GENERAL RELATIONS FOR MODE PARAMETERS OF COMPENSATED STRUCTURES IN THE VICINITY OF OPERATING POINT

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Abstract

In this report the general properties of neighbor modes in compensated accelerating structures in the vicinity of operating point are considered. The dispersion equation for arbitrary compensated periodical structure in the vicinity of operating point is derived. To obtain it, the field distributions and frequencies for operating mode and coupling one are necessary. The dispersion curve behavior and neighbor modes field distributions are investigated both for closed stop-band and for open one. The expressions for quality factors, sensitivity and so on are also obtained for both cases. Non-direct methods for the stop-band width evaluation are considered. The validity of conclusions was estimated in experiments and proved with direct numerical simulations

1 INTRODUCTION

The compensated accelerating structures are now widely used for acceleration of charged particles for high energies. Let remember, that a 'compensated' is named a structure in which at operating frequency coincide frequencies of two modes (0 or π type) with different parity of a field distribution with respect to symmetry plane (accelerating and coupling modes) [1]. Examples of compensated are such structures as side-coupled, annular-coupled, on-axis coupled, disk and washer, drift tube structure with posts and so on. These structures combine a high efficiency with a high stability of the field distribution to deviations in cells parameters and beam loading. In spite of these structures are different in a design, they have the common properties. The main properties of compensated structures are described in [1]. This report gives results of an additional investigation .

2 DISPERSION EQUATION

The general method of the field description in periodic structure is proposed in [2] and an eigenvalue equation (see [2]) can be considered as a dispersion one. Restricting consideration by four modes - two 0 modes with eigenvalues k_{01}, k_{02} and two π modes, accelerating mode with $k_a, \vec{E}_a$ and coupling one with $k_c, \vec{E}_c$, one get equation:

$$det \begin{pmatrix} k_{01}^2 - k^2 & 0 & \gamma_{1-a} & 0 \\ 0 & k_{02}^2 - k^2 & \gamma_{2-a} & 0 \\ \gamma_{1-a} & \gamma_{2-a} & k_a^2 - k^2 & \gamma_{ac} \\ 0 & 0 & \gamma_{ac} & k_c^2 - k^2 \end{pmatrix} = 0$$

where $\gamma_{1_a}, \gamma_{2_a}, \gamma_{ac}$ are coupling coefficients (see [2]). Remember E_a and E_c are normalized field distributions. This equation is still enough particular, because it approximates a total dispersion curve, taking in to account particularity of the structure (0 modes). More restricted for a particular structure, but more general for a compensated structures family is a case when consideration is restricted by two confining modes and describes a curve behavior in a vicinity of an operating point. This case we obtain equation with 0 or π modes:

$$(k_a^2 - k^2)(k_c^2 - k^2) - [(1 \pm \cos\theta)\gamma_{ac}]^2 = 0, \qquad (1)$$

$$\gamma_{ac} = \int_{S2} \vec{\nu}[\vec{E}_a, \frac{1}{\mu_0} rot \vec{E}_c] dS.$$

Here θ is a phase shift per structure period. Let rewrite this equation in terms of frequencies, assuming f_a and f_c, effective coupling coefficient $\gamma_e \sim k_c k_a \gamma_{ac}$, $f_a \approx f_c$ and a phase shift deviation $\xi = \theta$ for 0-mode structures and $\xi = \pi - \theta$ for π-mode ones.

$$(f_a^2 - f^2)(f_c^2 - f^2) + f_a^2 f_c^2 \gamma_e^2 \xi^2 = 0. \qquad (2)$$

In such definition the equation (2) is the same both for 0 and for π operating mode structure. And conclusions are valid also both for 0- and π- type operating modes. Further let assume operating π - mode.

