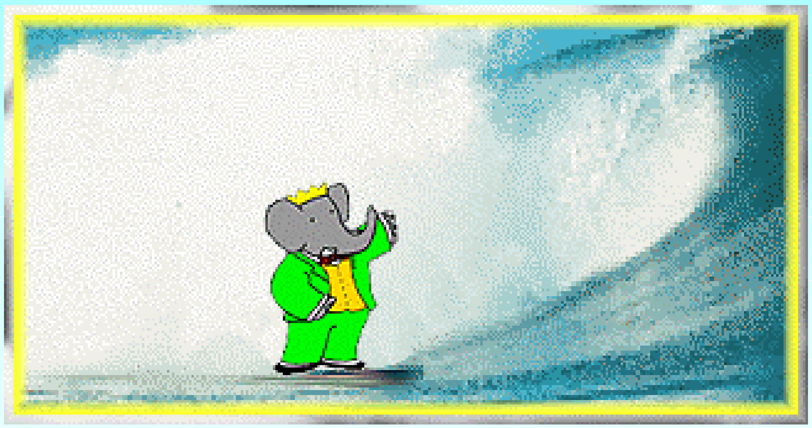
Tracking and Fitting

High Energy Physics



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BaBar Detector Physics Series

November 19, 1999

Outline

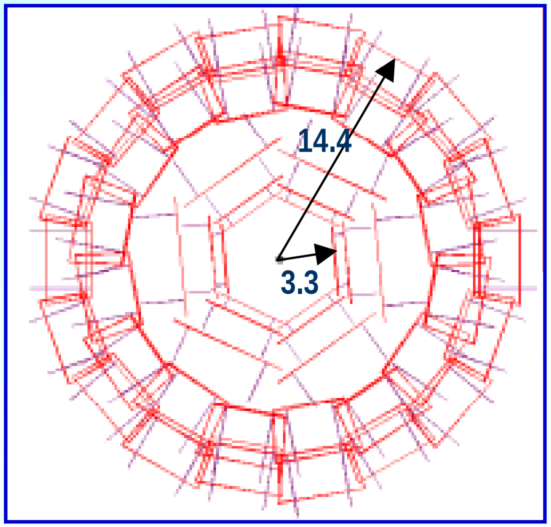
- BaBar tracking devices: SVT and DCH
- Track finding
  - SVT stand-alone
  - DCH stand-alone
  - An alternative approach
- Track fitting
  - Method of maximum likelihood
  - Kalman filtering
- Constrained fitting
  - vertexing

What Exactly IS Tracking?

- Tracking as such consists of two parts:
  - Track finding (pattern recognition)
  - Track fitting
    - really comes down to minimizing χ^2, which is a quantity that measures how close the measured parameters are to what they are assumed to be from a particular fit hypothesis (e.g., helical trajectory)
    - fitting would be trivial if it was not for complications arising because of multiple scattering, energy loss, non-uniform magnetic field, etc.., and if we understood our detectors perfectly.

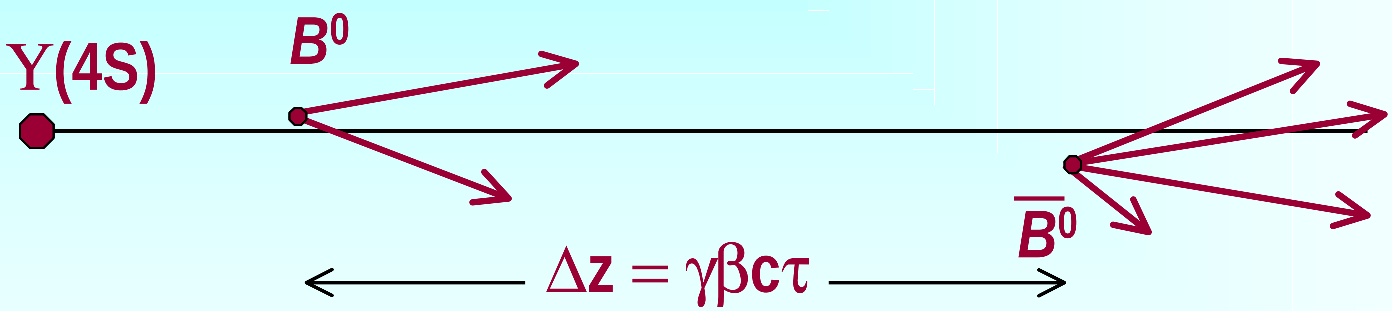
BaBar Silicon Vertex Detector

- 5 layers of double-sided silicon microstrip detectors
- ~0.94 m^2 of silicon
- ~150,000 electronics readout channels



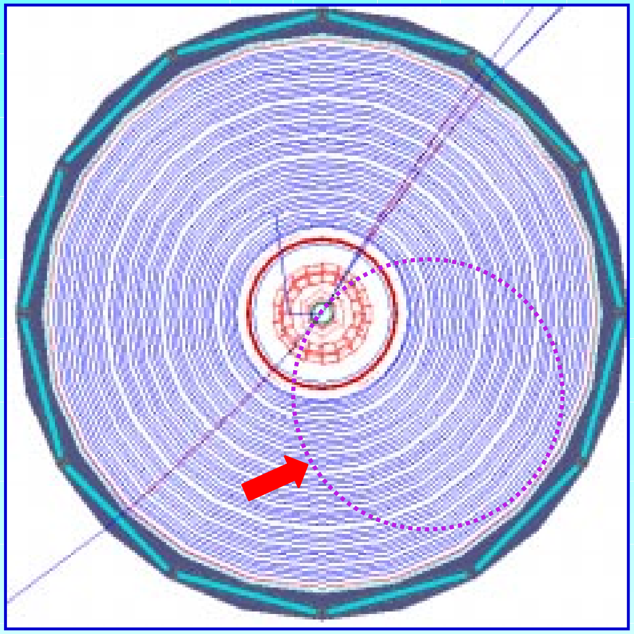
Main Goal of the Vertex Detector

- Good vertexing is crucial for many analyses, and indispensable for CP violation studies.
- The main purpose of the BaBar vertex detector is to determine the separation between the two *B* decay vertices along the z axis:



- However, it is not its only function!

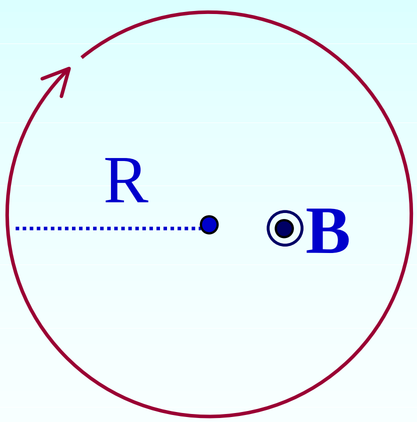
Tracks in SVT and DCH



- For a charged particle in a uniform magnetic field **B**:

$$|P_T|\ (\mathrm{GeV}/c) = 0.3\ |\mathbf{B}|(\mathrm{T})\ R(\mathrm{m})$$

where P_T = particle momentum component in a plane ⊥ to **B**, and R is the trajectory radius



Why 5 layers?

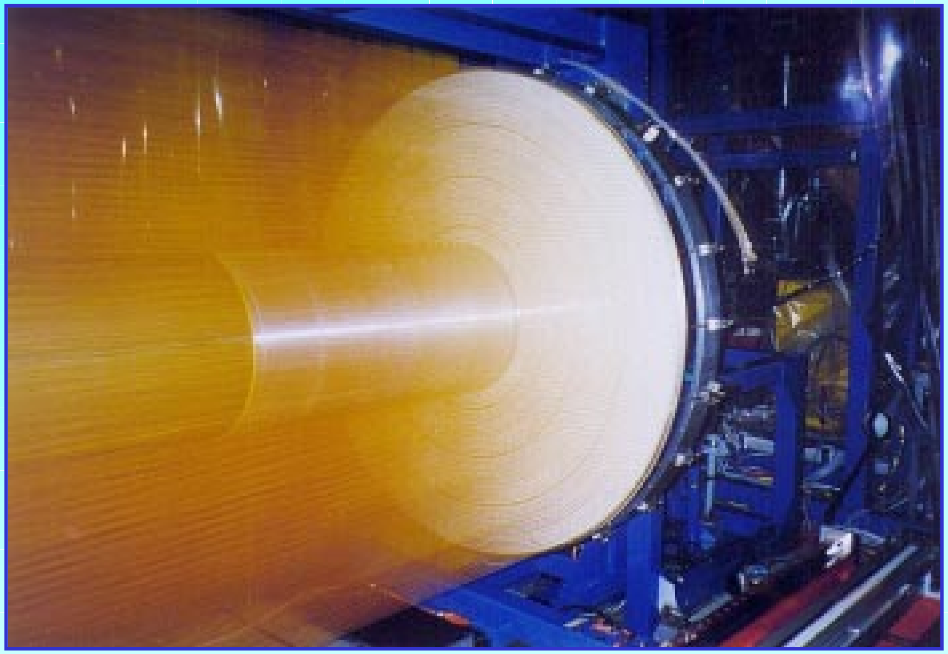
- Compare the following parameters for the three new *B*-factory detectors (all operating in a 1.5 T **B**-field):

