC) predictions for the branching ratios and mass spectra derived from e^+e^- data are in general consistent with the data on τ decays although some problems with the normalization can exist in the two pion channel. Possible applications of CVC to the calculation of the hadronic contribution to the muon anomalous magnetic moment are discussed.

1. INTRODUCTION

e^+e^- annihilation into isovector hadronic states and τ-lepton decays are related to each other via the hypothesis of the conserved vector current (CVC [1].

For the Cabibbo allowed vector part of the weak hadronic current the distribution over the mass of produced hadrons is given by

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ud}|^2 S_{EW}}{32\pi^2\alpha^2 m_\tau^3} \times (m_\tau^2 - q^2)^2 v(m_\tau^2 + 2q^2)\, v_1(q^2)$$

where G_F is the Fermi constant, $|V_{ud}|$ is the corresponding element of the CKM matrix, S_{EW} is a factor taking into account electroweak radiative corrections approximately equal to 1.02 [2] and $v_1(q^2)$ is a spectral function:

$$v_1(q^2) = \frac{q^2\sigma^{I=1}_{e^+e^-}(q^2)}{4\pi\alpha^2}.$$

The allowed quantum numbers for the relevant hadronic final states are:

$$J^{PG} = 1^{-+}, \quad \tau \to 2n\pi\nu_\tau, \quad \omega\pi\nu_\tau, \quad \eta\pi\pi\nu_\tau, \ldots$$

After integration, the branching ratio of the decay into some hadronic state X is

$$B(\tau^- \to X^-\nu_\tau) = \frac{3S_{EW}|V_{ud}|^2 B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}{2\pi\alpha^2 m_\tau^8} \times \int_{4m_\pi^2}^{m_\tau^2} dq^2 q^2 (m_\tau^2 - q^2)^2\, (m_\tau^2 + 2q^2)\, \sigma^{I=1}_{e^+e^-}(q^2).$$

Using experimental data on $e^+e^- \to$ hadrons with $I = 1$, one can confront the CVC predictions and τ-lepton data both for decay spectra and branching ratios. Such theoretical predictions for various decay modes of the τ based on CVC have been given before by different authors, see [3] and references therein.

Significant progress has been achieved in experimental studies of τ-lepton decays by CLEO as well as by four LEP detectors during past years. On the other hand, for more than five years, two new detectors CMD-2 [4] and SND [5] have been studying low energy e^+e^- annihilation at the e^+e^- collider VEPP-2M at Novosibirsk. The data samples collected by them are typically comparable or higher than those for similar modes of the τ decay. Therefore, these results make possible a new test of CVC by comparing the τ and e^+e^- data at a different level of accuracy. It is also interesting to address the question whether our understanding of CVC is adequate to use the τ decays in addition to e^+e^- data improving thereby the accuracy of the calculations of a_μ^{had}—the hadronic contribution to the anomalous magnetic moment of the muon as suggested in [6].

As in our previous works [3, 7–9], to calculate the branching ratios we directly integrate the experimental cross sections avoiding as much as possible any additional theoretical input or some approximations of the data. In such an approach one hopes to get an unbiased result and deal with statistical and systematic uncertainties of separate experiments in a straightforward manner.

Results of the calculations will be presented in terms of B($\tau^- \to X^-\nu_\tau$). To calculate it, we'll take the leptonic branching ratio $B(\tau^- \to e^-\bar{\nu}_e\nu_\tau) = (17.83 \pm 0.06)\%$ [10].

2. EXPERIMENTS AT VEPP-2M

Since 1974 VEPP-2M, the e^+e^- collider in the Budker Institute of Nuclear Physics in Novosibirsk [11], has been running in the energy range from the threshold of hadron production to 1400 MeV. Its luminosity reached 3×10^{30} cm^{-2}s^{-1} at the ϕ meson energy and by the end of the June 2000 two detectors (CMD-2 and SND) running at VEPP-2M collected about 30 pb^{-1} of data each.

