Extracting γ using multibody decays

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Technion

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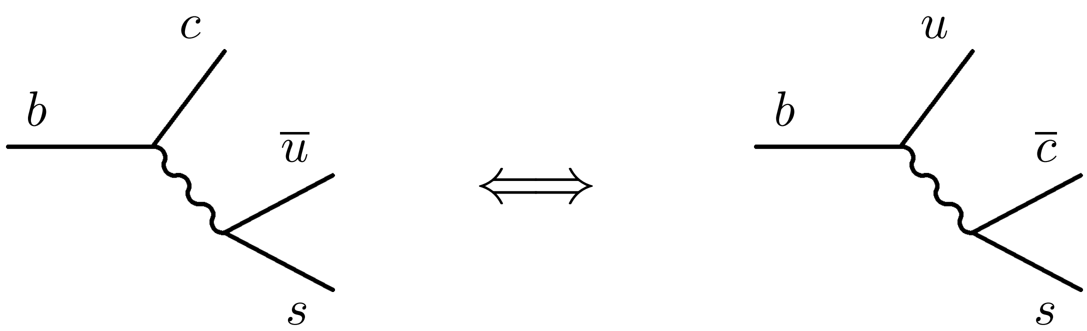
based on hep-ph/0303187

Obtaining γ

- many methods on the market, all statistically limited
- use interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$

