Measurement of Branching Fractions and Search for $CP$-Violating Charge Asymmetries in Charmless Two-Body $B$ Decays into Pions and Kaons

The BABAR Collaboration

<table>
<thead>
<tr>
<th>Authors</th>
<th>Institutions</th>
</tr>
</thead>
</table>

---

1. *Laboratoire de Physique des Particules, F-74941 Annecy-le-Vieux, France*
2. *Università di Bari, Dipartimento di Fisica e INFN, I-70126 Bari, Italy*
3. *Institute of High Energy Physics, Beijing 100039, China*
4. *Institute of Physics, University of Bergen, N-5007 Bergen, Norway*
5. *Lawrence Berkeley National Laboratory and University of California, Berkeley, CA 94720, USA*
We present measurements of the branching fractions and a search for CP-violating charge asymmetries in charmless hadronic decays of $B$ mesons into two-body final states of kaons and pions. The results are based on a data sample of approximately 23 million $B\bar{B}$ pairs collected by the BaBar detector at the PEP-II asymmetric $B$ Factory at SLAC. We find the following branching fractions: $\mathcal{B}(B^0 \to \pi^+\pi^-) = (4.1 \pm 1.0 \pm 0.7) \times 10^{-6}$, $\mathcal{B}(B^0 \to K^+\pi^-) = (16.7 \pm 1.6 \pm 1.3) \times 10^{-6}$, $\mathcal{B}(B^+ \to K^+\pi^0) = (10.8^{+2.1+1.0}_{-1.3-1.0}) \times 10^{-6}$, $\mathcal{B}(B^+ \to K^0\pi^+) = (18.2^{+3.3+2.0}_{-3.0-2.0}) \times 10^{-6}$, $\mathcal{B}(B^0 \to K^0\pi^0) = (8.2^{+3.2+2.1}_{-1.2-1.2}) \times 10^{-6}$. We also report the 90% confidence level upper limits $\mathcal{B}(B^0 \to K^+K^-) < 2.5 \times 10^{-6}$, $\mathcal{B}(B^+ \to \pi^+\pi^0) < 9.6 \times 10^{-6}$, and $\mathcal{B}(B^+ \to \bar{K}^0K^+) < 2.4 \times 10^{-6}$.

In addition, charge asymmetries have been measured and found to be consistent with zero, where the statistical precision is in the range of $0.10 \pm 0.18$, depending on the decay mode.

PACS numbers: 13.25.Hw, 13.25.-k, 14.40.Nd
are then kinematically fitted with their mass constrained to the nominal $\pi^0$ mass.

$B$ meson candidates are reconstructed in four topologies: $h^+h^-, h^+\pi^0, K^0\bar{h}^+$ and $K^0\pi^0$, where the symbols $h$ and $h'$ refer to $\pi$ or $K$. The kinematic constraints provided by the $Y(4S)$ initial state and relatively precise knowledge of the beam energies are exploited to efficiently identify $B$ candidates. We define a beam-energy substituted mass $m_{ES} = \sqrt{E^2 - \mathbf{p}_B^2}$, where $E_B = (s/2 + p_1 + p_2)/E_1$, $\sqrt{s}$ and $E_1$ are the total energies of the $e^+e^-$ system in the CM and lab frames, respectively, and $\mathbf{p}_1$ and $\mathbf{p}_B$ are the momentum vectors in the lab frame of the $e^+e^-$ system and the $B$ candidate, respectively. To improve the resolution in modes containing $\pi^0$ mesons, the $B$ candidate is kinematically fitted with the energy constrained to the CM beam energy. For all modes, the $m_{ES}$ resolution is dominated by the beam energy spread and is approximately 2.5 MeV/$c^2$. Candidates are selected in the range $5.2 < m_{ES} < 5.3$ GeV/$c^2$.