CMD-2 described in detail elsewhere [12] is a general purpose detector. Inside a superconducting solenoid with a field of $1T$ there are a drift chamber, a proportional Z-chamber and an endcap BGO calorimeter. Outside there is a barrel CsI calorimeter and muon streamer tube chambers. The main goal of CMD-2 is to perform a high precision measurement of the exclusive cross sections of various hadronic channels and determine parameters of the low lying vector mesons.

SND, described in detail elsewhere [13], is a nonmagnetic detector with drift chambers for tracking and a three layer electromagnetic NaI calorimeter. Outside it there are a muon streamer tube chamber and plastic scintillators. The main goal of SND is to study ρ, ω and ϕ decays as well as the main hadronic channels.

One should mention some special features of both experiments making high precision measurements feasible:

- large data samples due to the high integrated luminosity and large solid angle of detection
- multiple scans of the same energy ranges to avoid possible systematic effects; the step was 10 MeV for the continuum region and 1–2 MeV near the ω and ϕ peaks
- good space and energy resolution in combination with the low average multiplicity lead to small background
- redundancy—the unstable particles are independently detected via different decay modes ($\omega \to \pi^+\pi^-\pi^0, \pi^0\gamma$, $\eta \to 2\gamma, \pi^+\pi^-\pi^0, 3\pi^0, \pi^+\pi^-\gamma$)
- detection efficiencies and calorimeter response are studied by using "pure" experimental data samples rather than Monte Carlo events; more than 20 million ϕ meson decays can be used for that purpose.

New results are available on most of the hadronic channels. We'll briefly mention only those which are relevant to the CVC studies:

- CMD-2 collected more than 2 million events of the process $e^+e^- \to \pi^+\pi^-$ from 370 to 1380 MeV. The systematic uncertainty of less than 0.6% was achieved in the final analysis of the data set of about 100k events collected in the energy range 610 to 960 MeV in 1994–1995 [14]. Analysis is in progress for the rest of events and the expected systematic error ranges from 1% to 3% [15]. Figure 1 shows results of the pion form factor measurement coming from CMD-2.
- Both detectors observed production of four pions. CMD-2 showed that in the energy range above the ϕ, the $a_1(1260)^\pm\pi^\mp$ intermediate mechanism dominates in the $\pi^+\pi^-\pi^+\pi^-$ channel whereas both $a_1(1260)^\pm\pi^\mp$ and $\omega\pi$ contribute to the $\pi^+\pi^-\pi^0\pi^0$ final state [16]. The contribution of other possible states is small. The collected data sample includes about 60k events and the systematic uncertainty of the total cross sections is less than 15%. Below 1 GeV, CMD-2 reliably selected about 200 events of the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and showed that the cross section near the ρ peak is about 50 pb [17]. The measurement of the SND detector for which the data sample above the ϕ was about 80k events with the systematic uncertainty from 8 to 20% confirms the CMD-2 results on the production mechanisms [18]. However, in both 4π channels the SND cross sections are higher than those of CMD-2. The systematic uncertainties are still high and their further analysis is needed to clarify the picture. The corresponding cross sections are shown in Figure 2 and Figure 3 together with the results of the previous measurements at VEPP-2M, DCI and ADONE (for the references see [16]).
- Both detectors measured the cross section of the reaction $e^+e^- \to \omega\pi^0$ in the $\omega \to \pi^0\gamma$ channel collecting several thousand events each with the systematic error of 5% for SND [19] and 8.5% for CMD-2 [20]. Results of both groups are consistent within systematic errors.
- CMD-2 observed about 200 events of the process $e^+e^- \to \eta\pi^+\eta^-$ with the systematic accuracy of 15% [21].
- Analysis is in progress for the K^+K^- and $K^0_S K^0_L$ final states.