Gronau, Wyler, 1991



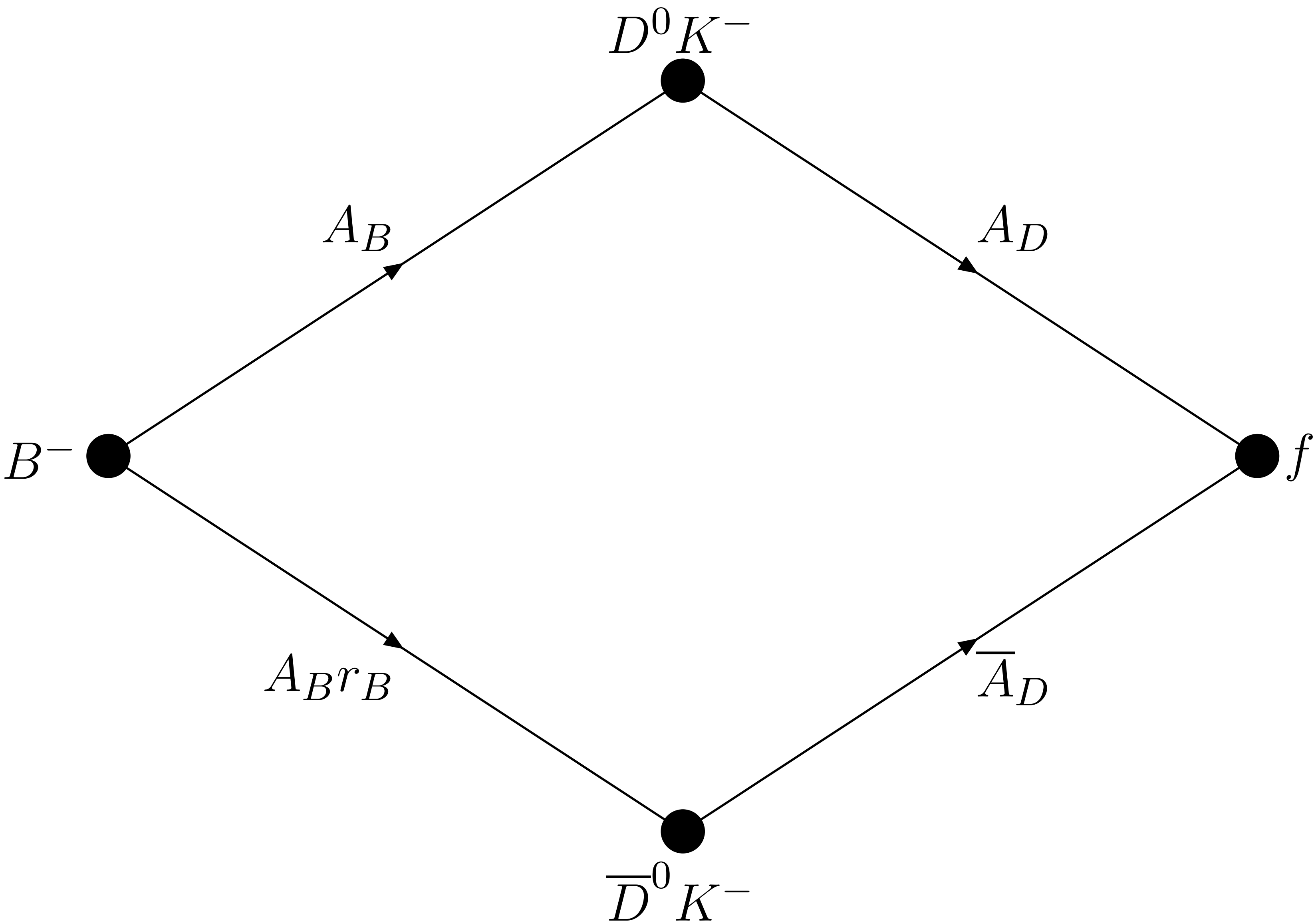
interference between

$$B^- \to DK^- \quad \text{followed by} \quad D \to f$$
$$B^- \to \bar{D}K^- \quad \text{followed by} \quad \bar{D} \to f$$

with f any common final state of D and $\bar{D}$

- no penguin contributions
- problems in measuring color suppressed $B^- \to \overline{D^0}K^-$

graphically...



Why multibody decays

- all model indep. methods need more than one channel
- different 2-body channels $\Leftrightarrow$ Dalitz plot analysis on the same channel
- many methods already use quasy two-body modes $\Rightarrow$ Dalitz plot analysis in effect already there
- significant portion of B and D branching ratios is into multibody final states $\Leftarrow$ needed to increase statistics
- reduces the number of discrete ambiguities

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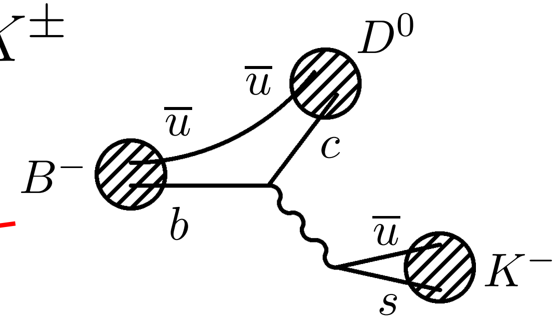
potential problem (!): need to integrate over phase space

- fits to Breit-Wigner forms for resonances
- develop a method that does it model independently

The method - definitions

for definiteness consider the decay chain

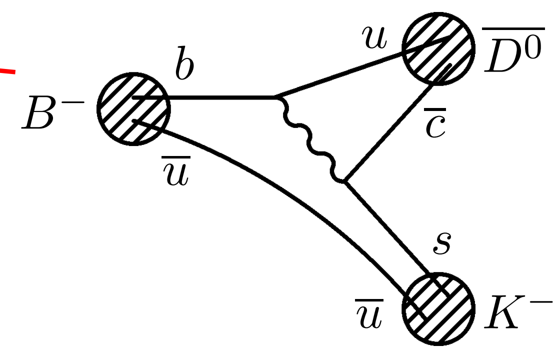
$$B^\pm \to DK^\pm \to (K_S\pi^-\pi^+)_D K^\pm$$



the amplitudes for B decays

$$A(B^- \to D^0 K^-) \equiv A_B$$

$$A(B^- \to \overline{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}$$



color suppression + CKM $\Rightarrow r_B \sim 0.1 - 0.2$

for three body D decay

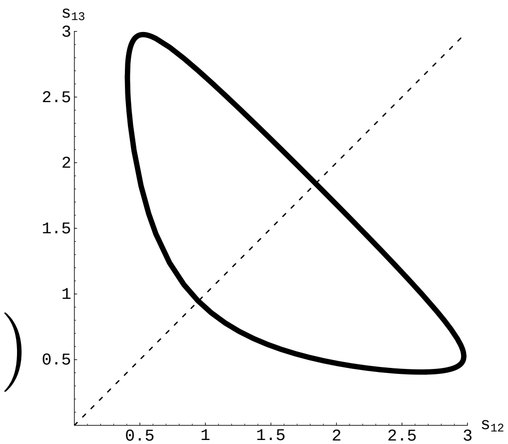
unknowns

$$A_D(s_{12}, s_{13}) \equiv A_{12,13}\, e^{i\delta_{12,13}} \equiv A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$

measurable

$$= A(\overline{D}^0 \to K_S(p_1)\pi^+(p_2)\pi^-(p_3))$$

The method - definitions II



the complete amplitude

$$A(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$

$$A_B \mathcal{P}_D \Big(A_D(s_{12}, s_{13}) + r_B e^{i(\delta_B-\gamma)} A_D(s_{13}, s_{12}) \Big)$$

$$B^- \to D^0 K^- + \overline{D^0} K^-$$

$$\hookrightarrow K_s \pi^- \pi^+ \hookleftarrow$$

interference term in the decay width:

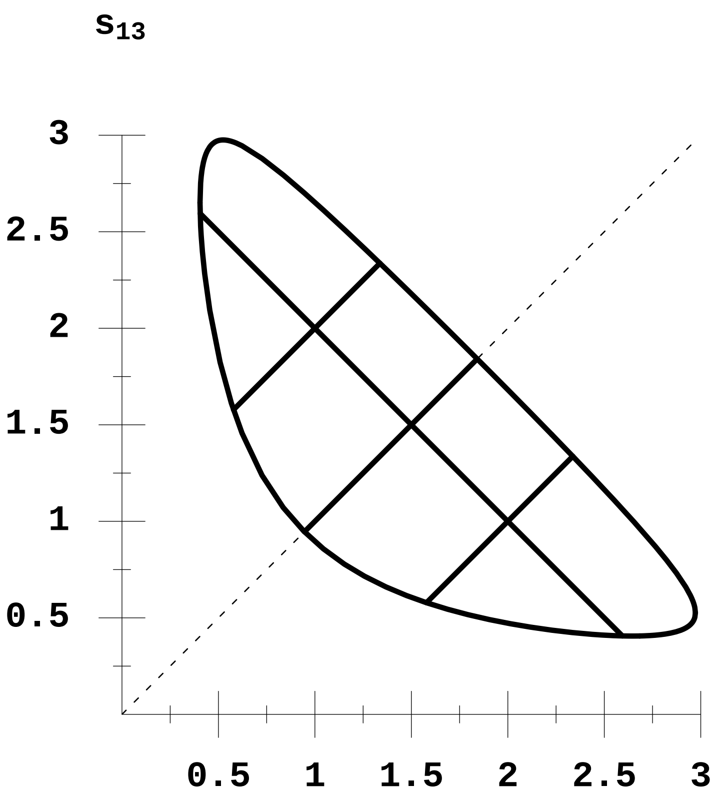
$$\mathcal{R}e\left[A_D(s_{12}, s_{13})\, A_D^*(s_{13}, s_{12})\, e^{-i(\delta_B-\gamma)}\right] = A_{12,13}\, A_{13,12} \times$$

$$\left[\cos(\delta_{12,13} - \delta_{13,12}) \cos(\delta_B - \gamma) + \sin(\delta_{12,13} - \delta_{13,12}) \sin(\delta_B - \gamma)\right]$$

unknown strong phases

The method - definitions III

- partition Dalitz plot in $2k$ bins
- label bins below symmetry axis i, above axis $\bar{i}$



unknowns

$$c_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \cos(\delta_{12,13} - \delta_{13,12})$$

$$s_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \sin(\delta_{12,13} - \delta_{13,12})$$

$$T_i \equiv \int_i dp\, A_{12,13}^2$$

measurable from tagged D

$$c_{\bar{i}} = c_i, \qquad s_{\bar{i}} = -s_i$$

$s_{12} = m^2_{K_s\pi^-}$ and $s_{13} = m^2_{K_s\pi^+}$

Master formulae

- a set of $4k$ equations
- the k equations for i bins

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S \pi^- \pi^+)_D K^-) =$$

$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i]$$

eqs. for $\hat{\Gamma}_{\bar{i}}^-$, $\hat{\Gamma}_i^+$, $\hat{\Gamma}_{\bar{i}}^+$ obtained by $12 \leftrightarrow 13$ and/or $\gamma \leftrightarrow -\gamma$

