Investigation of Twisted Waveguides as Slow-Wave Structures

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Abstract

Twisted hollow waveguides with noncircular cross sections may be considered as slow-wave structures. In a twisted waveguide, propagating radio frequency (rf) waveguide modes can have phase velocities slower than those of similar modes in a straight waveguide. This property is interesting since a twisted waveguide basically has a uniform cross section along the propagation direction. In such a waveguide, periodic standing waves may exist regardless of the mechanical periodicity. This type of waveguide structure may have applications in acceleration, bunch compression, and deflection of charged particles, etc. Results of the investigation on these waveguides for the rf properties are presented.

1 INTRODUCTION

Regular straight hollow waveguides have phase velocities greater than the free-space speed of light for propagating electromagnetic waves. Conventional slow-wave structures used for accelerating charged particles and other applications employ reactive loadings in hollow straight waveguides to reduce the phase velocity of electromagnetic fields in the specific mode to be used.

Twisted waveguide sections have been implemented in waveguide circuits for simple plumbing purposes with no attention to their phase properties. It has been found that the twisted hollow waveguides may support slow-wave waveguide modes. With an optimum shape of the cross section and pitch angle, a twisted waveguide can have the desired phase velocity of a specific mode. The waveguide can be an accelerating structure, if an accelerating mode is chosen. This type of accelerating structure may have many advantages over iris-loaded structures. The uniform structure may enable ease of manufacturing and tuning that can lead to cost reduction that is important for large-scale accelerators, such as the next-generation linear colliders.

In this paper, the slow-wave properties of the twisted waveguide for TE_{01}-like mode and TM_{01}-like mode, are discussed. The twisted waveguide structures have been modeled and simulated using MAFIA [1] and Agilent HFSS [2] codes, the 3-D electromagnetic solvers, to show the slow-wave properties.

2 TWISTED WAVEGUIDE

A twisted waveguide maintains a uniform cross section at any location on the beam axis except the bearing angle. Figure 1 (a) shows a section of twisted waveguide with end walls orthogonal to the twist. For a complete rf section, the end wall surfaces are no longer flat 2-D planes and must be twisted. The cross section of the waveguide in the x-y plane can be arbitrary, but a bow-tie like cross section is shown in Figure 1 (b).

Consider a TE_{01} mode in a rectangular waveguide. If a simple rectangular waveguide is twisted, both electric and magnetic fields in the waveguide will be twisted along the guide to satisfy the boundary conditions. The twist angle θ is a function of radial distance r as the normal vector $n(r)$ on an end wall changes the direction as a function of distance r.

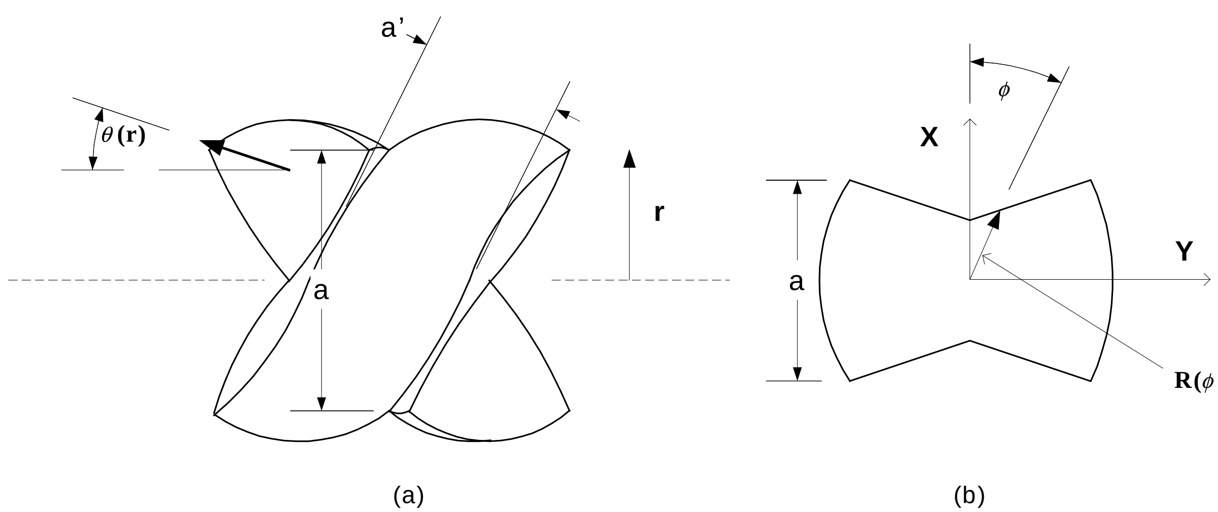

Figure 1: Geometry of a section of twisted waveguide. (a) a waveguide section with twisted port surfaces, (b) cross section of the bow-tie waveguide.