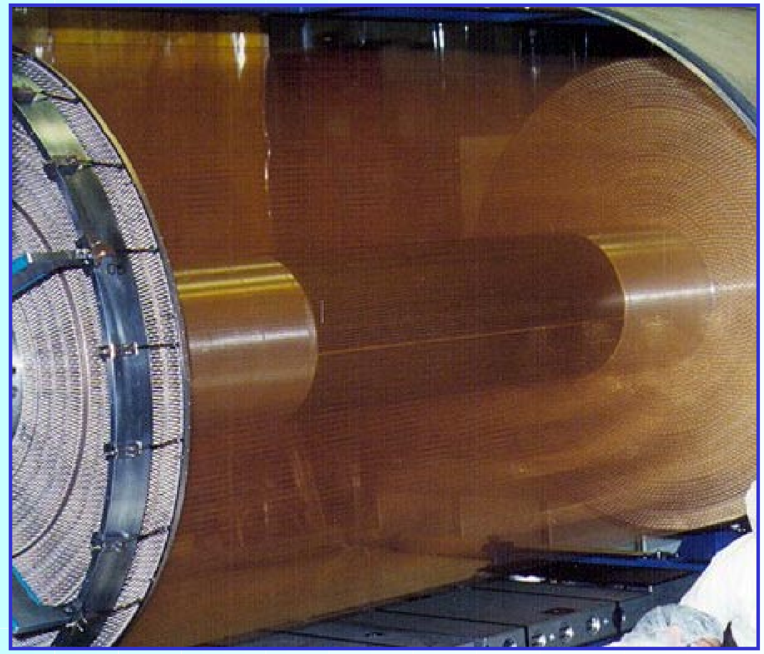
Experiment	# layers in SVT	Inner radius of DCH	Minimum P_T at inner radius of DCH
CLEO-III	4	17.5 cm	39 MeV/c
Belle	3	8.5 cm	19 MeV/c
BaBar	5	22.5 cm	51 MeV/c

courtesy Doug Roberts

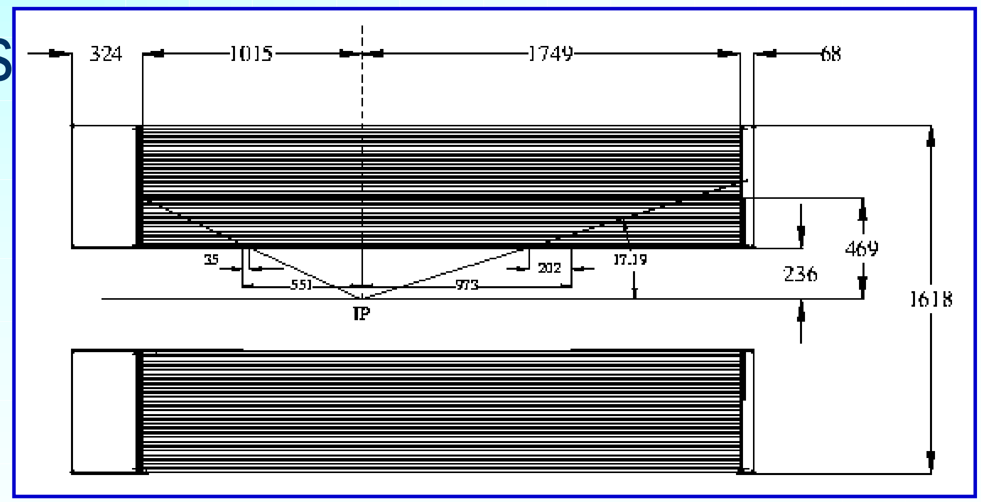
- BaBar SVT must not only do vertexing, but also perform tracking for low momentum tracks (up to $P_T \sim 120$ MeV/c).

BaBar Drift Chamber



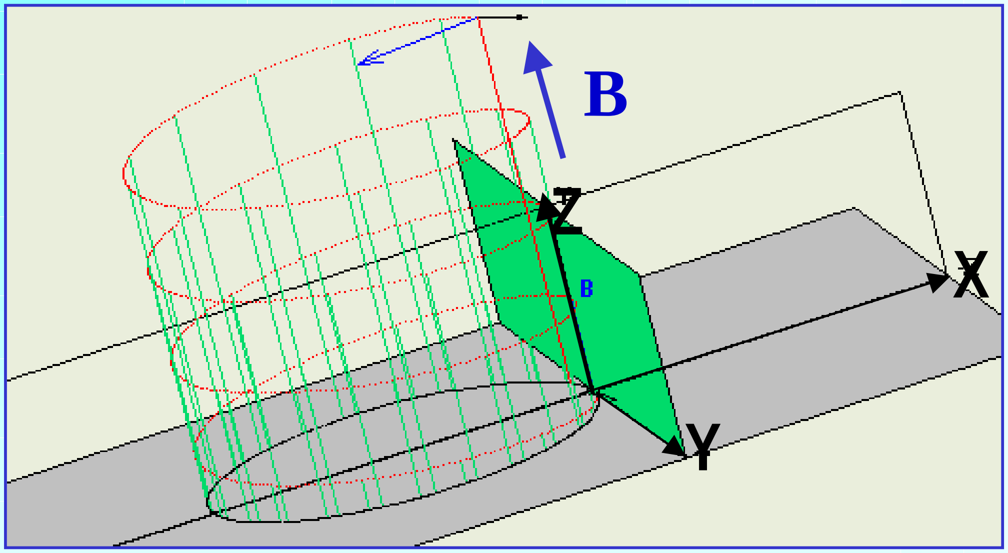


- 40 layers with 7,104 drift cells
- Layers organized into 10 superlayers, following the pattern AUVAUVAUVA *(axial (A)-stereo(U,V))*



Particle in a Uniform B-field

- Recall that a charged particle in a uniform magnetic field moves along a helix.

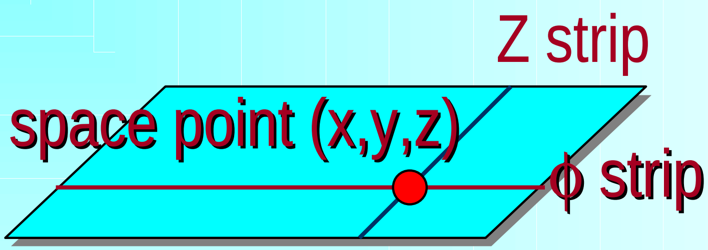


B || z

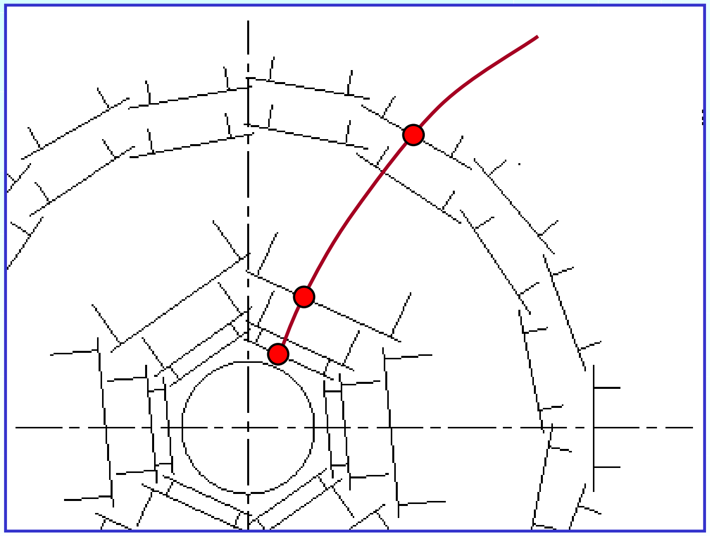
- But this can be decoupled into
  - moving along a circle in the xy-plane (need 3 points to define it)
  - and moving along a straight line in z (need 2 points to define it)

Track Finding in the SVT

- SVT stand-alone pattern recognition:
  - form space points from matching ϕ and Z hits:

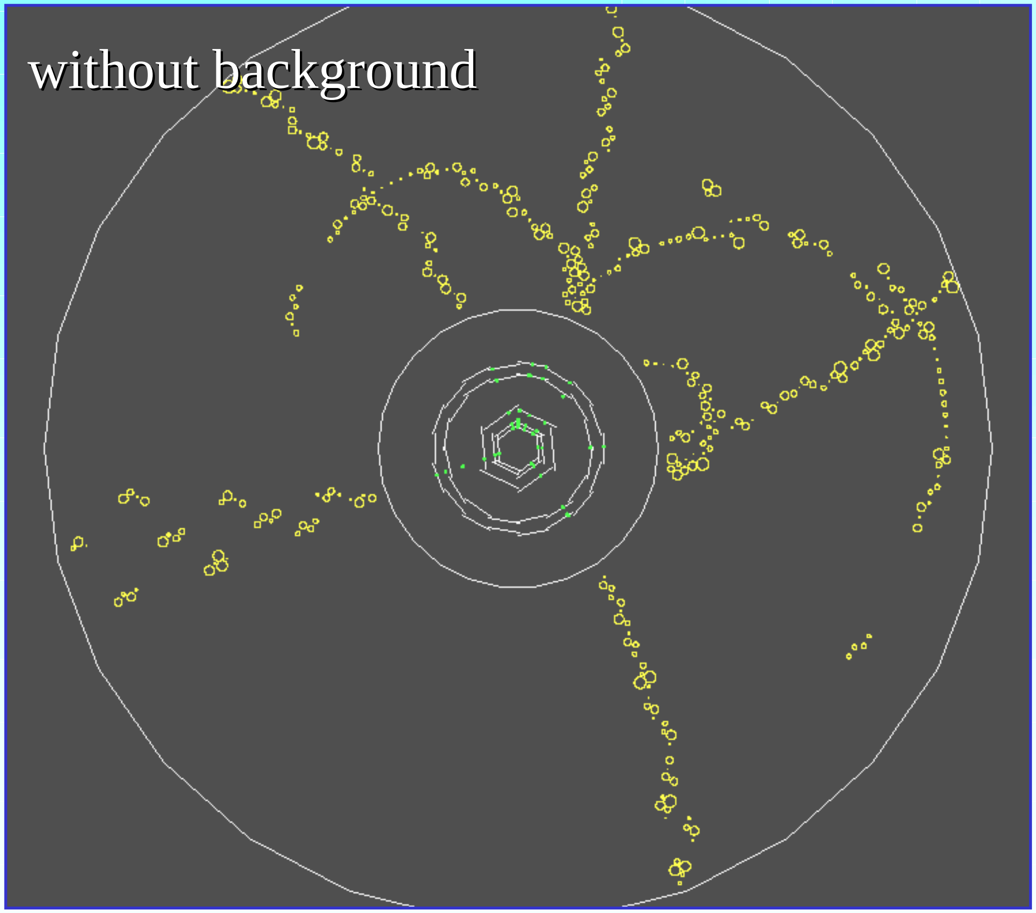


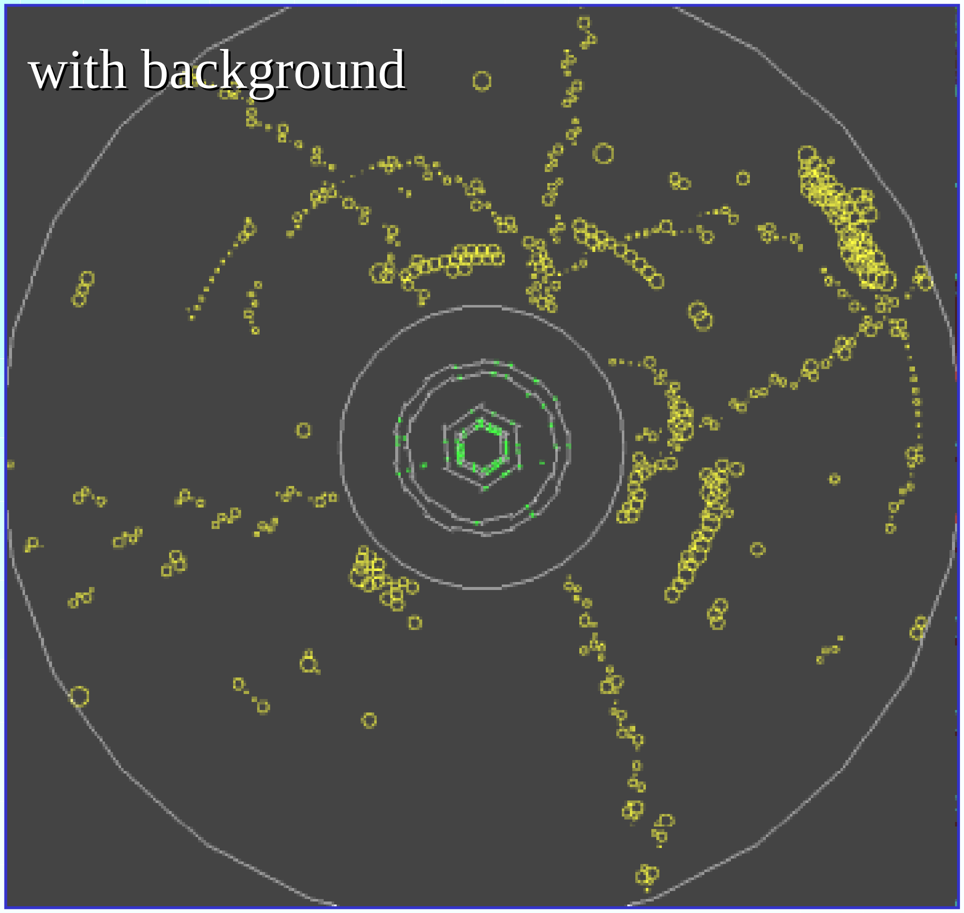
  - find 3 space points on different layers that might form a track:



Pattern recognition

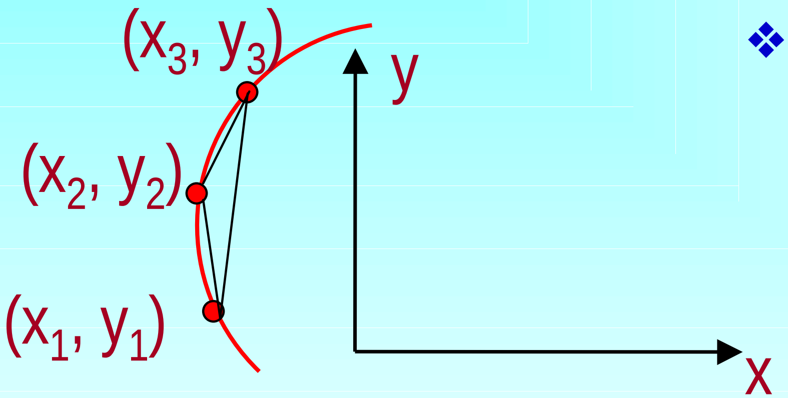
- Pattern recognition is sometimes easier for the eye than for the computer...





Track Fitting in the SVT

- Once three suitable points are found, it is possible to determine the r-ϕ parameters of a circular track

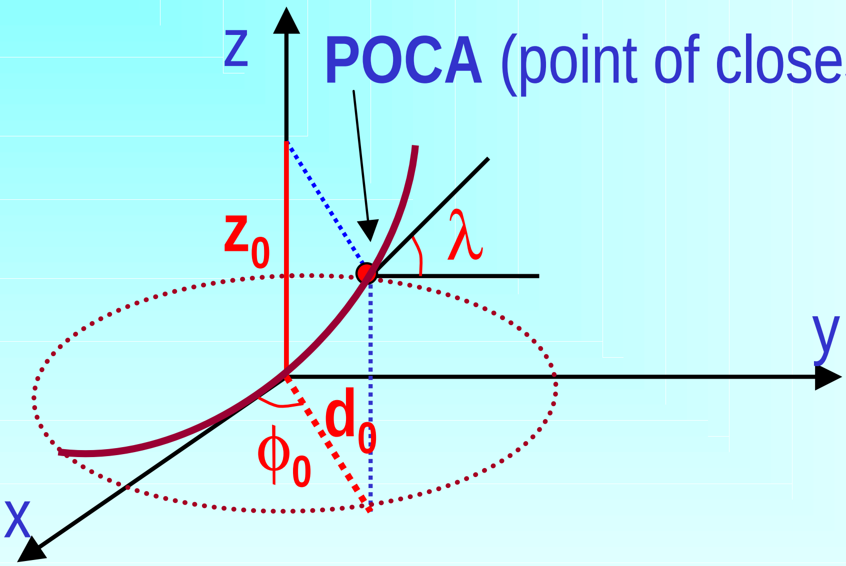


- The points are required:
  - to be close enough together azimuthally
  - to have consistent times
  - to lie in a "road" defined as **a + b/p_T**
  (2nd term accounts for multiple scattering)

- Then add z information (for at least two hits) to the circle… and turn it into a helix!

Track Fitting in the SVT (cont.)

- At the end, we end up with tracks fit to helices with 5 parameters: **(d_0 (cm), ϕ_0(rad), ω(cm^{-1}), z_0(cm), tanλ)**