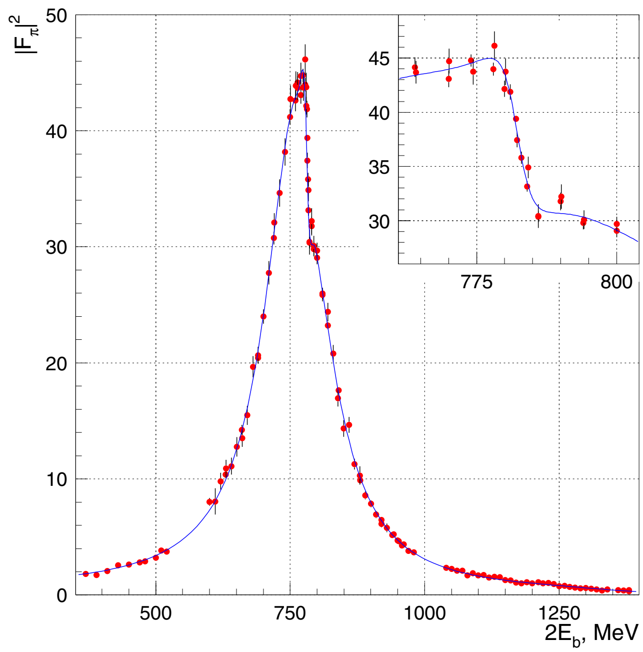

Figure 1: New data on the pion form factor

3. COMPARISON TO τ-LEPTON DECAYS

We'll now compare recent e^+e^- results to those from the τ decays. In the 2π channel, the spectral function of CLEO [22] is consistent with this from ALEPH [23] and in general well reproduces the picture observed in e^+e^- annihilation: the $\rho(770)$ meson peak followed by the $\rho(1450)$ and possibly $\rho(1700)$ (for obvious reasons there is no $\rho - \omega$ interference in the τ decay). The CLEO spectral function is by (3.2 ± 1.4)% higher than that in e^+e^- indicating some normalization problems. The deviation can probably decrease after CMD-2 completes its analysis since a new, more precise procedure

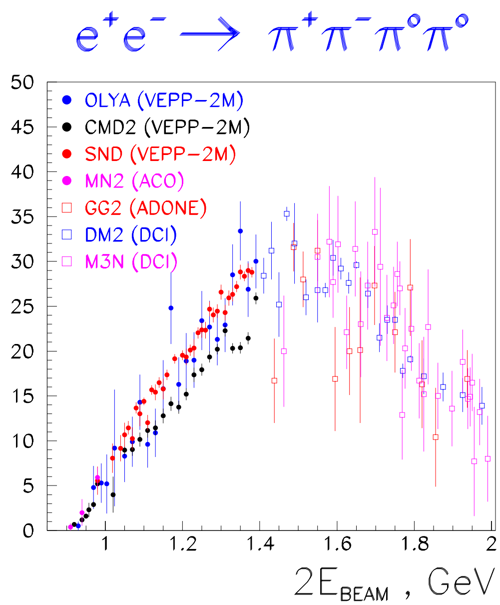

Figure 2: Cross section of the process $e^+e^- \to \pi^+\pi^-2\pi^0$

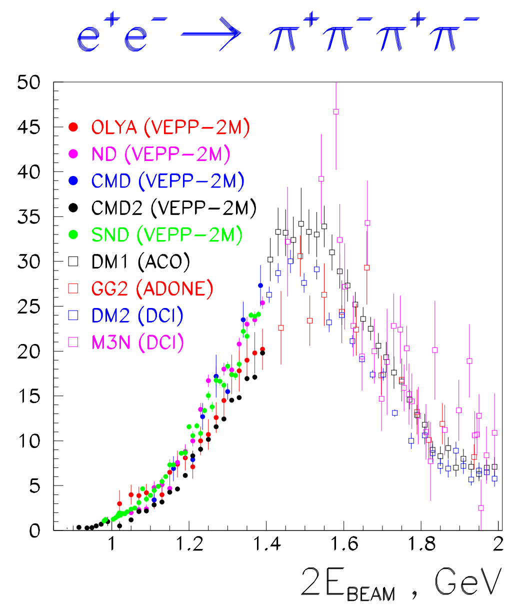

Figure 3: Cross section of the process $e^+e^- \to 2\pi^+2\pi^-$

of the radiative corrections tends to make the e^+e^- spectral function higher in the energy range below the ρ meson peak.