We define an additional kinematic parameter $\Delta E$ as the difference between the energy of the $B$ candidate and half the energy of the $e^+e^-$ system, computed in the CM system, where the pion mass is assumed for all charged decay products of the $B$. The $\Delta E$ distribution is peaked near zero for modes with no charged kaons and shifted on average $-45$ MeV ($-91$ MeV) for modes with one (two) kaons, where the exact separation depends on the laboratory kaon momentum. The resolution on $\Delta E$ is mode dependent. For final states that contain no $\pi^0$ mesons the resolution is about 26 MeV. For modes with $\pi^0$ mesons the resolution is about 42 MeV and is asymmetric due to underestimation of the $\pi^0$ energy in the EMC. Candidates are accepted in the following $\Delta E$ ranges (given in GeV): $[-0.15, 0.15]$ ($h^+h^-$), $[-0.2, 0.15]$ ($h^+\pi^0$), $[-0.115, 0.075]$ ($K^0\bar{h}^+$) and $[-0.2, 0.2]$ ($K^0\pi^0$).

Detailed Monte Carlo simulation, off-resonance data, and events in on-resonance $m_{ES}$ and $\Delta E$ sideband regions are used to study backgrounds. The contribution due to other $B$-meson decays, both from $b \rightarrow c$ and charmless decays, is found to be negligible. The largest source of background is from random combinations of tracks and neutrals produced in the $e^+e^- \rightarrow q\bar{q}$ continuum (where $q = u, d, s$ or $c$). In the CM frame this background typically exhibits a two-jet structure that can produce two high momentum, nearly back-to-back, particles, in contrast to the spherically symmetric nature of the low momentum $Y(4S) \rightarrow B\bar{B}$ events.

We exploit this topology difference by making use of two event-shape quantities. The first variable is the angle $\theta_s$ [11] between the sphericity axes of the $B$ candidate and of the remaining tracks and photons in the event. The distribution of $|\cos \theta_s|$ in the CM frame is strongly peaked near 1 for continuum events and is approximately uniform for $B\bar{B}$ events. We require $|\cos \theta_s| < 0.9$, which rejects 66% of the background that remains at this stage of the analysis.

The second quantity is a Fisher discriminant $F$ constructed from the scalar sum of the CM momenta of all tracks and photons (excluding the $B$ candidate decay products) flowing into nine concentric cones centered on the thrust axis of the $B$ candidate. Each cone subtends an angle of $10^\circ$ and is folded to combine the forward and backward intervals. Monte Carlo samples are used to obtain the values of the coefficients, which are chosen to maximize the statistical separation between signal and background events. The distributions of $F$ for Monte Carlo simulated $B^0 \rightarrow h^+h^-$ decays and background events in the $m_{ES}$ sideband region $5.20 < m_{ES} < 5.27$ GeV/$c^2$ are displayed in Fig. 1(a).

The final reconstruction efficiencies range from 31% to 45%, depending on the mode. The detection efficiencies, which include the branching fractions of $K^0 \rightarrow \pi^0 \rightarrow \pi^+\pi^-$ and $\pi^0 \rightarrow \gamma\gamma$ [12], are listed in Table I.

Signal yields are determined from an unbinned maximum likelihood fit that uses $m_{ES}$, $\Delta E$, $F$, and $\theta_c$ (where applicable). Separate fits are performed for each of the four topologies, where the likelihood for a given candidate $j$ is obtained by summing the product of event yield $n_i$ and probability $P_i$ over all the possible signal and background hypotheses $i$. The $n_i$ are determined by maximizing the extended likelihood function $L$:

$$L = \exp \left( -\sum_{i=1}^{M} n_i \prod_{j=1}^{N} \sum_{i=1}^{M} n_i P_i \left( \bar{x}_j; \bar{\alpha}_i \right) \right). \quad (1)$$

The probabilities $P_i(\bar{x}_j; \bar{\alpha}_i)$ are evaluated as the product of probability density functions (PDFs) for each of the independent variables $\bar{x}_j$, given the set of parameters $\bar{\alpha}_i$. Monte Carlo simulation is used to validate the
The yields are rewritten in terms of the sum $\theta$, the asymmetry, and $\Delta$. Equal branching fractions for the $K^0 \rightarrow K^+\pi^0$ and $K^0 \rightarrow K^-\pi^0$ terms, the yields are fitted terms of the sum $n_f + n_j$ and the asymmetry $A = (n_f - n_j)/(n_f + n_j)$, where $n_f$ ($n_j$) is the fitted number of events in the mode $B \rightarrow f$ ($\bar{f}$). The numbers of events, $N$, entering the maximum likelihood fit for each topology are 16032 ($h^+h^-$), 16452 ($h^+\pi^0$), 3623 ($K^0_s h^+$), and 1503 ($K^0_s \pi^0$).