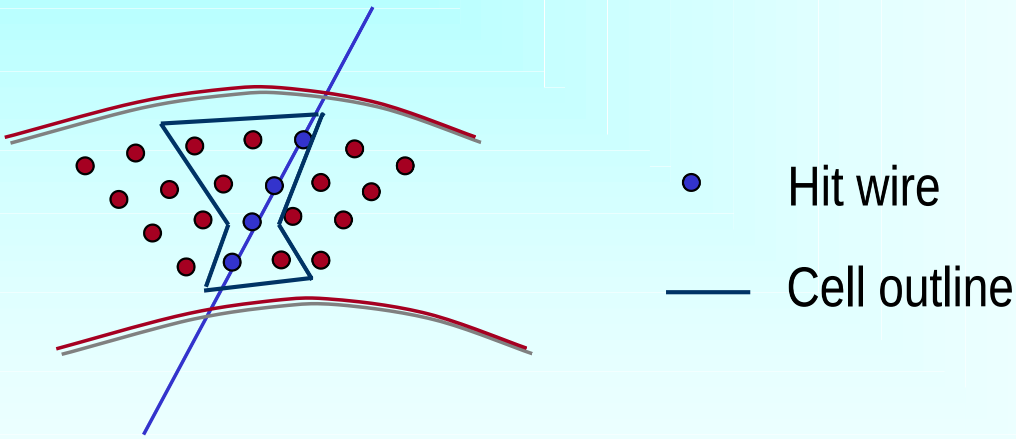
ω = *geometrical* curvature

r = *radius* of curvature

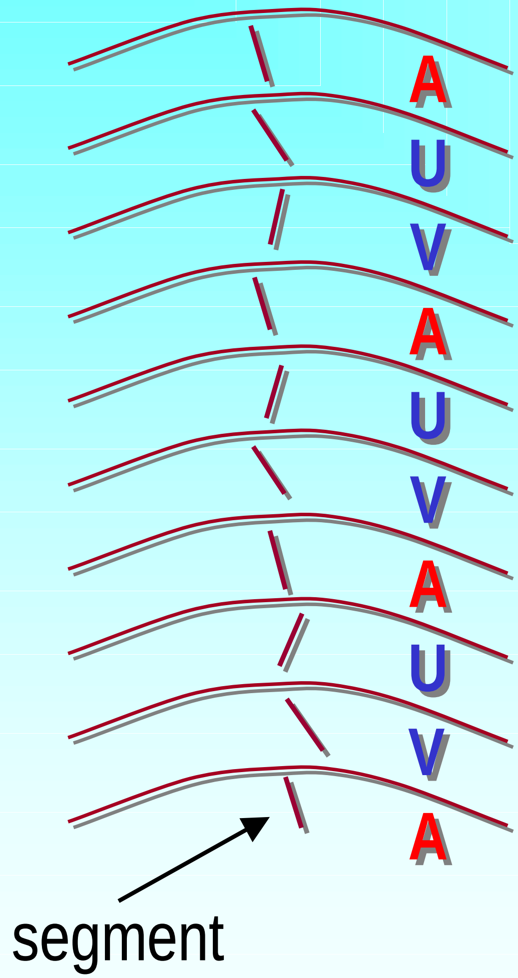
$$r = \frac{1}{\omega} = -\frac{10^{13} p_T}{q B_z c} = -\frac{333.567 p_T}{q B_z}$$

Track Finding in the DCH

- Tracking in the DCH is based on **segments**
- Segment finder looks for *patterns* of hit wires and calls the valid ones segments.
- Here is an example of a valid pattern:



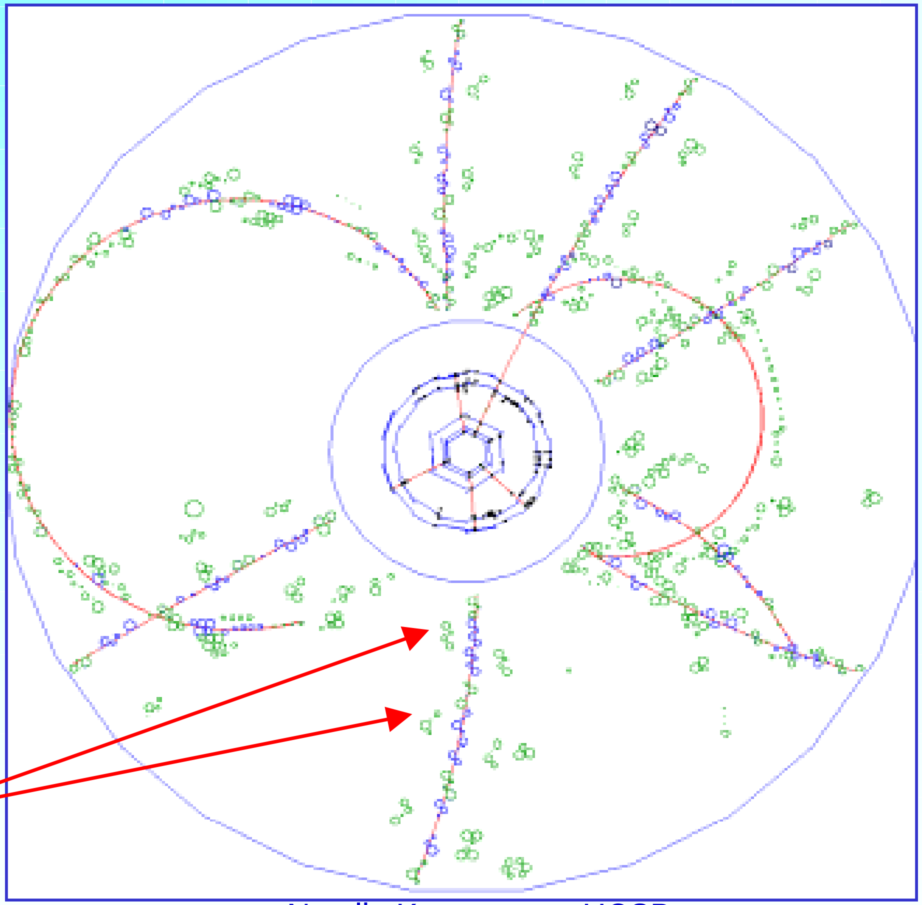
Stereo/Axial Layers in the DCH



- Three segments in the **axial** layers are used to form a circle in the xy plane
- Then z-measurements from the **stereo** layers are added… and we have a helix!

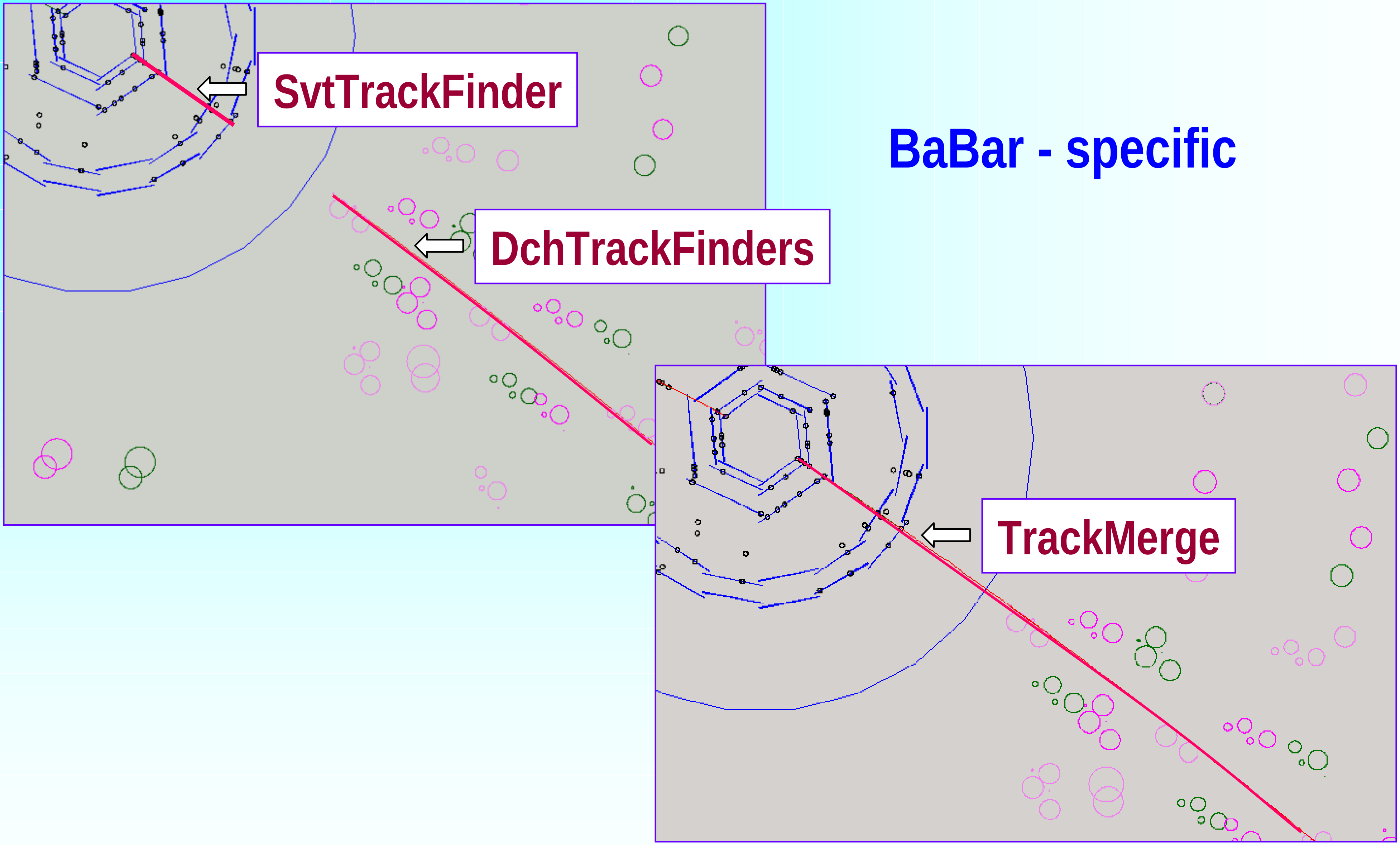
BaBar DCH Track Finders

- There are two independent segment-based track finders run in series
  - DchTrackFinder for tracks coming radially from origin ($|d_0| < 1$cm)
  - DcxTrackFinder for all tracks