- $2k + 3$ unknowns: $c_i, \ s_i, \ r_B, \ \delta_B, \ \gamma$

solvable for $k \geq 2$

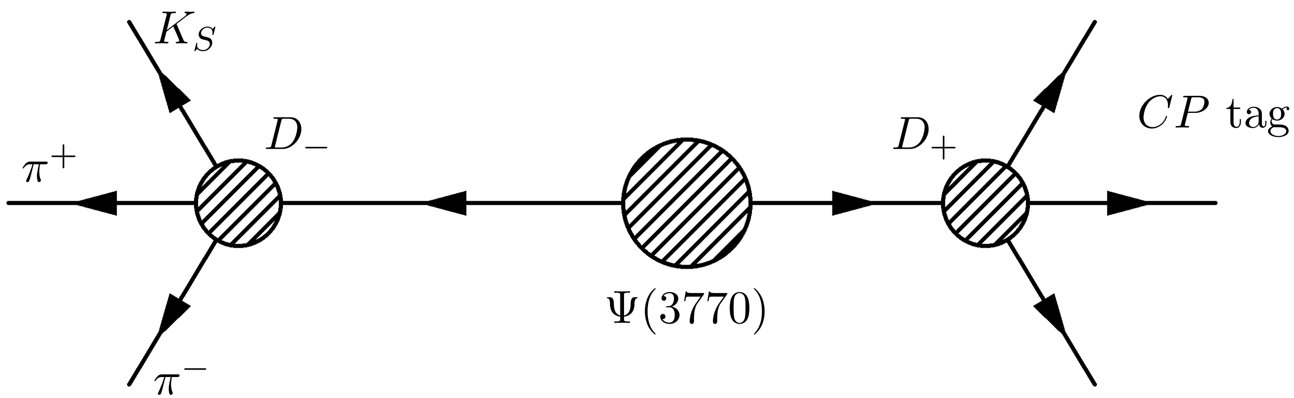
Special cases

- if $c_i = 0$ or $s_i = 0$ for $\forall i$, γ cannot be extracted. This is unlikely due to the presence of resonances.
- eqs. become degenerate for $\delta_B = 0$, γ can be extracted if some of c_i and/or s_i are independently measured
- c_i and s_i^2 can be measured at charm factories working at $\psi(3770)$
- they can be bound using tagged D^0 decays

$$|s_i|,\ |c_i| \le \int_i dp\, A_{12,13} A_{13,12} \le \sqrt{T_i\, T_{\bar{i}}}$$

Measuring c_i

- charm factory, $\psi(3770) \to D\bar{D}$
- use CP eigenstates $D^0_\pm \equiv (D^0 \pm \overline{D}^0)/\sqrt{2}$



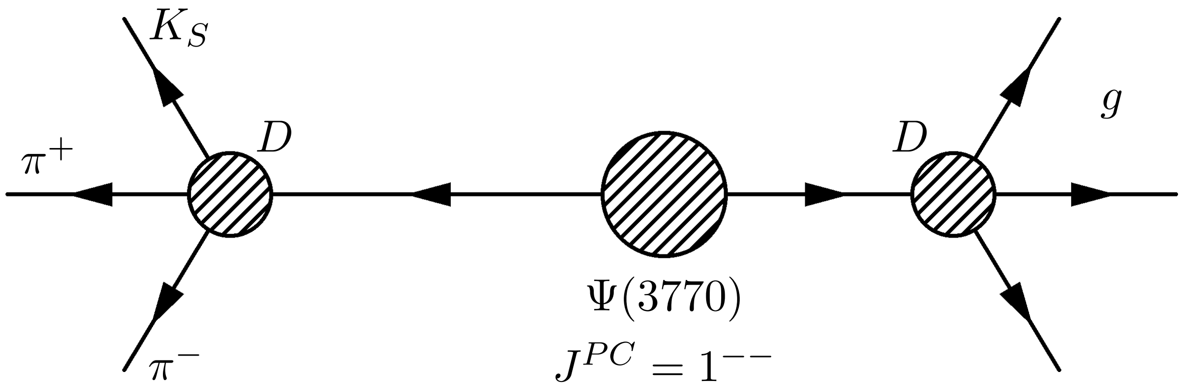
$$d\Gamma(D^0_\pm \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) =$$

$$\frac{1}{2}\left(A^2_{12,13} + A^2_{13,12}\right) \pm A_{12,13}A_{13,12}\cos(\delta_{12,13} - \delta_{13,12})dp$$

$$c_i = \frac{1}{2}\Big[\int_i d\Gamma(D^0_+ \to K_S\pi^-\pi^+) - \int_i d\Gamma(D^0_- \to K_S\pi^-\pi^+)\Big]$$

Measuring s_i^2

- instead of a CP tag consider a decay to general state g



ith bin of $K_S\pi^+\pi^-$ and jth bin of g

$$\Gamma_{i,j} \propto T_i T_{\bar{j}}^g + T_{\bar{i}} T_j^g - 2(c_i c_j^g + s_i s_j^g)$$