The parameters for background $m_{ES}$ and $\Delta E$ PDFs are determined from events in on-resonance $\Delta E$ sideband regions. The signal $m_{ES}$ and $\Delta E$ PDF parameters are determined from fully reconstructed $B^+ \rightarrow D^0\pi^+$ and $B^+ \rightarrow D^0\rho^+(\rho^+ \rightarrow \pi^+\pi^0)$ decays. Events in on-resonance $m_{ES}$ sideband regions and Monte Carlo simulated signal decays are used to parameterize the Fisher discriminant PDFs for background and signal, respectively (see Fig. 1(a)). Alternative parameterizations obtained from off-resonance data and Monte Carlo simulation are used as cross-checks and for determination of systematic uncertainties. The $\theta_c$ PDFs are derived from kaon and pion tracks in the momentum range of interest from approximately 42 000 $D^{*+} \rightarrow D^0\pi^+$ ($D^0 \rightarrow K^-\pi^+$) decays. This control sample is used to parameterize the $\theta_c$ resolution $\sigma_{\theta_c}$ as a function of track polar angle. The resulting $K^-\pi$ separation, defined as $|\theta_c^K - \theta_c^\pi|/\sigma_{\theta_c}$, where $\theta_c^K$ ($\theta_c^\pi$) is the expected Cherenkov angle for a kaon (pion), is shown as a function of momentum in Fig. 1(b).

The results of the fit are summarized in Table I, where the statistical error for each mode corresponds to a 68% confidence interval and is given by the change in signal yield $n_i$ that corresponds to a $-2\ln L$ increase of one unit. Signal significance is defined as the square root of the change in $-2\ln L$ with the corresponding signal yield fixed to zero. For the three modes that have statistical significance less than $4\sigma$ we report Bayesian 90% confidence level upper limits. In addition, for the purpose of combining with measurements from other experiments, we report the branching fractions corresponding to the fitted signal yields: $B(B^+ \rightarrow \pi^+\pi^0) = (5.1^{+2.0}_{-1.8} \pm 0.8) \times 10^{-6}$, $B(B^0 \rightarrow K^+K^-) = (0.85^{+0.66}_{-0.37} \pm 0.37) \times 10^{-6}$ and $B(B^+ \rightarrow K^0K^+) = (-1.3^{+1.4}_{-0.7} \pm 0.7) \times 10^{-6}$. The upper limit on the signal yield for mode $i$ is given by the value of $n_i^0$ for which $\sum_{i=0}^{n_0} L_{\text{max}} d n_i/\int_0^{\infty} L_{\text{max}} d n_i = 0.90$, where $L_{\text{max}}$ is the likelihood as a function of $n_i$, maximized with respect to the remaining fit parameters. Branching fraction upper limits are calculated by increasing the signal yield upper limit and reducing the efficiency by their respective systematic errors.

Figure 2 shows the distributions in $m_{ES}$ and $\Delta E$ for events passing the selection criteria, as well as requirements on likelihood ratios, which are used to increase the relative fraction of signal events of a given type. These likelihood ratios are defined for a given topology as $R_{\text{sig}} = \sum_i n_i P_i/\sum_i n_i P_0$ and $R_k = n_k P_k/\sum_i n_i P_i$, where $\sum_i$ denotes the sum over the probabilities for signal hypotheses only, $\sum_i$ denotes the sum over all the probabilities (signal and background), and $P_k$ denotes the probability for signal hypothesis $k$. These probabilities are constructed from all the PDFs except that describing the displayed variable. The likelihood fit projections, scaled by the relative efficiencies for the likelihood ratio requirements, are overlaid on each distribution.

Systematic uncertainties arise from: imperfect knowledge of the PDF shapes, uncertainties in the detection efficiencies, and potential charge bias in track reconstruction and particle identification. Uncertainties in the PDF shapes affect both branching fraction and charge asymmetry measurements.