segments

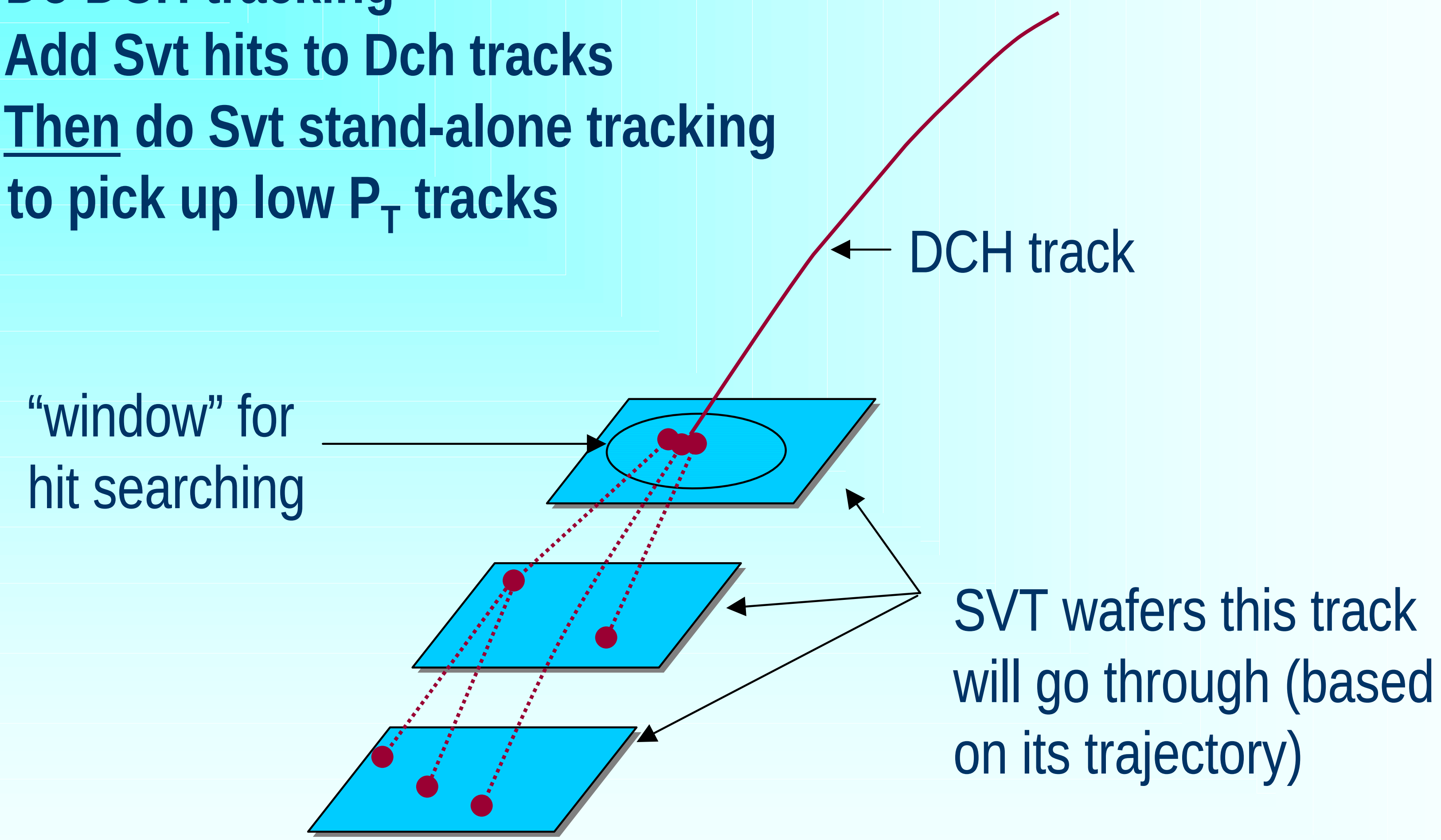
Combining SVT and DCH Tracks



BaBar - specific

An Alternative Approach

- Do DCH tracking
- Add Svt hits to Dch tracks
- Then do Svt stand-alone tracking to pick up low P_T tracks



What Defines This Window?

- One can show that:
  - if we have **n** measuring planes extending to max radius **R**
  - each with intrinsic spatial resolution ε
  - and evenly spaced
- then the track parameter resolutions will be:

$$\sigma_{d0} = \sqrt{\frac{9}{n}}\varepsilon \qquad \sigma_{\phi 0} = \sqrt{\frac{192}{n}}\frac{\varepsilon}{R} \qquad \sigma_{z0} = \sqrt{\frac{4}{n}}\frac{\varepsilon}{\mu}$$

stereo angle (μ)

- so, for **n = 40**, **ε = 140 μm**, **R = 81 cm**, **μ = 50 mrad**, we get:

  σ_{d0} = 66 μm, $\sigma_{\phi 0}$ = 0.4 mrad, σ_{z0} = 885 μm

What about the SVT?

- The SVT gives much better resolution on d_0, z_0, $\tan\lambda$!
  - The intrinsic resolutions are of the order of 10-15 μm in xy plane, and 15-30 μm in z.
  - The track parameter resolutions are dominated by multiple scattering, not by intrinsic resolution, for most of the momentum range.
  - at some **r** $(=x^2+y^2)$:

$$\sin(\phi - \phi_0) = \frac{(b/r)(1+\omega d_0) + \omega r}{1 + 2\omega d_0}$$

➨ For angles, need good resolution on the momentum, which comes (mostly) from the DCH!

Momentum Resolution in the DCH

- For many (n>10) position measurements, the curvature resolution is:

$$\sigma_{\omega} = \frac{\varepsilon}{L'^{2}} \sqrt{\frac{720}{n+4}}$$

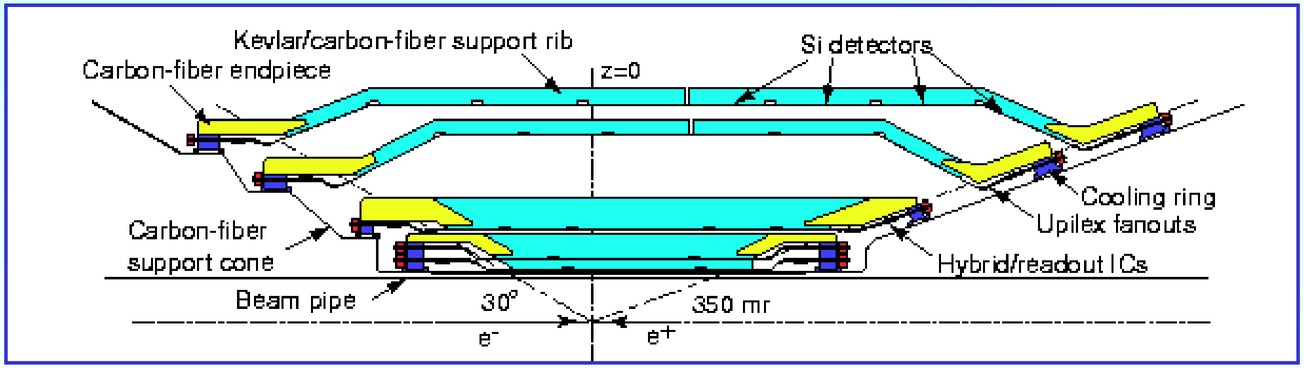
projected length of track onto bending plane

- Assuming intrinsic resolution **ε = 140 μm**, **n = 40** measurements, and **L' = 52.2 cm**, we get

$$\sigma(P_T)/P_T = 0.46\%\ P_T$$

The more measurements the better?

- One can see that the resolutions ~ $1/\sqrt{n}$
- Therefore, the more measurements one makes the better?
  - Not always! If there is significant **multiple scattering**, adding measurements may *degrade* the resolution on some track parameters.
    - e.g., when one fits for the impact parameters, the **innermost** precision measurement from the SVT provides most useful information



Track Parameter Resolution

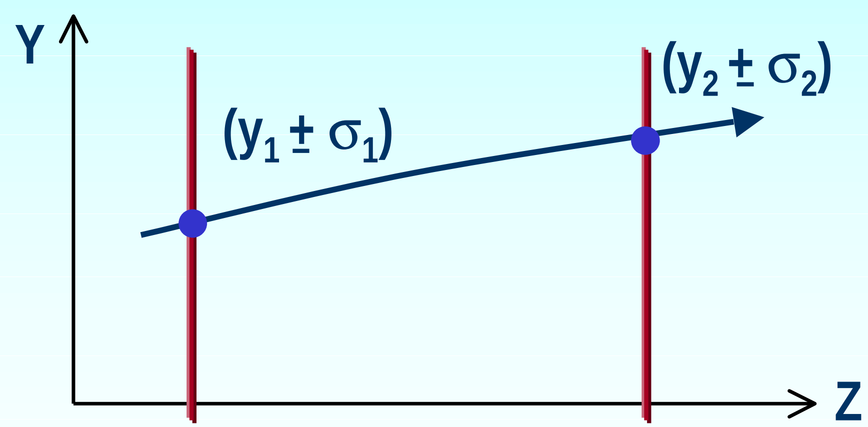
- The DCH information dominates the momentum resolution
- The SVT information dominates the impact parameter resolution, both in xy and z directions.

- But how exactly does one obtain track parameter resolutions?
  - = How does fitting really work?

Least Square Track Fitting: an Example

- In order to start fitting a track, one needs two things:
  - a **model** which approximates the track's trajectory
  - an understanding of the **detector accuracy** (resolution)

- Let's consider an example: two measurements to be fit to a straight line



Assume that the two points can be fit to

$$f_i = a + z_i b, \quad i = 1,2$$

Some General Considerations

- Suppose we have **n** data values y_l **(l=1,..,n)**
  - ➟ assume they are functions of **m** variables α_i **(m ⩽ n):**

$$y_l = f_l(\alpha_i)$$

  - ➟ assume each measurement y_l has a Gaussian measurement error σ_l .