- if $g = K_S\pi^+\pi^-$ and $j = i$ ($j = \bar{i}$) one measures s_i^2
- if g a CP even (odd) eigenstate, $s_j^g = 0$, $T_j^g = T_{\bar{j}}^g = \pm c_j^g$, no sensitivity fo s_i

Discussions

- CP asymmetries sizeable at least in part of Dalitz plot

$$a^i_{\rm CP} \equiv \hat\Gamma^-_i - \hat\Gamma^+_{\bar i} = 4 r_B \sin\gamma \left[c_i \sin\delta_B - s_i \cos\delta_B\right]$$

- fourfold ambiguity in model-independent γ extraction

$$P_\pi \equiv \{\delta_B \to \delta_B + \pi, \gamma \to \gamma + \pi\}$$
$$P'_\pi \equiv \{c_i \to -c_i, s_i \to -s_i, \gamma \to \gamma + \pi\}$$
$$P_- \equiv \{\delta_B \to -\delta_B, \gamma \to -\gamma, s_i \to -s_i\}$$

← resolved by BW fits

- use of other D decay modes possible along the same lines
  - Cabibbo allowed: $D \to K_S \pi^- \pi^+ \pi^0$, $K^- K^+ K_S$
  - Cabibbo supp.: $D \to K^- K^+ \pi^0$, $\pi^- \pi^+ \pi^0$, $K_S K^+ \pi^-$
  - (almost) flavor eigenst.: $D \to K^- \pi^+ \pi^0$, $K^- \pi^+ \pi^- \pi^+$

Discussions II

- does not involve absolute branching ratios (i.e. can normalize by setting $T_i = 1$ for some i)
- r_B suppression lifted in $B^- \to D X_s^- \to (K_S \pi^- \pi^+)_D X_s^-$, same formalism applies with trivial changes
- can be extended to $B^0(\bar{B}^0) \to D K_S$ multibody D decays

Life is not always so simple...

in general multibody system such an analysis may not be possible...

- separation of B and D decay observables essential $\Rightarrow T_i$ measured separately from tagged D decays
- only two interfering amplitudes (e.g. no penguins)

example:
obtaining α from $B \to 3\pi$ in same spirit (+ isospin analysis) is not possible

many unknowns: $\int_i \mathrm{Tree_a Tree_b^*}, \int_\mathrm{i} \mathrm{Tree_a Peng_b^*}$

$\Rightarrow$ too few observables to fit all

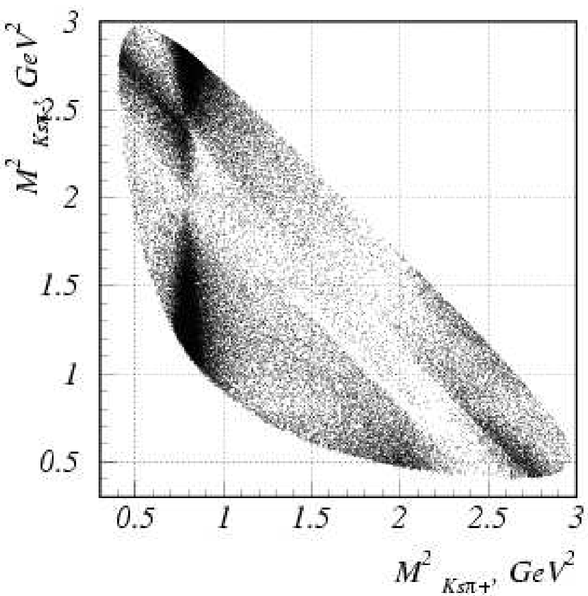
Assuming Breit-Wigner dependence

a fit to Breit-Wigner forms:

$$A_D(s_{12}, s_{13}) = A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) =$$

$$= a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

$$\mathcal{A}_r(s_{12}, s_{13}) = {}^J\mathcal{M}_r \times \frac{1}{s - M_r^2 + iM_r\Gamma_r(\sqrt{s})}$$



