HIGH - BRIGHTNESS BEAM CURRENT LIMIT IN RF FIELD

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Abstract

An analytical solution for a self-consistent particle equilibrium distribution in an RF field with uniform transverse focusing is applied to the problem of the high brightness beam current limit. The distribution function in phase space is determined as a stationary function of the energy integral. Particle distribution of the bright beam always tends to such a shape that the space charge beam potential is opposite to the external potential regardless of the applied field. Analytical expressions for r-z equilibrium beam profile and beam current limit in an RF field are obtained.

1 INTRODUCTION

The stationary self-consistent particle distribution in an RF field provides an estimation of the maximum beam current in an accelerating field. The approximation of the bunched beam as an ellipsoid gives the most simple way to determine the maximum beam current. However, an ellipsoid is not a self-consistent solution for a bunched beam in an RF field. In Ref. [1] the self-consistent bunched beam was approximated by a uniformly populated cylinder with density dependent on the longitudinal coordinate. In Ref. [2] the spatial particle distribution in a 3-dimensional configuration space was calculated numerically. In Ref. [3] an analytical solution for a self-consistent high brightness bunched beam distribution was found. The latest approach allows us to determine the self-consistent maximum accelerated beam current.

2 SELF-CONSISTENT SPACE CHARGE POTENTIAL OF THE BEAM

The Hamiltonian for particle motion in an RF field with continuous transverse focusing is given by [1]

$$H = \frac{p_x^2 + p_y^2}{2\, m\, \gamma} + \frac{p_z^2}{2\, m\, \gamma^3} + q\, U_{ext} + q\, \frac{U_b}{\gamma^2}, \qquad (1)$$

$$U_{ext} = \frac{E}{k_z}\, [I_o(\frac{k_z r}{\gamma})\sin(\varphi_s - k_z\zeta) - \sin\varphi_s + k_z\zeta\cos\varphi_s] + \frac{G_t r^2}{2}, \qquad (2)$$

where p_x and p_y are transverse particle momentum, $p_z = p - p_s$ and $\zeta = z - z_s$ are deviations from longitudinal momentum and position of synchronous particle, respectively, U_{ext} is the potential of an external field, U_b is the space charge potential of the beam, E is the amplitude of the accelerating field, φ_s is the synchronous phase, G_t is the gradient of the focusing field, r is the particle radius, $k_z = 2\pi/(\beta\lambda)$ is the wave number and λ is the wavelength. In Ref. [3] the first approximation for a self-consistent potential of the beam was found:

$$U_b = -\frac{\gamma^2}{1 + \delta}\, U_{ext}\ , \qquad (3)$$

where $\delta \approx b^{-1}$ is a small parameter, inversely proportional to the dimensionless beam brightness, b,

$$b = \frac{2}{\beta\gamma}\, \frac{I}{I_c}\, \frac{R^2}{\varepsilon_t^2}, \qquad (4)$$

I is the beam current, R is the beam radius, I_c is the characteristic value of the beam current

$$I_c = 4\pi\varepsilon_o \frac{mc^3}{q}\ , \qquad (5)$$

and ε_t is the transverse beam emittance. Equation (3) indicates that the particle distribution of the bright beam has such a shape that the space charge potential is opposite to the external potential. This fact is well known for a stationary distribution of a transported beam in a linear focusing channel [1]. Equation (3) generalizes this statement for a 3-dimensional beam distribution.

Taking the first approximation to the space charge potential of the beam, Eq. (3), the Hamiltonian corresponding to the self-consistent bunch distribution is as follows:

$$H = \frac{p_x^2 + p_y^2}{2\, m\, \gamma} + \frac{p_z^2}{2\, m\, \gamma^3} + q\, (\frac{\delta}{1 + \delta})\, U_{ext}\ . \qquad (6)$$

Equation (6) indicates that in the presence of an intense, bright bunched beam ($\delta << 1$), the stationary longitudinal phase space of the beam becomes narrow in momentum spread, remaining, in the first approximation, the same in coordinate. This is in a qualitative agreement with the study of Ref. [1].

3 SELF-CONSISTENT BEAM PROFILE

A self consistent space charge distribution of a matched beam in a channel is attained from the Poisson's equation:

$$\rho(r,\zeta) = -\,\varepsilon_o\, [\frac{1}{r}\frac{\partial}{\partial r}(r\,\frac{\partial U_b}{\partial r}) + \frac{\partial^2 U_b}{\gamma^2 \partial\zeta^2}] = 2\, \varepsilon_o\, \frac{\gamma^2}{1 + \delta}\, G_t. \qquad (7)$$

The space charge density of a high brightness beam is nearly constant within the bunch.