3 DISPERSION CURVE BEHAVIOR

The dispersion curve behavior of a compensated structure in the operating point vicinity strongly depends on a stop-band width $\delta f = f_c - f_a$ and a group velocity β_g. In general case

$$\beta_g = \frac{2\pi d}{c} \frac{\partial f}{\partial \theta} = \frac{dP_T}{cW_T}, \qquad (3)$$

where P_t is a traveling wave power flux (proportional to the boundary coupling integral in (2), W_t - is a traveling wave stored energy, d is the structure period length. If the π-mode structure has a mirror symmetry planes, the expression for β_g can be modified [3]:

$$\beta_g = |\frac{\pi\beta \int_{V_2} (\mu_0 H_a H_c - \epsilon_0 E_a E_c) dV}{\sqrt{2 W_a W_c}}| \qquad (4)$$

(For 0-mode structure expression (4) is not valid.) Here V_2 is a volume of on half of the structure period. This expression has been obtained in [3] basing on another approach and there was a good guide in the development of a new structure with high coupling coefficient [4].

The closed stop-band. For the case $\delta f = 0, f_a = f_c$ one can find directly from (2) for the upper $f^u(\xi)$ and the bottom $f^b(\xi)$ branches of the dispersion curve (see Fig. 1):

$$\frac{\partial^{(2n-1)} f^u}{\partial \xi^{(2n-1)}} = -\frac{\partial^{(2n-1)} f^b}{\partial \xi^{(2n-1)}}, \frac{\partial^{(2n)} f^u}{\partial \xi^{(2n)}} = -\frac{\partial^{(2n)} f^b}{\partial \xi^{(2n)}}. \quad (5)$$

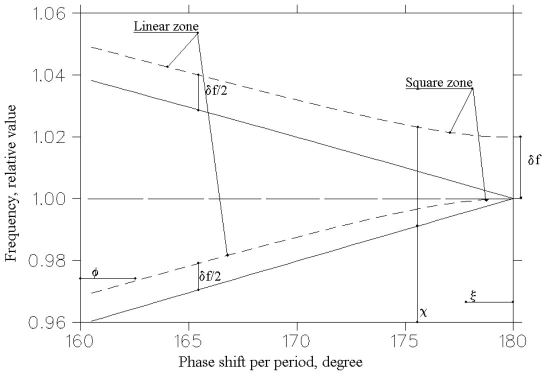

Figure 1. A dispersion curve behavior in the vicinity of operating (π) mode. Solid line - $\delta f = 0$. Dashed line - $\delta f = 0.02 f_a$.

Let define for modes f_m^u, f_m^b of the $\theta_m = \frac{m\pi}{N}, \xi_m = \pi - \theta_m$ type, where N is a number of periods in the section,

$$\Delta F^m = (f_m^u - f_a) - (f_a - f_m^b) = f_m^u + f_m^b - 2f_a. \quad (6)$$

Let ΔF_0^m corresponds to the $\delta f = 0$ case:

$$\Delta F_0^m \approx \frac{\partial^2 f^{u,b}}{\partial \xi^2}\xi_m^2 = \frac{G m^2 f_a}{N^2}, G = \frac{\pi^2}{f_a}\frac{\partial^2 f^{u,b}}{\partial \xi^2}. \quad (7)$$

and the upper and the bottom branches of the curve can be approximated as:

$$f^{u,b} \approx f_a \pm \frac{\beta_g c \xi}{2\pi d} + \frac{G f_a \xi^2}{2\pi^2} \quad (8)$$

The opened stop-band. For the case $\delta f = f_c - f_a$ (for further consideration let suppose $\delta f \geq 0, f_a \geq f_c$) there are two regions at the dispersion curve, depending on χ value:

$$\chi = \frac{\pi \beta \delta f}{2\beta_g f_a} = \frac{2\delta f}{\gamma_e f_a}, \quad \beta_g = \beta\frac{\pi \gamma_e}{4}, \quad (9)$$

For $\xi \leq \chi$ (a nearest to the operating point region) the behavior of the upper and the bottom curve branches is approximated as:

$$f^b \approx f_a - \frac{f_a \beta_g \xi^2}{\pi \beta \chi}, \quad (10)$$
$$f^u \approx f_a + \delta f + \frac{f_a \beta_g \xi^2}{\pi \beta \chi}.$$