We can also compare results in the $\omega\pi$ channel. From Figure 4 which in addition to e^+e^- data shows recent results from CLEO [24] recalculated to the e^+e^- case, it is clear that in the VEPP-2M energy range (below 1400 MeV) results from the τ sector are compatible with e^+e^- whereas above 1400 MeV the τ spectral function is systematically higher than that from DM2 measurements [25].

The CMD-2 analysis of intermediate mechanisms in the 4π production is consistent with the conclusions of CLEO. The model used by CMD-2 to describe their results has been successfully applied [26] to describe various two pion and three pion distributions for both CLEO and ALEPH. However, there is an obvious problem with the normalization since the $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ spectral function is higher than that from CMD-2 in the whole energy range (Figure 5), the difference reaching 20%. The spectral functions calculated from OLYA [27] and SND [18] better match the CLEO result. As already noted, the ongoing analysis of the systematic uncertainties in the e^+e^- case can clarify the picture.

We summarize the comparison in Table I showing the expected branching ratios for various τ decays which were obtained by assuming that CVC is correct and averaging recent results from VEPP-2M with those from the previous measurements.

From Table I it can be seen that with the exception of the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay mode CVC predictions do not contradict the world average results [10]. Although CVC predictions are slightly lower than τ measurements, the difference is not statistically significant in most of the cases. The difference in the 2π channel is higher than in our last year analysis [28] because there we used the preliminary data set from CMD-2 and a slightly different procedure of taking into account the $\rho - \omega$ interference strongly exhibiting itself in the e^+e^- channel and absent in the τ decay case. The total branching ratio predicted for CVC modes is $(29.99 \pm 0.33 \pm 0.13)$% where the first error comes from the uncertainties of the e^+e^- data and the second is due to the relative error of 5.4×10^{-3} caused by the uncertainties of the quantities S_{EW}, $|V_{ud}|$ and B_e entering the expression for the CVC prediction and common for all considered decay modes ($B_e = B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$). This value should be compared to the corresponding sum of the world average τ branching fractions which equals (30.99 ± 0.31)% and the difference between the world average value and CVC prediction is (1.00 ± 0.47)%. Better understanding of the CVC validity can be expected after very high data sample experiments at B-factories in the τ sector as well as the future improvement of the accuracy after two groups at VEPP-2M complete their analysis. Moreover, experiments at the upgraded VEPP-2M machine VEPP-2000 which will be able to cover the whole energy range from threshold of hadron production up to 1800-2000 MeV, will make possible a consistent CVC analysis based on the results of one experiment thus avoiding the problems of matching results in various energy ranges obtained by several detectors.

In addition, serious theoretical input is needed to clarify how important the isospin breaking effects are. In contrast to the original analysis of such effects in [6], a recent result from [29] may indicate some more significant effects of $m_u \neq m_d$ in the 2π channel. For example, in [29] it is shown that the ratio of the e^+e^- spectral function to that from τ depends on s and is 0.74% lower than 1 near the ρ peak. However, after taking into account the correct s dependence of the ρ^0 and $\rho^\pm$ widths the integrated effect is rather small and the CVC prediction should be increased by 0.06% only. After inserting this correction and ascribing an additional model error of the same size, our final CVC prediction becomes