Belle, hep-ex/0308043

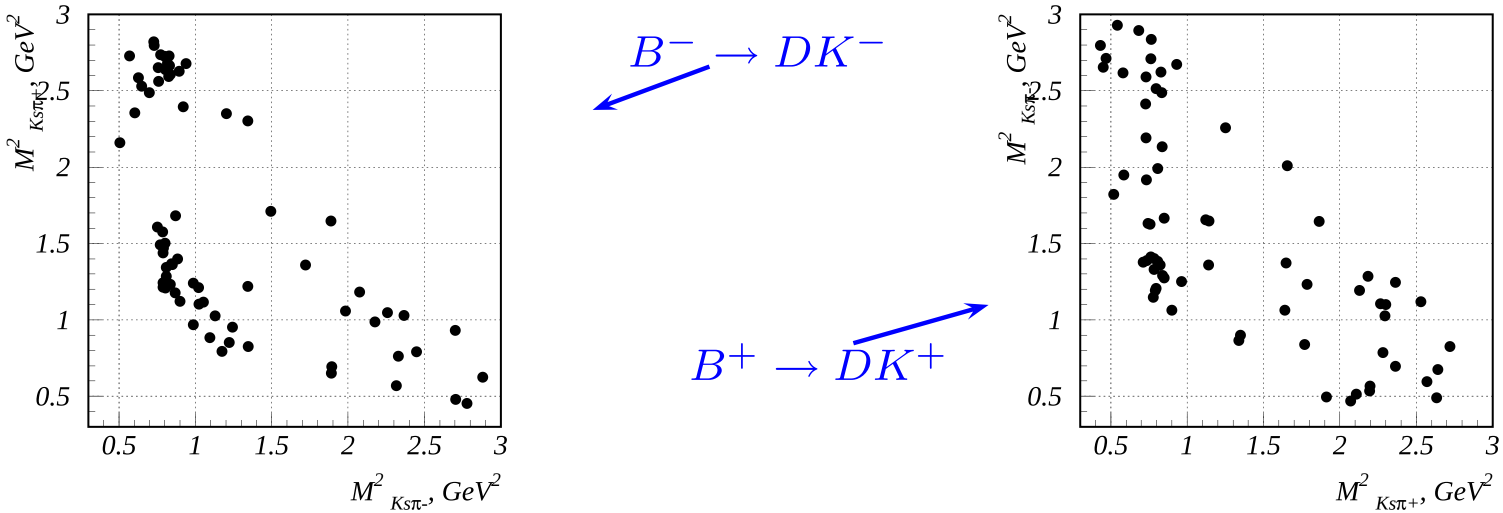
- high statistics D^0 tagged decay data available from B-factories
- in $B^\pm \to (K_S\pi^-\pi^+)_D K^\pm$ only r_B, δ_B and γ to be fit

Belle results

Belle, hep-ex/0308043

- fit to BW for D decays $\Leftarrow$ largest system. error
- MC study: $1000 fb^{-1} \Rightarrow 10^3 B^\pm \to [K_S\pi^+\pi^-]_D K^\pm$ $\Rightarrow 10^\circ$ error on γ
- now: $140 fb^{-1} \Rightarrow 107 \pm 12$ events

$61^\circ < \gamma/\phi_3 \mod \pi < 142^\circ$ at 90% C.L.



Conclusions

- have provided formalism for model independent γ extraction using $B^\pm \to K^\pm (K_S \pi^- \pi^+)_D$ cascade decay
- measurement of γ (combining with other methods/decay modes) is within experimental reach

Backup slides

CP asymmetry or γ extraction?

one does not need nonzero CP asymmetry to extract γ

consider two interfering amplitudes, $a \ll A$

$$\alpha_{CP} \simeq 4\frac{a}{A}\sin\delta_B \sin\gamma$$

$\delta_B = 0 \Rightarrow \alpha_{CP} = 0$, yet γ extraction in GW still possible

but cannot resolve discrete ambiguities...

Statistical significance

- CP asymmetry: $\frac{S}{N} \sim \frac{1}{\sqrt{1-\alpha_{CP}^2}} a$
- γ extraction: $\frac{S}{N} \sim a$

for γ extr. “large” a preffered $\Rightarrow$ Cabibbo allowed D decays

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all methods statistically limited $\Rightarrow$ important to use all modes & methods + find new methods

Decay width

reduced partial decay width

$$d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) = \Big(A_{12,13}^2 + r_B^2\, A_{13,12}^2$$
$$+2r_B \mathcal{R}e\big[A_D(s_{12}, s_{13})\, A_D^*(s_{13}, s_{12})\, e^{-i(\delta_B-\gamma)}\big]\Big)dp$$

