THE METHOD FOR THE FIELDS DISTRIBUTION DESCRIPTION IN PERIODICAL STRUCTURES

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Abstract

The variational method for numerical simulation of rf parameters of periodical structures with arbitrary phase shift per structure period is developed. At any phase shift the field distribution is represented as a sum over trial functions. As trial functions we use numerically simulated field distribution, considering with different boundary conditions only one structure period. By using standard variational technique and Floquet boundary conditions, the generalized symmetrical eigen-value problem is formed. Due to successful choice of trial functions, which keep a lot of information about the real structure shape, the method has fast convergence. Not so many trial functions are needed to describe good the field distribution. The method combines the powerful ideas of variational approach with possibilities of modern computer codes for direct simulations. A lot of applications, both for structure parameters calculations and for general properties investigation, are possible.

1 INTRODUCTION

Now we have a powerful codes, like MAFIA, for numerical simulations of the field distributions in electromagnetic systems with possibilities of simulations in periodical structures of arbitrary shape by using Floquet boundary conditions. But a code output, as a role, is a number in a mesh point. It has a practical value, but, even with powerful interface, sometimes it is difficult to generalize results. Moreover, the direct numerical simulations for periodical structures is still consuming in computer resources.

Fifty years ago a variety of another methods were in use (see, for example, [1]). One of them, very general, with a well developed theory [2], is the variational technique. It allows to treat results in terms of interacting trial functions. The main disadvantage of the variational method were in poor choice of a basis of trial functions, which have to satisfy to a set of conditions. A lot of work were needed to obtain a practical value.

The main idea of this work is to combine a power of modern codes with general possibilities of variational approach to get a tool both for simulations and for clarified treatment of results.

2 PROCEDURE

Let consider a symmetrical periodic structure. At Fig. 1 periods of the Disk and Washer (DAW) structure and the Disk Loaded waveguide are shown together with definition for further explanations. The mirror symmetry plane is at $z = 0$.

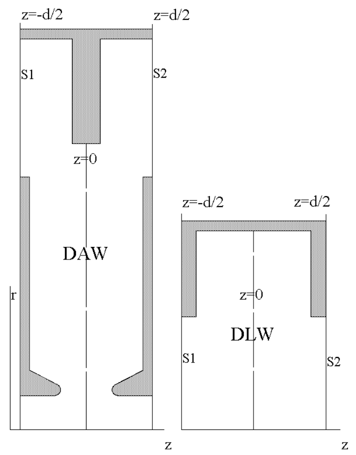

Figure 1. A geometry and definitions for the Disk and Washer structure (a) and Disk Loaded Waveguide (b).

The travelling wave complex amplitude $\vec{E} = \Re\vec{E} - \imath\Im\vec{E}$ has components with different parity with respect $z = 0$ [3]. Suppose real part $\vec{E}$ satisfy boundary condition of 'electric wall' (denoted further as **(e)**) $[\nu, \Re\vec{E}] = 0$ at $z = 0$. Than imaginary part must satisfy to condition of 'magnetic wall' (denoted further as **(m)**), $(\nu, \Im\vec{E}) = 0$.

By using a numerical code, let simulate and store for the structure under investigation a set of 0- and π-type modes. In the structure with symmetry plane 0-type modes can be calculated considering one half of the period in conditions **(ee)** (**(e)** at $z = 0$ and **(e)** at $z = d/2$) and **(mm)**. The π-type modes should be calculated in conditions **(em)** and **(me)**. For the DAW structure an example of such set is shown in Fig. 2.

Supposing all modes are normalized $2W = \int_V \epsilon_0 \vec{E}_n^* \vec{E}_n dV = 1$, let consider these four families of modes (functions), **(ee)**, **(em)**, **(me)** and **(mm)** as a basis of trial functions for variational approach. Each function satisfies to the Maxwell equations and boundary conditions at the metal surface in the real shape of the structure. Each function is a limit for the travelling wave field distribution when a phase shift θ per structure period $\theta \to 0$ or $\theta \to \pi$ and described the field well in the vicinity of 0 or π mode. This basis is not ortohonal, it is not dangerous. More important this basis is complete and no potential functions are need for travelling wave field description.

The idea to represent field of an arbitrary θ as a sum over several modes is not new. It is usual for the lumped circuit method. For two limiting modes there are electrodynamic result in [1]. The question is only in the precision of description and ability to investigate quite different structures.

Let represent $\Re\vec{E}$ and $\Im\vec{E}$ as:

$$\Re\vec{E} = \sum_{n}^{Nee} c_n^{ee}\vec{E}_n^{ee} + \sum_{n}^{Nem} c_n^{em}\vec{E}_n^{em}, \quad (1)$$

$$\Im\vec{E} = \sum_{n}^{Nme} c_n^{me}\vec{E}_n^{me} + \sum_{n}^{Nmm} c_n^{mm}\vec{E}_n^{mm}.$$

This representation (2) satisfies to parity conditions for $\Re\vec{E}$ and $\Im\vec{E}$.