From Eq. (3) it follows, that, in the first approximation, space charge potential of the beam is the same function of coordinates, as the external potential, with opposite sign. Therefore, equation $U_{ext}(r, \zeta)$= const gives the family of equipotential lines of space charge field of the beam:

$$I_o\left(\frac{k_z r}{\gamma}\right)\sin(\varphi_s - k_z\zeta) - \sin\varphi_s + k_z\zeta\cos\varphi_s + \frac{G_t k_z}{2\,E}\, r^2 = \text{const.} \qquad (8)$$

In general case, bunch boundary does not create an equipotential surface. Consider uniformly populated bunch with boundary $R(\zeta)$, defined by nonlinear equation

$$I_o\left(\frac{k_z R}{\gamma}\right)\sin(\varphi_s - k_z\zeta) - \sin\varphi_s + k_z\zeta\cos\varphi_s + C(k_z R)^2 = \text{const.} \qquad (9)$$

Space charge potential of the bunch with boundary, Eq. (9), is close to that, given by Eq. (3). Parameter C is used to adjust the shape of the bunch in such a way, that space charge field of the bunch is opposite to the external field with the specific values of accelerating field, E, and focusing gradient, G_t.

The value of constant in Eq. (9) can be determined from the condition, that longitudinal bunch size is, in the first approximation, the same as for zero - current mode. Therefore, at $R(\zeta) = 0$, the left bunch boundary is $k_z\zeta = 2\varphi_s$ and the value of constant is

$$\text{const} = 2\varphi_s \cos\varphi_s - 2\sin\varphi_s\,. \qquad (10)$$

Substitution of Eq. (10) into Eq. (9) gives the first approximation to the beam profile, $R = R(\zeta)$, defined by the expression (see Fig. 1 b):

$$I_o\left(\frac{k_z R}{\gamma}\right)\sin(\varphi_s - k_z\zeta) + \sin\varphi_s - (2\varphi_s - k_z\zeta)\cos\varphi_s + C(k_z R)^2 = 0. \qquad (11)$$

For a long bunch, $\beta\lambda >> R_{max}$, one can assume $I_o(\xi) \approx 1$ and transverse bunch size, R_{max} is:

$$R_{max} = \frac{1}{k_z}\sqrt{\frac{2\,(\varphi_s\cos\varphi_s - \sin\varphi_s)}{C}}\,. \qquad (12)$$

Parameter C can be expressed as a function of the ratio of transverse, G_t, and longitudinal, G_z, gradients of electric field within the bunch, $C = C\,(G_t/G_z)$, see Fig. 2. Because of Eq. (3), the ratio of gradients, G_t/G_z, is the same for the external field and for the space charge field of a stationary bunch. The gradient of accelerating field in the vicinity of synchronous particle is

$$G_z = 2\pi\,\frac{E\sin\varphi_s}{\beta\lambda}\,. \qquad (13)$$

The values of gradients of space charge field of the bunch are obtained from numerical solution of the Poisson's equation for a uniformly populated bunch with boundary, Eq. (11), for every specific value of the parameter C. Inverse function, $C = C\,(G_t/G_z)$, gives the dependence, presented in Fig. 2.