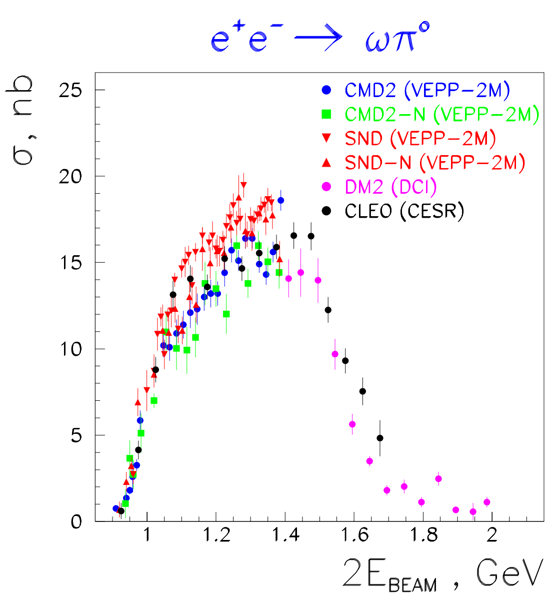

Figure 4: Cross section of the process $e^+e^- \to \omega\pi^0$

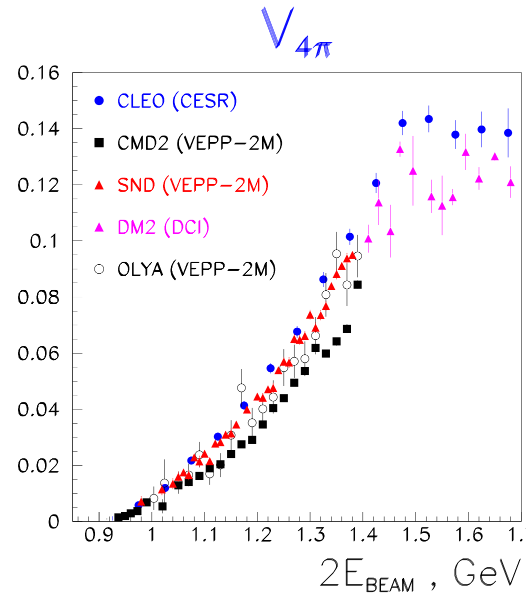

Figure 5: Spectral function of the decay $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$

$$B(\tau^- \to \pi^-\pi^0\nu_\tau) = (24.64 \pm 0.25 \pm 0.13 \pm 0.06)\%,$$

still below the world average value so that their difference is $(0.67 \pm 0.34)\%$. Another place worth theoretical efforts is the calculation of the electroweak radiative corrections. If S_{EW} appears to be slightly higher than its currently accepted value of 1.02, most of the problems with the normalization mentioned above will disappear. Note recent analysis of [30] in which both isospin breaking and radiative effects are taken into account and it is shown that corresponding corrections are rather large (about 1% near the ρ peak and 3% or even higher far from it).

4. APPLICATION OF CVC TO a_μ

It is interesting to study whether recent improvements both in τ and e^+e^- sectors can influence the accuracy with which we know a_μ^{had}—the hadronic contribution to the muon anomalous magnetic moment a_μ.

It is well known that the current uncertainty of the theoretical value of a_μ is dominated by the contribution of strong interactions. In its turn, the latter can be expressed by dispersion relations in terms of the integral of the total cross section of $e^+e^- \to$ hadrons with the kernel emphasizing the role of low energies. As a result, the largest contribution to $a_\mu^{\rm had}$ comes from the energy range from threshold of hadron production to 2 GeV. Careful analysis of all available e^+e^- data performed in [31] gave a conservative estimate of the uncertainty of $a_\mu^{\rm had}$ of 15×10^{-10}, mostly due to the insufficient accuracy with which the cross section of the process $e^+e^- \to \pi^+\pi^-$ is known. In [6] it was suggested to increase the accuracy of the $a_\mu^{\rm had}$ calculation by using data on the decay modes $\tau^- \to \pi^-\pi^0\nu_\tau$, $(4\pi)^-\nu_\tau$ in addition to those from e^+e^-. The idea is to apply CVC to convert the hadronic mass spectrum from the τ decays to the cross section of e^+e^- and benefit from the high accuracy of the τ data by averaging the calculated contribution from the τ data sample with that from e^+e^-. The first attempt of this kind showed an impressive improvement of the accuracy from 15 to 9 in units of 10^{-10}. The improvement can be even higher so that the uncertainty becomes as low as 6×10^{-10} if one relies not on the data only, but additionally uses some theoretical assumptions like the validity of perturbative QCD and QCD sum rules [32]. Here, however, we confine ourselves to the calculations based on the experimental data only.