In a periodic structure the field distributions must satisfy to the Floquet condition:

$$\vec{E}_{z=d/2} = \vec{E}_{z=-d/2}e^{-\iota\theta}. \quad (2)$$

In symmetrical structures the condition for electric field tangential components:

$$(\vec{\nu}(\vec{E}e^{\iota\theta/2} + \vec{E}^{*}e^{-\iota\theta/2}))_{z=d/2} = 0, \quad (3)$$

is equivalent to (2). One can proof it directly, involving $\Re\vec{E}$ and $\Im\vec{E}$ parities or see [3] for particular case of the TM_0 modes. The expression similar (3) may be obtained and for magnetic field tangential components. Remember, only tangential components are necessary to define the internal electrodynamic problem.

One can check directly, the expression:

$$\omega^2 = \frac{\int_V \frac{1}{\mu_0} rot\vec{E}^{*} rot\vec{E} dV + I_S}{\int_V \epsilon_0 \vec{E}^{*}\vec{E} dV}, \quad (4)$$

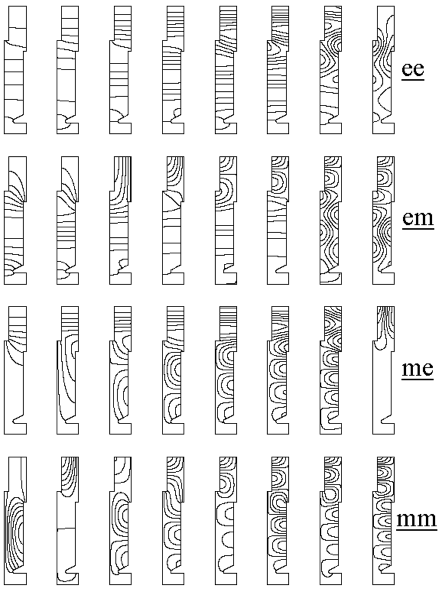

Figure 2. The basis of trial functions for the DAW structure. 0 modes - **(ee), (mm)** and π modes **(em), (me)**.

with

$$I_S = \frac{1}{4}\int_{S1} \vec{\nu}[(\vec{E}e^{-\iota\theta/2} + \vec{E}^{*}e^{\iota\theta/2})\frac{1}{\mu_0}(rot\vec{E}^{*}e^{\iota\theta/2} + \quad (5)$$
$$+ rot\vec{E}e^{-\iota\theta/2})]dS + \frac{1}{4}\int_{S2} \vec{\nu}[(\vec{E}e^{\iota\theta/2} + \vec{E}^{*}e^{-\iota\theta/2})$$
$$\frac{1}{\mu_0}(rot\vec{E}e^{\iota\theta/2} + rot\vec{E}^{*}e^{-\iota\theta/2})]dS,$$

is variational one for the problem of the travelling wave propagation in symmetrical periodic structure.

Substituting (2) in (6) and following to a standard variational technique (the Ritz technique), one come to the generalized symmetrical eigen-value problem:

$$AC - k^2BC = 0, \quad (6)$$

where C is the column of unknown coefficients c_n in (2), A and B are square block-type matrixes:

$$\mathbf{A} = \begin{pmatrix} A_{ee}^{ee} & A_{ee}^{em} & 0 & A_{ee}^{mm} \\ A_{ee}^{em} & A_{em}^{em} & A_{em}^{me} & 0 \\ 0 & A_{em}^{me} & A_{me}^{me} & A_{me}^{mm} \\ A_{ee}^{mm} & 0 & A_{me}^{mm} & A_{mm}^{mm} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} B_{ee}^{ee} & B_{ee}^{em} & 0 & 0 \\ B_{ee}^{em} & B_{em}^{em} & 0 & 0 \\ 0 & 0 & B_{me}^{me} & B_{me}^{mm} \\ 0 & 0 & B_{me}^{mm} & B_{mm}^{mm} \end{pmatrix}$$

Coefficients of these matrixes are for blocks

$$(A_{ee}^{ee}, A_{em}^{em}, A_{me}^{me}, A_{mm}^{mm}), a_{ij} = \delta_{ij}k_ik_j, \quad (7)$$
$$(B_{ee}^{ee}, B_{em}^{em}, B_{me}^{me}, B_{mm}^{mm}), b_{ij} = \delta_{ij},$$
$$(B_{ee}^{em}), b_{ij} = \epsilon_0 \int_V \vec{E}_i^{ee}\vec{E}_j^{em} dV,$$
$$(B_{mm}^{me}), b_{ij} = \epsilon_0 \int_V \vec{E}_i^{mm}\vec{E}_j^{me} dV,$$

for block (A_{ee}^{em}):

$$a_{ij} = k_ik_jb_{ij} + \sin\theta \int_{S2} \vec{\nu}[\vec{E}_i^{em}, \frac{1}{\mu_0} rot\vec{E}_j^{ee}]dS, \quad (8)$$

for block (A_{mm}^{me}):

$$a_{ij} = k_ik_jb_{ij} - \sin\theta \int_{S2} \vec{\nu}[\vec{E}_i^{mm}, \frac{1}{\mu_0} rot\vec{E}_j^{me}]dS, \quad (9)$$

for block (A_{em}^{me}):

$$a_{ij} = -(1+\cos\theta) \int_{S2} \vec{\nu}[\vec{E}_i^{em}, \frac{1}{\mu_0} rot\vec{E}_j^{me}]dS, \quad (10)$$

for block (A_{ee}^{mm}):

$$a_{ij} = (1-\cos\theta) \int_{S2} \vec{\nu}[\vec{E}_i^{mm}, \frac{1}{\mu_0} rot\vec{E}_j^{ee}]dS, \quad (11)$$