- Then the probability density for the measurements is:

$$g(y_1, y_2, \ldots.\, y_n) = \prod_l \frac{1}{\sqrt{2\pi\sigma_l}} \left(e^{-\sum_l (y_l - f_l)^2 / 2\sigma_l^2} \right)$$

$$\equiv \prod_l \frac{1}{\sqrt{2\pi\sigma_l}} \left(e^{-\sum_l \chi^2} \right), \text{ where } \chi^2 \equiv \sum_l \frac{(y_l - f_l(\alpha))^2}{\sigma_l^2}$$

➡ maximazing the probability density = minimizing the χ^2

Fitting to a line

- We need to minimize

$$\chi^2 \equiv \frac{(y_1 - f_1(a,b))^2}{\sigma_1^2} + \frac{(y_2 - f_2(a,b))^2}{\sigma_2^2} = \frac{(y_1 - a - bz_1)^2}{\sigma_1^2} + \frac{(y_2 - a - bz_2)^2}{\sigma_2^2}$$

- So:

$$\frac{\partial \chi^2}{\partial a} = \frac{y_1 - a - bz_1}{\sigma_1^2}(-2) + \frac{y_2 - a - bz_2}{\sigma_2^2}(-2) = 0$$

$$\frac{\partial \chi^2}{\partial b} = \frac{y_1 - a - bz_1}{\sigma_1^2}(-2z_1) + \frac{y_2 - a - bz_2}{\sigma_2^2}(-2z_2) = 0$$

Two equations with two unknowns (a, b) give a unique solution:

$b = (y_1 - y_2)/(z_1 - z_2)$

$a = (z_1 * y_2 - y_1 * z_2)/(z_1 - z_2)$

(who would have thought?!)

Generalization

- n <u>independent</u> measurements **Y** which we would like to fit to function **F** with ***l*** parameters Θ:

$$Y = \begin{pmatrix} y_1 \\ \dots \\ y_n \end{pmatrix} \quad F = \begin{pmatrix} f_1 \\ \dots \\ f_n \end{pmatrix} \quad \Theta = \begin{pmatrix} \vartheta_1 \\ \dots \\ \vartheta_l \end{pmatrix}$$

measurements (y_i) **predictions (f_i)** **parameters (a,b)**

$$V \equiv \langle \delta y_i \delta y_j \rangle = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \dots & \\ 0 & & \sigma_n^2 \end{pmatrix} \quad A = \begin{pmatrix} a_{11} & & a_{1l} \\ & \dots & \\ a_{n1} & & a_{nl} \end{pmatrix}$$

covariance or error matrix **coefficients (z_i, 1)**

- The solution is: $\Theta = (A^T V^{-1} A)^{-1} A^T V^{-1} \mathbf{Y}$ ➡ need $\mathbf{V^{-1}}$!

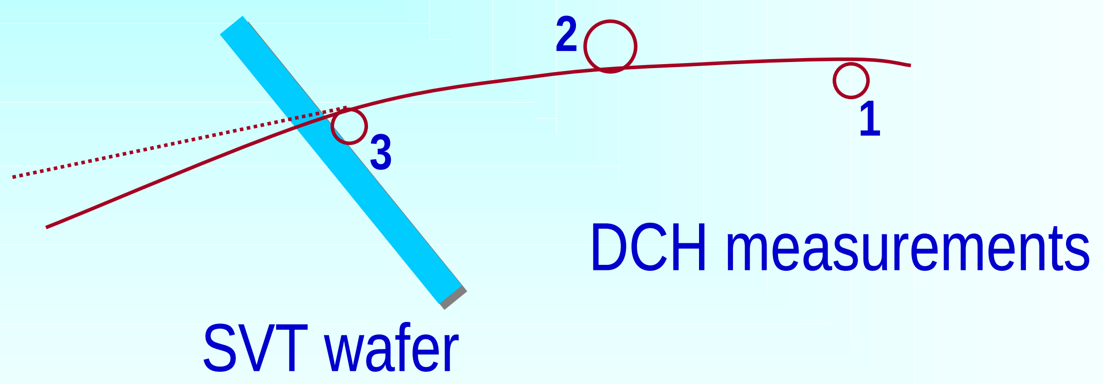
Problems With This Approach

- This method is **global** in the sense that it fits all the measurements at the same time
- If all measurements are independent of each other, the execution time is ~ n (the # of measurements)

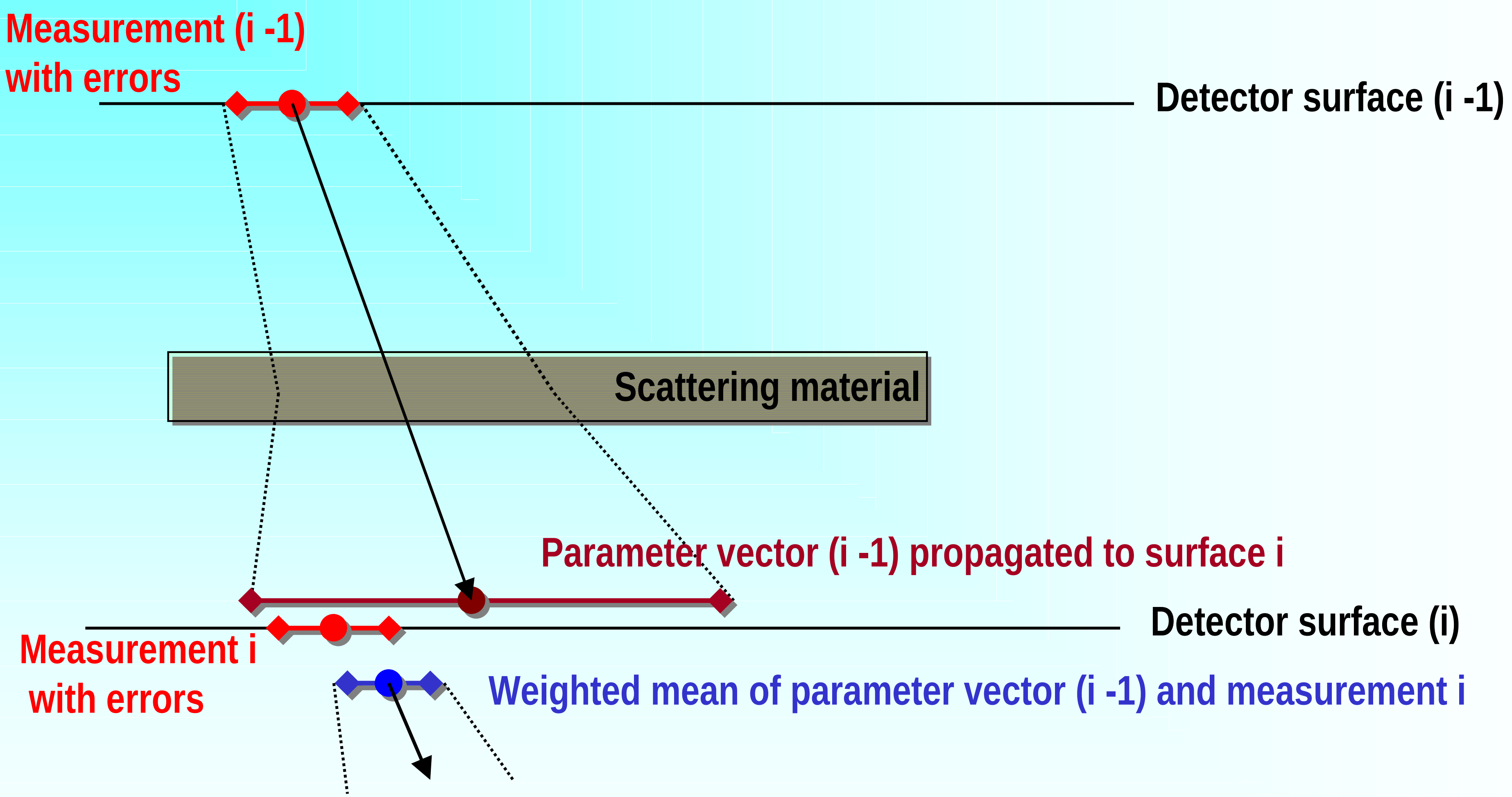
- But what if we have correlations between measurements?
  - the covariance matrix will contain non-diagonal terms
  - and inverting it becomes VERY time consuming for large n
    - ➤ e.g., a track has 40 hits in the DCH and 5 in the SVT
    - ➤ that's 40 + 2*5 (2 views in SVT) + 2*20 (~20 pieces of material) = 90!
    - ➤ inverting a 90 x 90 matrix is no fun!