Table II shows results of the estimate of $a_\mu^{\pi\pi}$ based on e^+e^- data only (the first line uses the data sample from [31] whereas the second one additionally uses recent data from CMD-2 [14, 15]), a corresponding value extracted through CVC from ALEPH [6] and CLEO [22] and finally the average of the most recent e^+e^- and τ-lepton contributions. Note that we ignore some of the above mentioned problems and implicitly assume that CVC is 100% correct or, in other words, using τ data does not introduce any additional model uncertainty. This assumption seems to be rather strong in view of the observed systematic excess of the τ spectral function over that from e^+e^- data. This excess is reflected in the larger values obtained by ALEPH or CLEO compared to the estimate based on purely e^+e^- data. As noted above, this effect, at least partly, can be accounted for by the isospin breaking and radiative effects. For example, the authors of [29, 30] claim that the τ based prediction for $a_\mu^{\pi\pi}$ should be decreased by approximately 0.6%.

Analysis of the Table shows that the current accuracy of the e^+e^- based estimate is already slightly better than that of the τ based one and the resulting significant improvement of the

Table I Branching ratio B($\tau^- \to X^- \nu_\tau$), %

Hadronic State X	CVC, 2001	World Average	WA - CVC
$\pi^-\pi^0$	24.58 ± 0.25	25.31 ± 0.18	0.73 ± 0.31
$\pi^- 3\pi^0$	1.07 ± 0.05	1.08 ± 0.10	0.01 ± 0.11
$2\pi^-\pi^+\pi^0$	3.84 ± 0.17	4.19 ± 0.23	0.35 ± 0.29
$\omega\pi^-$	1.82 ± 0.07	1.92 ± 0.07	0.10 ± 0.10
$\eta\pi^-\pi^0$	0.13 ± 0.02	0.17 ± 0.02	0.04 ± 0.03
Others	0.37 ± 0.11	0.24 ± 0.02	-0.13 ± 0.11
Total	29.99 ± 0.33	30.99 ± 0.31	1.00 ± 0.45

Table II Calculations of $a_\mu^{\pi\pi}$

Data	Source	$a_\mu^{\pi\pi}$, 10^{-10}
e^+e^-	EJ, 1995 ADH, 1997	495.9 ± 12.5
e^+e^-	This work, 2000	498.0 ± 5.5
τ	ALEPH, 1997	502.2 ± 6.9
τ	CLEO, 1999	513.1 ± 5.8
$e^+e^- + \tau$	ALEPH, 1997	500.8 ± 6.0
$e^+e^- + \tau$	CLEO, 1999	510.0 ± 5.3
$e^+e^- + \tau$	This work, 2000	504.4 ± 3.5

accuracy is due to the utilization of both data sets. This necessitates further thorough analysis of all possible differences between the e^+e^- and τ sectors and quantitative assessment of the corresponding corrections.

5. CONCLUSIONS

- Two detectors at Novosibirsk studied various hadronic channels at $\sqrt{s} < 1400$ MeV with large data samples and small systematic uncertainties
- CVC predicts the shape of spectral functions and production mechanisms qualitatively well
- There are normalization problems:
    1. τ spectral functions are regularly slightly higher than in e^+e^-
    2. The total predicted Br($\tau^- \to X^-_{I=1}\nu_\tau$) is (29.99 ± 0.35)% compared to the world average of (30.99 ± 0.31)%, the difference mostly coming from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay
    3. More theoretical input is necessary to calculate radiative corrections and effects of isospin breaking
- Joint efforts of τ and e^+e^- groups started which will hopefully make possible CVC tests as well as improvements of the accuracy of $a_\mu^{\rm had}$
- VEPP-2000 (the upgraded VEPP-2M collider in Novosibirsk) will cover the energy range from threshold to 1800-2000 MeV, able to test CVC in the whole energy range ($2m_\pi - m_\tau$) in one experiment

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