For most of the branching fraction measurements, the PDF shapes contribute the largest systematic error. The exception is the $B^+ \rightarrow K^+\pi^0$ mode, where the largest systematic error is due to the 5% uncertainty in the $\pi^0$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\varepsilon$ (%)</th>
<th>$N_S$</th>
<th>$S(\sigma)$</th>
<th>$B(10^{-6})$</th>
<th>$A$</th>
<th>$A$ 90% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>45</td>
<td>$41 \pm 10 \pm 7$</td>
<td>4.7</td>
<td>$4.1 \pm 1.0 \pm 0.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^+\pi^-$</td>
<td>45</td>
<td>$169 \pm 17 \pm 13$</td>
<td>15.8</td>
<td>$16.7 \pm 1.6 \pm 1.3$</td>
<td>$-0.19 \pm 0.10 \pm 0.03$</td>
<td>$[-0.35, -0.03]$</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>43</td>
<td>$8.2^{+7.8}_{-6.4} \pm 3.5$</td>
<td>1.3</td>
<td>$&lt; 2.5$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^-\pi^0$</td>
<td>32</td>
<td>$37 \pm 14 \pm 6$</td>
<td>3.4</td>
<td>$&lt; 9.6$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^+\pi^0$</td>
<td>31</td>
<td>$75 \pm 14 \pm 7$</td>
<td>8.0</td>
<td>$10.8^{+2.1}_{-1.5} \pm 1.0$</td>
<td>$0.00 \pm 0.18 \pm 0.04$</td>
<td>$[-0.30, +0.30]$</td>
</tr>
<tr>
<td>$K^0\pi^+$</td>
<td>14</td>
<td>$59^{+11}_{-10} \pm 6$</td>
<td>9.8</td>
<td>$18.2^{+3.3}_{-3.0} \pm 2.0$</td>
<td>$-0.21 \pm 0.18 \pm 0.03$</td>
<td>$[-0.51, +0.09]$</td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>14</td>
<td>$-4.1^{+4.5}_{-3.8} \pm 2.3$</td>
<td></td>
<td>$&lt; 2.4$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>10</td>
<td>$17.9^{+6.8}_{-5.8} \pm 1.9$</td>
<td>4.5</td>
<td>$8.2^{+3.1}_{-2.7} \pm 1.2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
reconstruction efficiency. PDF systematic errors are estimated either by varying the PDF parameters within 1σ of their measured uncertainties or by substituting alternative PDFs from independent control samples. The systematic errors in the signal yields due to PDF uncertainties depend on decay mode as shown in Table I.

The $D^{*+}$ control sample of kaon and pion tracks is used to estimate systematic uncertainties in the asymmetries arising from possible charge biases in the $\theta_c$ quality requirements, as well as from differences in $\theta_c$ reconstruction for different charge species. From these studies we conservatively assign a systematic uncertainty of ±0.01 on $A$ for all the modes. Charge biases in the detector and track reconstruction chain are studied in high statistics samples of charged tracks in multihadron events. These studies show differences in reconstruction efficiencies for positively and negatively charged tracks of less than 0.005. We assign an overall systematic uncertainty of ±0.01 on $A$ for possible charge-correlated biases in track reconstruction and particle identification. All measured background asymmetries are consistent with zero with statistical uncertainties less than 0.03. The fitted signal yields and asymmetries for off-resonance data and on-resonance $\Delta E$ sidebands are also consistent with zero.

The overall systematic errors on the branching fractions and charge asymmetry measurements are computed by adding in quadrature the PDF systematic uncertainties and the systematic uncertainties on the efficiencies or due to possible charge biases, respectively.

In summary, we have measured branching fractions for the rare charmless decays $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow K^+\pi^-$, $B^+ \rightarrow K^+\pi^0$, $B^+ \rightarrow K_0^0\pi^+$, and $B^0 \rightarrow K^0\pi^0$, and set upper limits on $B^0 \rightarrow K^+K^-$, $B^+ \rightarrow \pi^+\pi^0$, and $B^+ \rightarrow K^0K^+$. We find no evidence for direct $CP$ violation in the observed decays and set 90% C.L. intervals. These measurements are in good agreement with previous results [6, 7].

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from the Swiss National Science Foundation, the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

---

* Also with Università di Perugia, Perugia, Italy.
† Also with Università della Basilicata, Potenza, Italy.
[5] Charge conjugate states are assumed throughout, except where explicitly noted.