Compared to the fields in a straight waveguide, the fields in the twisted waveguide will be twisted accordingly, which is a function of the radial distance *r*. The effective height of the waveguide can be approximated as

$$a'(r) = a(r)\cos\theta(r) \ . \qquad (1)$$

Therefore, if a regular rectangular waveguide is twisted, the twisting effectively squeezes the waveguide height down to a lower height as *r* increases. This suggests that the cross section defined in the *x-y plane* of a regular rectangular coordinate system must be modified to have an effective cross section similar to that of a rectangular waveguide. This goal may be achieved if the cross section in the *x-y plane* has a bow-tie shape as shown in Figure 1 (b). The fields near the narrow walls of the waveguide are stretched as the waveguide is twisted. Therefore, the volume of the waveguide is increased when the waveguide height is increased with the angle $\theta(r)$.

If the effective waveguide height is made uniform, the twisted waveguide will have greater volume at the narrower walls. This structure may be treated as a wall-perturbed cavity, considering the stretched walls as an outward perturbation. An approximate expression for the change of resonance frequency of a waveguide cavity with a wall perturbation is given as [3]

$$\frac{\omega - \omega_o}{\omega_o} \approx \frac{\iiint_{\Delta\tau} (\mu |H_o|^2 - \varepsilon |E_o|^2) d\tau}{\iiint_{\tau} (\mu |H_o|^2 + \varepsilon |E_o|^2) d\tau} \ , (2)$$

where E_o and H_o are the unperturbed electric and magnetic fields, and $\Delta\tau$ is the volume change. In a hollow waveguide, the stored magnetic energy is greater near the narrow walls of the waveguide. The above expression suggests that a fixed length cavity structure gets lower resonance frequency as it is twisted, which results in a decrease in the phase velocity. Note that the envelope of the longitudinal cross section of the twisted waveguide can be determined by transforming the radial distance $R(\phi, z=z_o)$ into $R'(z)$, where ϕ is the azimuth angle in a circular cylindrical coordinate system.

3 RF SIMULATION

The direction of the normal vector in Figure 1 (a) is related to the directions of field vectors and wave propagation. Since existing commercial electromagnetic codes use orthogonal curvilinear coordinate systems, the twisted structure can not be modeled accurately. In computer simulations, modeling capability of the true transverse boundary planes in the code is needed for more accurate analysis of the structure.

3.1 TE_{01} –like mode in Rectangular Waveguide

To show the slow-wave property for a TE mode, the phase delay of the dominant TE_{01} mode of a straight waveguide was compared to the phase delays through the free space and the rectangular twisted waveguides. The Agilent HFSS code was used for the computation. The twist angle is varied with a fixed rectangular cross section that measures 0.15m x 0.30m and a fixed length of 3.5 cm. The ports modeled in the code are not the true boundary surfaces for the twisted waveguides. Figure 2 shows the phase delay vs. the frequency for the various cases. The result shows the wave velocity decreases as the twisted angle increases.

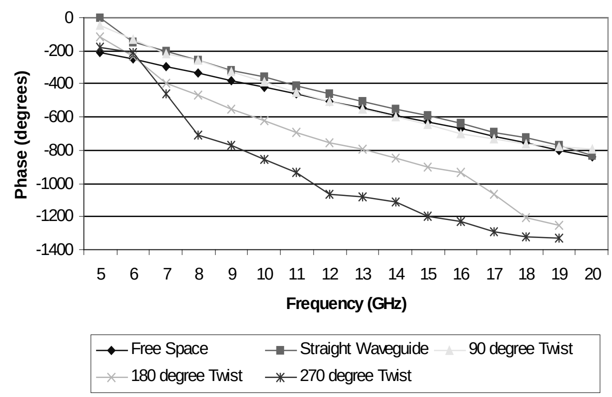

Figure 2: Phase delays of TE_{01} mode through free space and twisted rectangular waveguides. For all cases, cross sections are 3 cm x 1.5 cm and lengths are 3.5 cm.

3.2 TM_{01} –like mode in Circular Waveguide

For TM mode properties, the twisted waveguide with the bow-tie shaped cross section has been modeled in the MAFIA code using stacked waveguide slices, as shown in Figure 3. The end walls used in the model are defined in rectangular coordinates, which are not the true boundary walls for the fields of a sufficiently long twisted waveguide. The displacement angle of the slices is adjusted until the free-space half wavelength of the TM_{01}-like resonance frequency times β becomes identical to the length of the twisted structure. This was done to find the twist angle for the accelerating mode for particles with the speed $v_p = \beta c$.

In an example for 3 GHz with $\beta=1$, the 5 cm long copper waveguide can support a TM_{01}-like mode with Q = 12,620 and the shunt impedance R_s = 2.6 MΩ. For another example with a 1.15” long structure and $\beta=0.87$, a resonant frequency of 4.5GHz was obtained for a desired TM_{01}-like mode. A three times longer structure was modeled, and a field variation of 3π was confirmed at the similar frequency to show if the end walls are valid. This structure is shown in Figure 3.