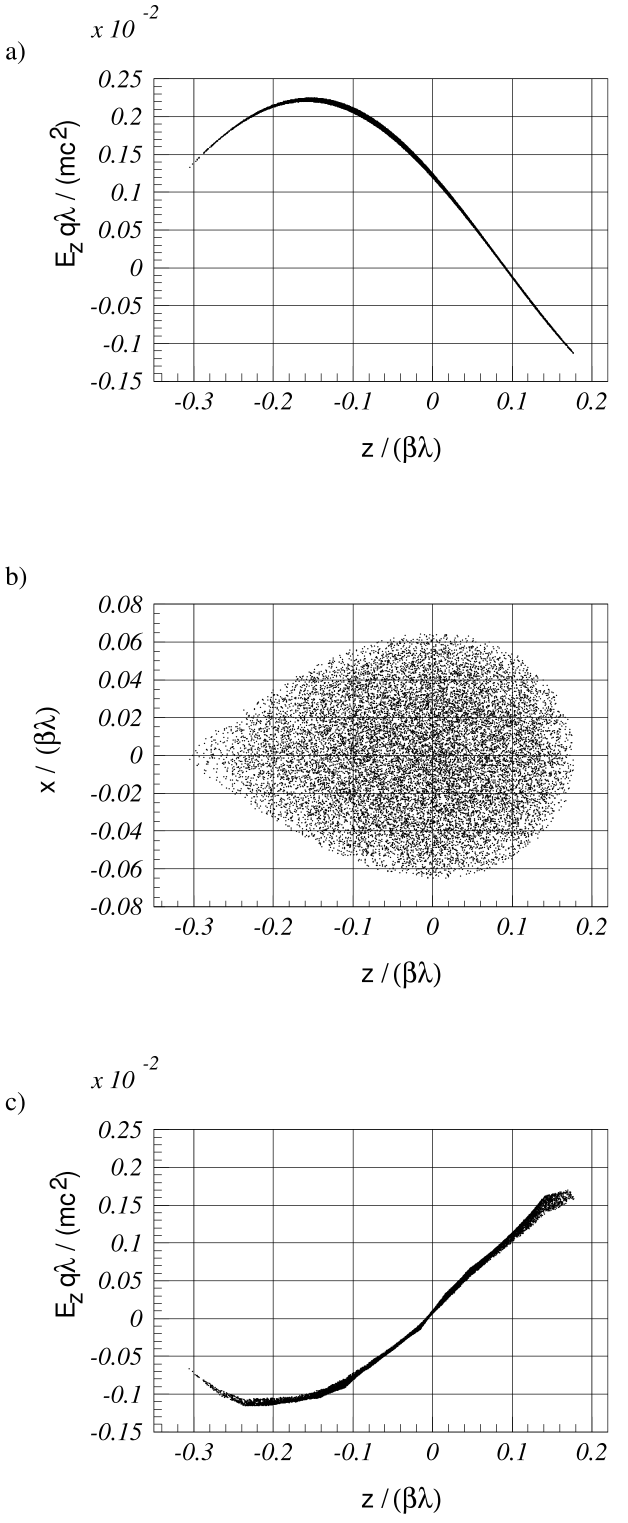

Fig 1. Stationary distribution of bunched beam, $\varphi_s = -1$, $C = 3.8$, $\delta = 0.2$: a) RF accelerating field, b) particle distribution, c) longitudinal space charge field of the bunch.

4 MAXIMUM BEAM CURRENT

The above analysis allows us to determine the value of maximum beam current which can be accelerated for given values of an accelerating field, E, and focusing gradient, G_t. The volume of the bunch is obtained by integration of the bunch shape along the z-coordinate:

$$V = \pi \int_{z_{min}}^{z_{max}} R^2(z)\, dz \quad . \qquad (14)$$

For a long bunch, $\beta\lambda >> R_{max}$, integration gives:

$$V = \frac{(\beta\lambda)^3}{8\pi^2 C} f(\varphi_s) , \qquad (15)$$

$$f(\varphi_s) = 3\varphi_s \sin\varphi_s - \frac{9}{2}\varphi_s^2 \cos\varphi_s + \cos\varphi_s - \cos 2\varphi_s . \qquad (16)$$

The total charge of the bunch is $Q = \rho V$, and the beam current, $I = Q c/\lambda$, is therefore,

$$I = I_c \left(\frac{\beta^3\gamma^2}{16\pi^3 C (1 + \delta)} \right) \left(\frac{G_t q \lambda^2}{m c^2} \right) f(\varphi_s) . \qquad (17)$$

Eq. (17) gives a unique expression for the limited beam current for every combination of the values of RF field, E, and focusing gradient, G_t. The expression of $f(\varphi_s)$, Eq. (16), is close to a cubic function of the synchronous phase, φ_s^3, (see Fig. 3). It indicates, that the maximum beam current is proportional to the cube of the synchronous phase, which is in qualitative agreement with the study of Ref. [1].

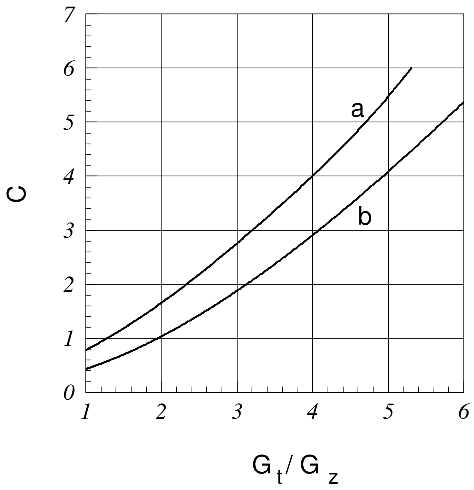

Fig. 2. Coefficient C in bunch shape as a function of ratio of transverse and longitudinal gradients: a) $\varphi_s = -60^0$, b) $\varphi_s = -30^0$.

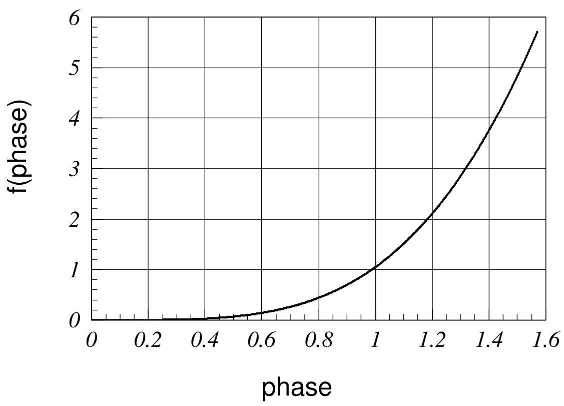

Fig. 3. Function $f(\varphi_s) = 3\varphi_s\sin\varphi_s - \frac{9}{2}\varphi_s^2\cos\varphi_s + \cos\varphi_s - \cos 2\varphi_s$.

5 REFERENCES

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