The branches come to the operating mode with square low in ξ and further will refer this region as square region.
For $\xi \geq \chi$ (a more far from the operating point region) the behavior of the upper and the bottom curve branches is approximated as:

$$f^{u,b} \approx f_a \pm \frac{\beta_g c \xi}{2\pi d} + \frac{G f_a \xi^2}{2\pi^2} + \frac{\delta f}{2}, \quad (11)$$

In this region the branches of the curve are shifted by $\delta f/2$ value, but come parallel with respect to the branches for an ideal case $\delta f = 0$ (practically linearly, see (8)) and this region will name further as a linear one. If the mode θ_m belongs to the linear region, the parameter $\beta_g^{(m)}$:

$$\beta_g^{(m)} = \frac{\beta N (f_m^u - f_m^b)}{2 m f_a}, \quad (12)$$

doesn't depend on m. Deviation of $\beta_g^{(m)}$ from a constant value allows to determine the upper boundary of the linear region.

4 FIELDS DISTRIBUTIONS

In the case of the closed stop-band $\delta f = 0$ and for modes from linear region with an open stop-band the field distributions for modes in the operating point vicinity are so, that:

$$\lim_{\xi \to 0} \frac{E_a}{E_c} = 1. \quad (13)$$

In a travelling ($E_T^{u,b}$) or a standing ($E_S^{u,b}$,in j-th section period) wave regimes the field distributions are composed from accelerating and coupling modes in equal parts,

$$E_T^b = \frac{E_a - \iota E_c}{\sqrt{2}}, E_S^b = \frac{E_a \cos j\theta_m - E_c \sin j\theta_m}{\sqrt{2}}, \quad (14)$$
$$E_T^u = \frac{E_a + \iota E_c}{\sqrt{2}}, E_S^u = \frac{E_a \cos j\theta_m + E_c \sin j\theta_m}{\sqrt{2}}$$

as one can find from (2).
For modes in the square region, similar to (14), one can derive from (2):

$$\frac{E_c^b}{E_a^b} = \frac{\xi}{\chi} = \frac{E_a^u}{E_c^u}, \quad (15)$$
$$E_T^b = \frac{E_a - \iota\frac{\xi}{\chi}E_c}{\sqrt{(1 + \frac{\xi}{\chi}^2)}}, E_T^u = \frac{\frac{\xi}{\chi}E_a + \iota E_c}{\sqrt{(1 + \frac{\xi}{\chi}^2)}},$$
$$E_S^b = \frac{E_a \cos j\theta_m - \frac{\xi}{\chi}E_c \sin j\theta_m}{\sqrt{(1 + \frac{\xi}{\chi}^2)}},$$
$$E_S^u = \frac{\frac{\xi}{\chi}E_a \cos j\theta_m + E_c \sin j\theta_m}{\sqrt{(1 + \frac{\xi}{\chi}^2)}}.$$

For modes in the square region one component dominate in the field distribution - the accelerating component at the bottom branch and coupling component at the upper branch, because we assume $f_c > f_a$.

5 ANOTHER PROPERTIES

Because the fields distributions for modes in linear region are practically the same as for the ideal case of the closed stop-band (14), below we will distinguish the parameters for modes in the linear region and in the square one, assuming modes parameters in the linear region similar to the modes parameters for the case $\delta f = 0$.

Quality factor. For θ_m modes in the linear region, basing on (14), one can derive for quality factor Q_m:

$$Q_m^u \approx Q_m^b \approx \frac{2Q_a Q^c}{Q_a + Q_c} \tag{16}$$

The expression (16) can be used to estimate the coupling mode quality factor Q_c. For the modes in the square region, taking into account (15):

$$Q_m^u = \frac{(1+\frac{\xi_m^2}{\chi^2})Q_a Q_c}{Q_a + \frac{\xi_m^2}{\chi^2}Q_c}, \lim_{\xi_m \to 0} Q_m^u = Q_c, \tag{17}$$

$$Q_m^b = \frac{(1+\frac{\xi_m^2}{\chi^2})Q_a Q_c}{\frac{\xi_m^2}{\chi^2}Q_a + Q_c}, \lim_{\xi_m \to 0} Q_m^b = Q_a,$$