The Solution: Kalman Filtering

- The idea:
  - estimate track parameters at every given point using previously obtained information + guess about the contributions from various physical processes (multiple scattering, energy loss,…)



Kalman Filter

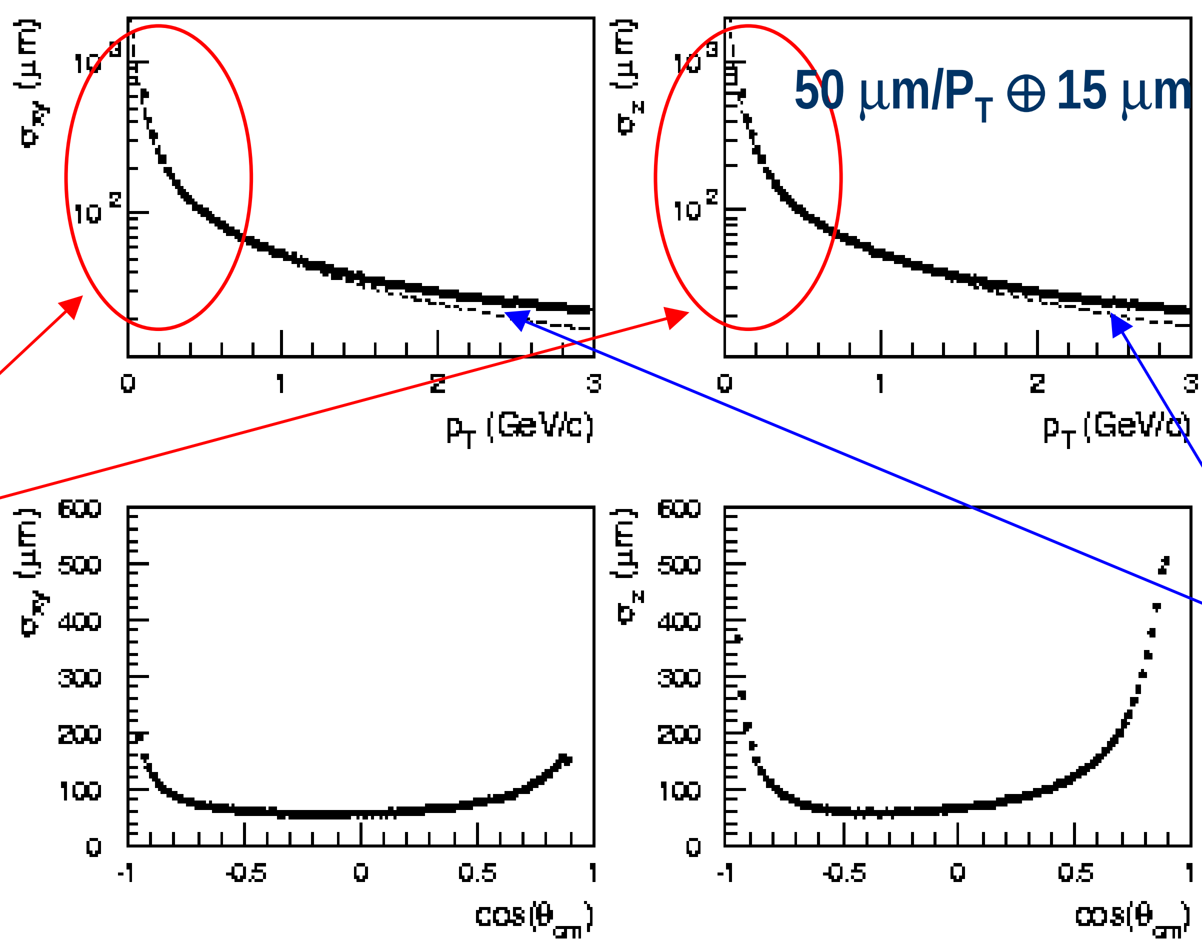


Kalman Filter (cont.)

- Kalman fitting is really about taking weighted averages!
- It's the simplest way to get the track errors right
  - with global fits, the errors cannot always be trusted
    - e.g., multiple scattering is most often ignored for speed...
    - ...and recall that in BaBar, the particles come from B's decaying almost at rest in the CM frame, so they have low momenta -- and multiple scattering becomes a very serious effect!

Track Parameter Resolutions

Kalman-fit tracks



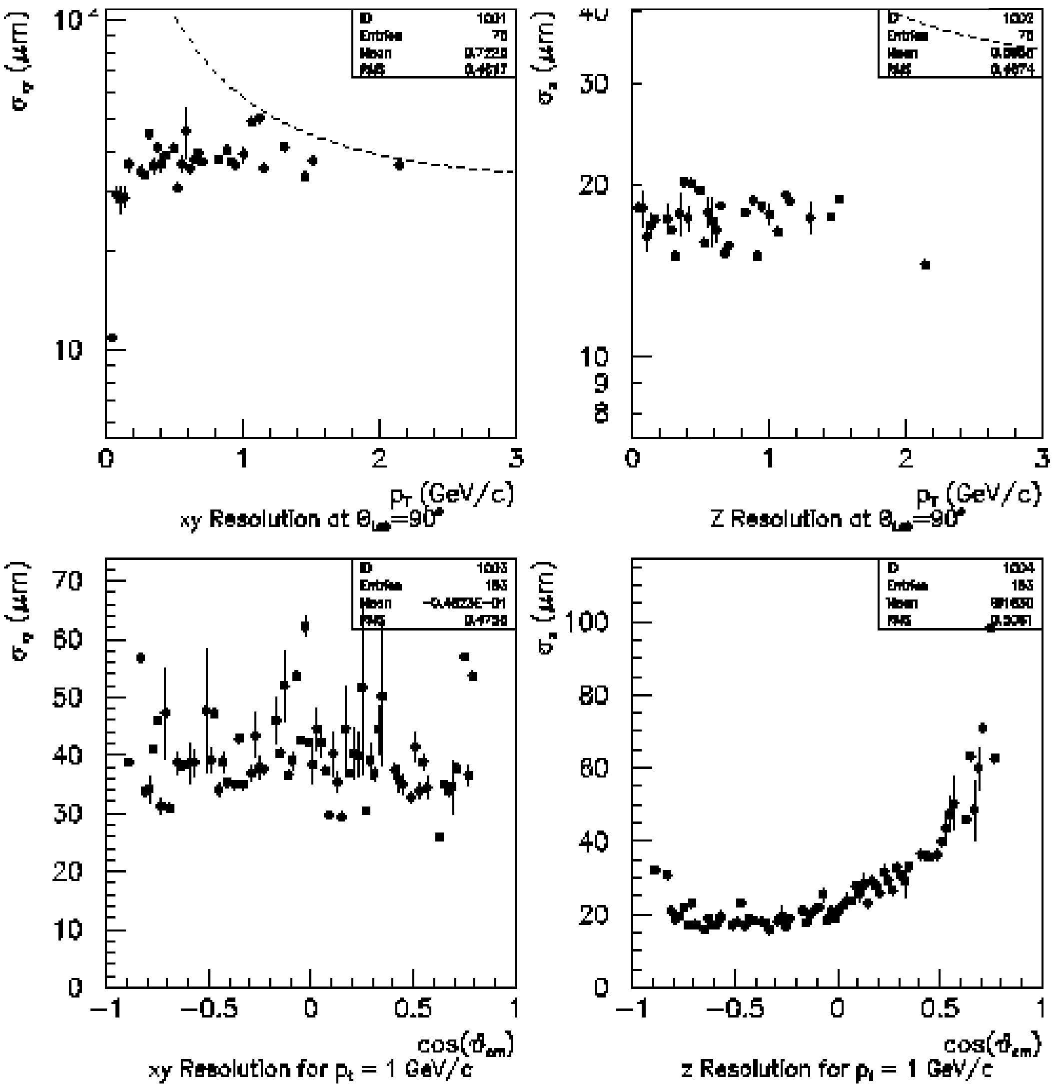
courtesy Doug Roberts

multiple scattering degrades resolution!

intrinsic resolution becomes limiting factor

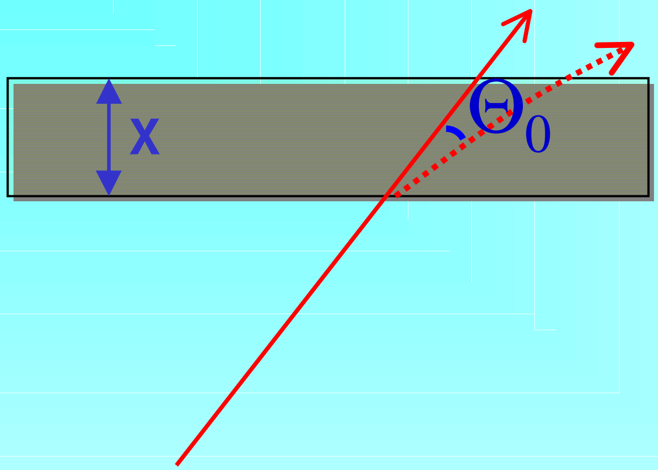
Track Parameter Resolution (cont.)