The off-diagonal blocks describe mode coupling due to both non-ortohonal basis (like B_{ee}^{em}) and surface coupling (like A_{em}^{me}). Dimension of the A and B matrixes $N = N_{ee} + N_{em} + N_{me} + N_{mm}$ is not so large ($40 \div 100$) and numerical solution is not a question by using modern computing libraries.

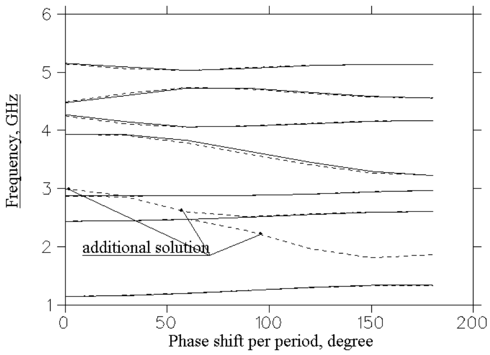

Figure 3. The Brillouin diagram for the DLW. The solid lines - direct calculations, the dashed lines - the decomposition.

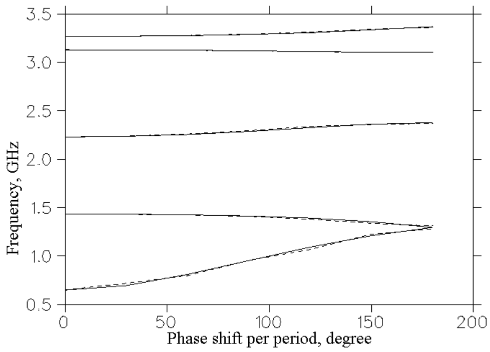

Figure 4. The Brillouin diagram for the DAW. The solid lines - direct calculations, the dashed lines - the method presented.

Table 1. The relative frequency diffferences for the DLW passbands between direct simulations and decomposition (multiplied by 10^4).

θ	**1**	**2**	**3**	**4**	**5**	**6**
0.0	0.66	0.33	0.94	0.03	4.88	2.81
30.0	4.46	1.85	0.66	3.13	9.41	8.16
60.0	4.45	3.12	0.61	11.8	0.92	2.44
90.0	1.51	4.15	0.06	13.8	1.83	3.21
120.0	2.48	4.07	0.97	12.7	2.28	3.83
150.0	6.13	2.50	1.38	7.79	1.45	2.30
180.0	9.41	0.15	2.94	1.70	0.59	0.65

3 APPLICATIONS

Structure parameters calculations. At present the code using this method of the field decomposition is realized at the base of our 2D FEM code set [4]. For all numerical experiments we took $N_{ee} = N_{em} = N_{me} = N_{mm} = 8$. In general, according theory, the number of modes have to be taken so that a highest eigenvalues for each family are approximately equal.

In Fig. 3 the Brillouine diagram for DLW (see Fig. 1) is shown with seven passbands of TM_0 type. As for the computing resources, it is cheaper to get the total diagram shown in Fig. 3 (starting with basis storage) then do two direct numerical simulations for different θ values. In numbers a relative differences between results of direct simulations and by using decomposition are presented in Table 1 for 6 DLW TM_0-type passbands.

One can see a good coincidence between results of different methods. It is interesting also, that the relative error doesn't relate obviously with the passband number. Usually for numerical methods the error rises for higher order passbands.

But DLW is a narrow-band (in the main passband) structure. For wide-band DAW structure in Fig. 4 are shown five passband, calculated in the same way as for DLW in Fig. 3. The relative errors are several times larger and drop fast with the basis extension.

In general, the problem (6) has N eigenvalues. But physical sense has only $min(N_{ee} + N_{mm}, N_{em} + N_{me})$ solutions, describing real passbands. And the problem (6) in general is not-positive defined. This problem has 'additional' solutions without physical sense. The main part of 'additional' solutions either have $k^2 < 0$ or $k^2 \gg max(k_i^2)$, so, can be filtered easily. But can be a case when an 'additional' passband is the frequency region under investigation (see Fig. 3). This case the correct filtering algorithm is not developed jet. But real passband must start from one of the modes in the basis. 'Additional' passbands depend strongly on N. It allows us to distinguish real and 'additional' passbands.

Structure investigations. The equation (6) can be considered as a dispersion equation of the structure for arbitrary passband. From direct solution (6) with large N we can, comparing c_n values in (1) estimate, which kind of modes is important for the field description in a given passband and remove non-important modes from consideration, reducing dimension of the equation (6). It allows to do analytica estimations [5]. The restricted equation (6) is also a base for the structure equivalent scheme development.

4 REFERENCES

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