Frequencies sensitivity. Suppose x is an arbitrary geometrical parameter of the structure. By using usual perturbation theory and basing on (14), one can show the frequency sensitivity coefficients $\frac{\partial f_m^{u,b}}{\partial x}$ for modes in the linear region satisfy to:

$$\frac{\partial f_m^b}{\partial x} \approx \frac{\partial f_m^u}{\partial x} \approx \frac{1}{2}(\frac{\partial f_a}{\partial x} + \frac{\partial f_c}{\partial x}). \tag{18}$$

For the modes in the square region, taking into account (15), one get::

$$\frac{\partial f_m^u}{\partial x} = \frac{\frac{\partial f_a}{\partial x} + \frac{\xi_m^2}{\chi^2}\frac{\partial f_c}{\partial x}}{1+\frac{\xi_m^2}{\chi^2}}, \frac{\partial f_m^b}{\partial x} = \frac{\frac{\xi_m^2}{\chi^2}\frac{\partial f_a}{\partial x} + \frac{\partial f_c}{\partial x}}{1+\frac{\xi_m^2}{\chi^2}}, \tag{19}$$

For all modes, both in the linear region and in the square one, next statement is valid:

$$\frac{\partial f_m^b}{\partial x} + \frac{\partial f_m^u}{\partial x} = \frac{\partial f_a}{\partial x} + \frac{\partial f_c}{\partial x}. \tag{20}$$

Field perturbations. Suppose the j-th period of the structure has a perturbation ΔV, leading to the deviation of the accelerating mode frequency Δf_a. By using the perturbation theory for a multi-cell cavities [5]:

$$E = E_a + \sum_{m'} E_{m'} \frac{f_{m'}^2 \int_{\Delta V}(\eta^2 H_a H_{m'} - E_a E_{m'})dV}{W(f_a^2 - f_{m'}^2)}, \tag{21}$$

one can find, referring with (11, 14) for the contribution only of two modes θ_m^u, θ_m^b from the linear region into the perturbed field E:

$$E = E_a(1 + \frac{4\delta f \Delta f_a N \beta^2 \pi^2 \cos j\theta_m \cos i\theta_m}{f_a^2 m^2 \beta_g^2}) + \tag{22}$$
$$+E_c \frac{8\beta \Delta f_a \sin j\theta_m \cos i\theta_m}{m \beta_g f_a}.$$

These contributions are partially compensated and a residual (a slope in the perturbed field distribution) is proportional to the δf value. All time exists the coupling mode contribution in the perturbed field.

For modes θ_m^u, θ_m^b contributions in the square region, taking into account (11),(15), one get:

$$E = E_a + E_a \frac{\Delta f_a N^2 \beta \pi^2 \cos i\theta_m \cos i\theta_m}{f_a m^2 \beta_g}. \tag{23}$$

If there are modes in the square region, the structure loose the properties of the compensated one. **Stop-band width determination**. In practice, the square region at the dispersion curve exists only in untuned structures and should be removed in tuning by the stop-band removing. To close the stop-band, one need to know f_c value. A direct f_c measurement is not all time possible, especially for structures with high γ_e value. Let suppose modes θ_m, θ_n belong to the linear region. From (11) it follows:

$$\Delta F^m - \Delta F_0^m = \Delta F^n - \Delta F_0^n = \delta f. \tag{24}$$

Taking into account $n^2 \Delta F_0^m = m^2 \Delta F_0^n$ (see (7)), one get:

$$\delta f = \frac{m^2 \Delta F^n - n^2 \Delta F^m}{m^2 - n^2}, G = \frac{(\Delta F^m - \Delta F^n)N^2}{(m^2 - n^2) f_a}. \tag{25}$$

The stop-band width δf, defined with (25), has a sense "in average", but it is the value which we need during tuning the structure with high γ_e value, when intermediate cells tuning is avoided and a structure section tunes "in average".

6 SUMMARY

In very condensed form the general properties of the modes in the operating point vicinity are considered. Results are general for an arbitrary compensated structure and may be used for different structures comparison and particularity understanding. The useful application for results were during the tuning of structures with high coupling coefficient, such as the Disk and Washer structure and the Cut Disk [4] one.

7 REFERENCES

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