Simple helix-fit tracks



➡ **multiple scattering is ignored, the errors are wrong!!**

Multiple Scattering

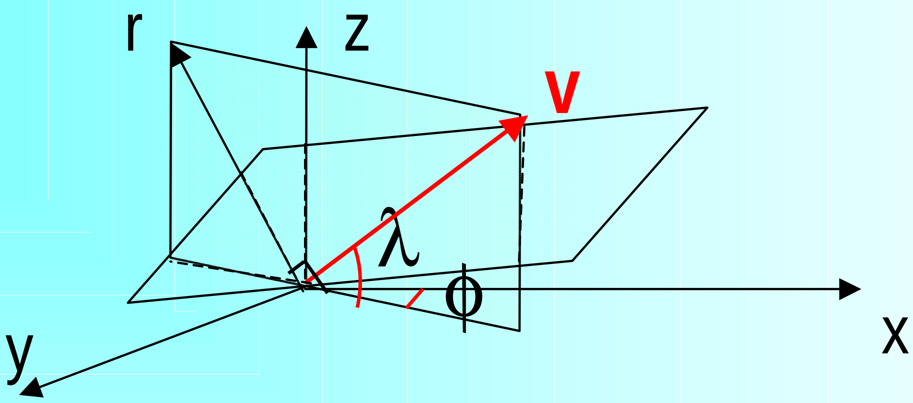


- Mostly due to Coulumb scattering from nuclei
- For small angles roughly Gaussian distribution:

$$\Theta_0 = \frac{13.6 MeV}{\beta c p} z \sqrt{x / X_0} \left[1 + 0.038 \ln(x / X_0) \right]$$

➤ *x/X_0 is the thickness of the scattering material in radiation lengths*

Multiple Scattering in Track Fitting



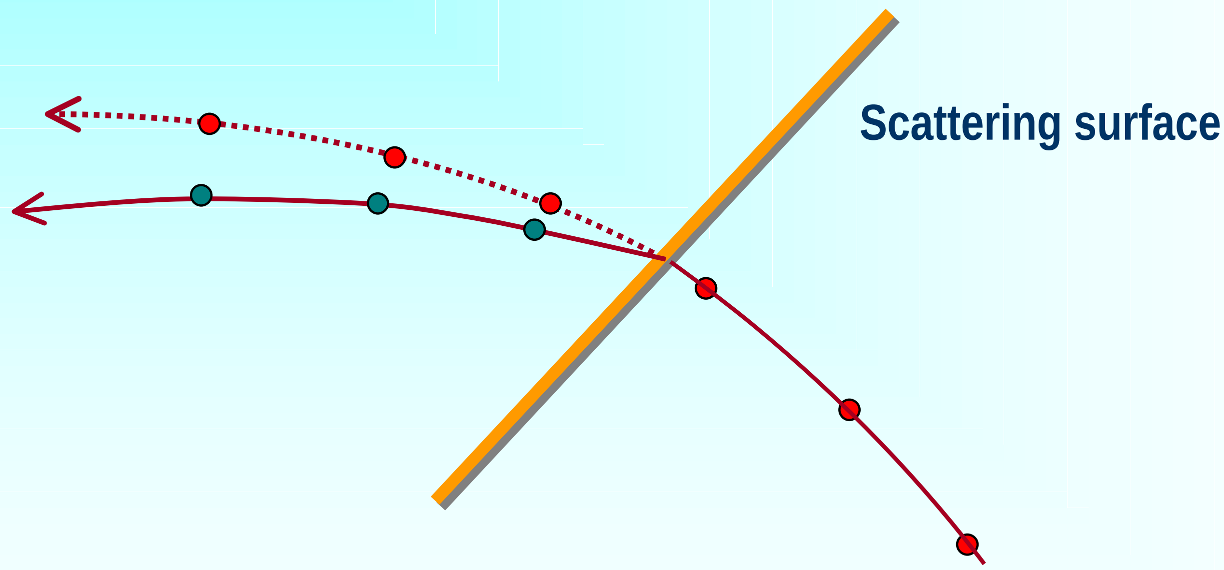
- Multiple scattering does not affect the track's momentum, nor the track's fit parameters
- It only affects the error matrix:

$$V_{\lambda\lambda} \rightarrow V_{\lambda\lambda} + \Theta_0^2 \qquad V_{\omega\lambda} \rightarrow V_{\omega\lambda} + \omega\, \Theta_0^2 \sin\lambda / \cos^3\lambda$$

$$V_{\phi\phi} \rightarrow V_{\phi\phi} + \Theta_0^2/\cos^2\lambda \qquad V_{\omega\omega} \rightarrow V_{\omega\omega} + \omega^2 \tan^2\lambda\, \Theta_0^2$$

Correlations

- Scatter in one point, and all points from then on get shifted, so in this sense they become **correlated**.
- This is reflected in the non-zero off-diagonal elements of the covariance matrix



Energy Loss

- The processes:
  - ionization described by Bethe-Bloch formula:

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

  - for *e*$^-$'s, *bremsstrahlung* is significant
- Basis of particle ID (see DCH talk)

Energy Loss in Track Fitting

- Energy loss affects the track parameters (momentum):

$$\omega' \rightarrow \omega\, p / \sqrt{p^2 + 2E\Delta E + \Delta E^2}$$

- It also affects the error matrix (process is not deterministic):

$$V_{\omega\omega} \rightarrow V_{\omega\omega} + \omega^2 E^2 \delta E^2 / P^4$$

Residuals

- **Residual** for track parameter α:

$$r = \alpha_{meas} - \alpha_{track}$$

where α_{track} is the result of the fit

  – Fitters can typically provide two types of hit residuals:
    - when the hit is included in the track fit
    - when it's excluded

- Note that the χ^2 can be written as

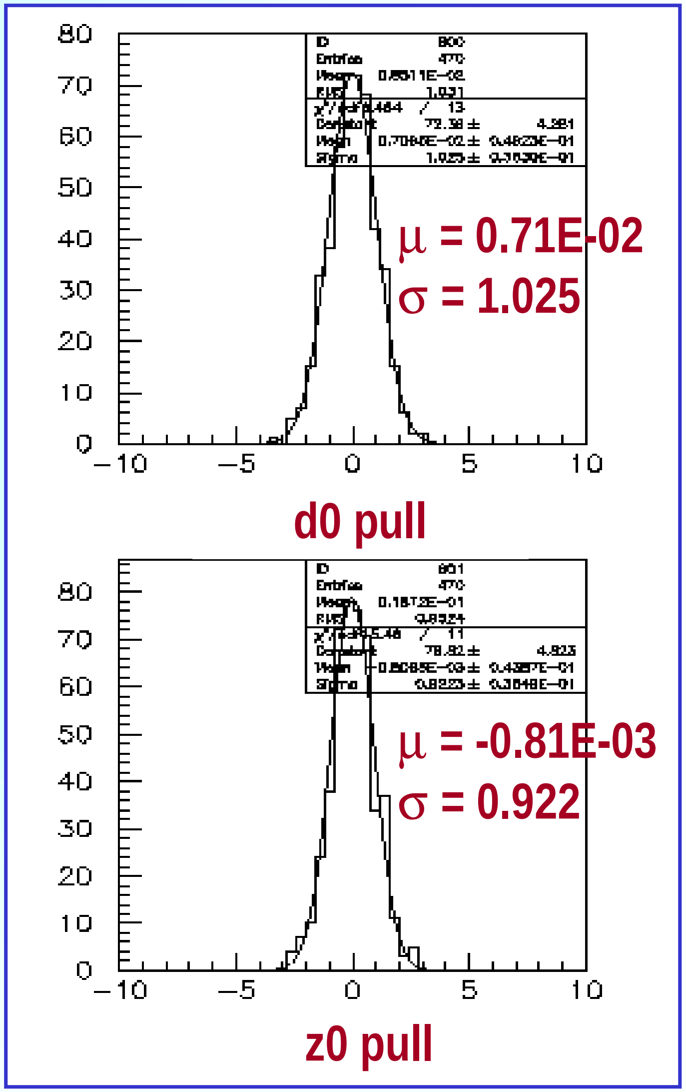
$$\chi^2 \equiv \sum_l \frac{r_l^{\,2}}{\sigma_l^{\,2}}$$

Pulls

- A **pull** for a track parameter α is defined as:

$$pull_{\alpha} \equiv \frac{\alpha_{meas} - \alpha_{track}}{\sigma_{\alpha}}$$

- If a fit is reasonable and errors are estimated correctly, expect a Gaussian with σ = 1 and μ = 0
  - So by plotting pulls can see if errors are correct or over/under estimated!



Kinematic Fitting

- The idea of kinematic fitting is to use the known properties (**constraints**) of a given physical process to improve the measurements describing the process.
- E.g.:

$$B^0 \to D^{*+} \pi^-$$
$$\hookrightarrow D^0 \pi^+$$
$$\hookrightarrow K^- \pi^+$$

- D^0 mass constraint for K, π (1)
- D^0 vertex (1)
- D^{*+} mass constraint for D^0, π (1)
- *B* vertex (3)
- E_{total} = beam energy (1)

7 constraints total

Lagrange Multiplier Method

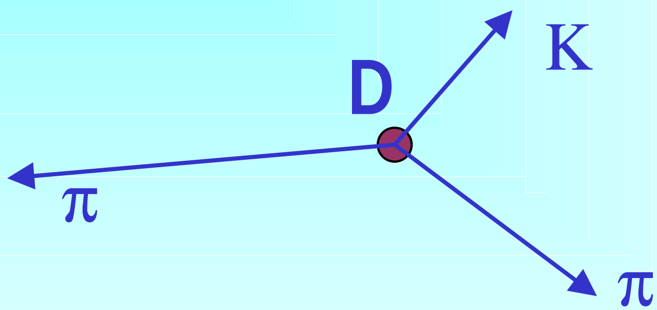
- The general algorithm for constrained fitting is based on the Lagrange multiplier method.
  - The idea is to incorporate the process' constraints into the calculation of the parameters of interest.
  - Simple example: back to back particles

Constraint equations:

$$p_{x1} + p_{x2} = 0$$
$$p_{y1} + p_{y2} = 0$$
$$p_{z1} + p_{z2} = 0$$

Vertexing

- A particular application of constrained fitting is **vertexing** when the tracks are required to come from the same point in space



  - we know that K, π, π must come from the same vertex
  - this fact can be used to improve the mass resolution of the *D*

Conclusion

- The goal of tracking is to determine the 5 helical parameters of a track as precisely as possible in the presence of complicated physical effects (multiple scattering, energy loss, non-uniform **B**-field, etc.).

- BaBar tracking devices, the SVT and the DCH, complement each other and allow us to get tracks over a large range of p_T (from ~40 MeV/c to a few GeV/c) .