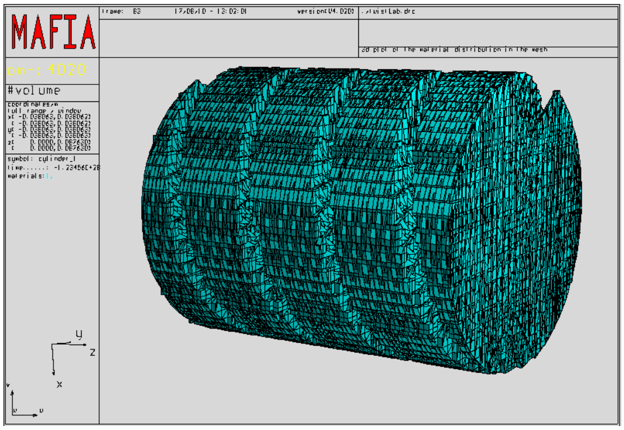

Figure 3: A twisted waveguide modeled in MAFIA with a stack of slices turned by a constant angle.

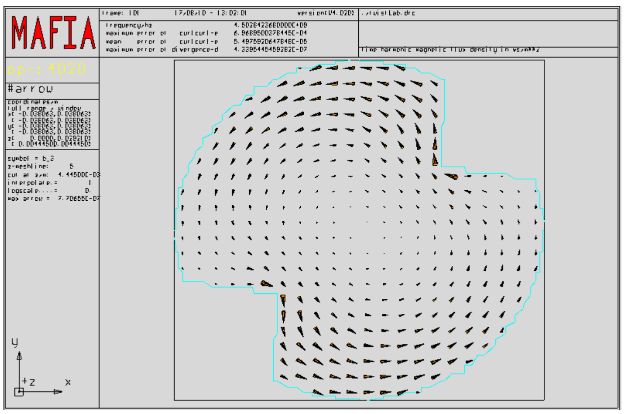

Figure 4: Magnetic field of the TM_{01}-like mode in the bow-tie twisted waveguide.

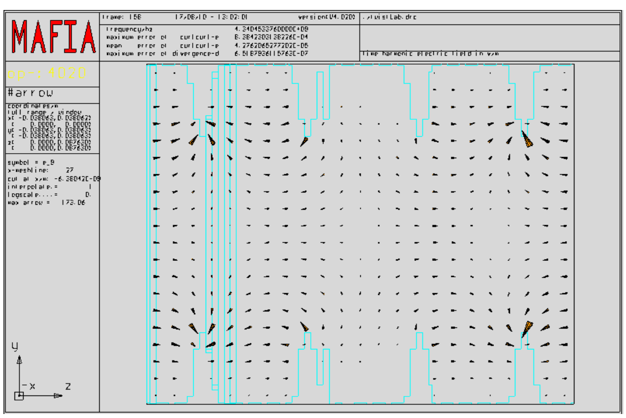

Figure 5: Electric field of the TM_{01}-like mode in the bow-tie twisted waveguide.

Figures 4 and 5 show the magnetic and electric fields, respectively, of the TM_{01}-like mode in the twisted waveguide with $\beta=0.87$. In a regular rectangular waveguide, the magnetic field vectors of the TM_{11} mode are lying on the transverse plane. However, the magnetic field of the present TM_{01}-like mode is twisted. That is, the H_z portion of the magnetic field vector increases as radial distance r increases.

The electric field will have E_z only on the beam axis but will have E_ϕ if $r\neq0$. It is interesting to note that the cavity envelope and the field configurations look similar to those of the conventional iris-loaded rotationally symmetrical structure.

5 CONCLUSION

Commercially available MAFIA and HFSS codes have been used to show that the phase velocities can be lowered in twisted waveguides. The results of the MAFIA simulations for a specific TM_{01}-like mode have shown that such twisted waveguide structures can support an accelerating mode with $\beta_b\leq1$. Although approximations are made in the modeling with nonorthogonal end walls, it is clear that having the slow wave is possible. A slow wave for a specific β_b can be excited in a twisted waveguide by choosing an optimum shape of the cross section and pitch angle.

Investigation on the dispersion properties of the twisted waveguides is under way for further understanding of the structures and for feasibility studies for accelerator applications. The inner surface of the structure is smooth and free from any sharp corners. This means that, in the accelerating structure, damping the higher-order modes is easier and the particle beam properties may be enhanced. This also means that the maximum field inside the waveguide may be greater so that the field gradient limit could be raised.

Development of new code for the specific coordinate system, which is conformal to the twisted structure, may be needed. Using this type of new code may aid in making more accurate assessments of rf and beam properties in the structure.

Since the twisted waveguide structure has a uniform cross section, unlike the conventional iris-loaded accelerating cavity structures, it may be built without welding or brazing many parts. A special extrusion technique or electroforming may be used for mass production of the waveguide at significantly lower cost.

5 ACKNOWLEDGEMENT

The authors are grateful to the APS management for encouragement on this investigation. This work was supported by the U. S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.

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