Master formulae- complete

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S \pi^- \pi^+)_D K^-) =$$

$$T_i + r_B^2 T_{\bar{i}} + 2 r_B [\cos(\delta_B - \gamma) c_i + \sin(\delta_B - \gamma) s_i],$$

$$\hat{\Gamma}_{\bar{i}}^- \equiv \int_{\bar{i}} d\hat{\Gamma}(B^- \to (K_S \pi^- \pi^+)_D K^-) =$$

$$T_{\bar{i}} + r_B^2 T_i + 2 r_B [\cos(\delta_B - \gamma) c_i - \sin(\delta_B - \gamma) s_i],$$

$$\hat{\Gamma}_i^+ \equiv \int_i d\hat{\Gamma}(B^+ \to (K_S \pi^- \pi^+)_D K^+) =$$

$$T_{\bar{i}} + r_B^2 T_i + 2 r_B [\cos(\delta_B + \gamma) c_i - \sin(\delta_B + \gamma) s_i],$$

$$\hat{\Gamma}_{\bar{i}}^+ \equiv \int_{\bar{i}} d\hat{\Gamma}(B^+ \to (K_S \pi^- \pi^+)_D K^+) =$$

$$T_i + r_B^2 T_{\bar{i}} + 2 r_B [\cos(\delta_B + \gamma) c_i + \sin(\delta_B + \gamma) s_i].$$

The case of $B \to 3\pi$

Snyder-Quinn method for α det.

- $B^0 \to \rho^0\pi^0$, $B^0 \to \rho^-\pi^+$, $B^0 \to \rho^+\pi^-$ + isospin analysis + time dependent measurement of $B^0 \to \pi^+\pi^-\pi^0$
- sensitivity to α comes from ρ overlaps
- only tails of BW (!)
- can one do model indep. analysis?

The case of $B \to 3\pi$ II

- consider isospin decomp. of $B^0 \to \pi^+\pi^-\pi^0$ and $B^+ \to \pi^+\pi^+\pi^0$
- demand up to one π^0 in f.s. (to be competitive with SQ)
- counting the reduced amplitudes $A_{\Delta I}^{I_f, I_{2\pi}}$:

$$A_{1/2}^{0,1} A_{1/2}^{1,0}, A_{1/2}^{1,1}, A_{1/2}^{1,2}$$
$$A_{3/2}^{1,0}, A_{3/2}^{1,1}, A_{3/2}^{1,2} A_{3/2}^{2,1}, A_{3/2}^{2,2}$$
$$A_{5/2}^{2,1}, A_{5/2}^{2,2}, A_{5/2}^{3,2}$$
$$A_{7/2}^{3,2}$$

- $A_{\Delta I=3/2}^{I_f, I_{2\pi}}$ only tree, $A_{\Delta I=5/2}^{I_f, I_{2\pi}}$ also peng., other EWP~ 0

The case of $B \to 3\pi$ III

counting unknowns

- α, $9+9$ moduli& strong phases from tree, 2×4 moduli & strong ph. from penguins (assuming that top dominated, i.e. weak ph. known), gives 26 unknowns
- three observables for each point in the $B^0 \to \pi^+\pi^-\pi^0$ Dalitz plot $|A_f| \equiv |A(B^0 \to f)|$, $|\bar{A}_f| \equiv |A(\bar{B}^0 \to f)|$ and

$$\mathcal{I}m(\lambda_f) \equiv \mathcal{I}m\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right)$$

- isospin relates 6 point in the Dalitz plot of $B^0 \to \pi^+\pi^-\pi^0$ (3 in B^+), so $6\times 3 + 3\times 2 = 24$ observables

:-(

The case of $B \to 3\pi$ IV

assume one can project to even angular momenta (of the two pions)

- the two pions can only be in $I = 0, 2$
- in notation $A^{+-0}(s_{12}, s_{13})$ for $p_{\pi^+} = p_1,\, p_{\pi^-} = p_2,\, p_{\pi^0} = p_3$ the isospin decomp.

$$\begin{aligned} A_L^{++-} &= T_1 + 2T_2 + 2P - T_3 \\ A_L^{\{-0\}+} &= -T_1 + T_2 + P + T_3 \\ A_L^{\{+0\}-} &= T_1 + T_2 + P + T_3 \end{aligned}$$

+CP conj. modes

The case of $B \to 3\pi$ V

- only neutral B's:
  6 measurements:

  $$|A^{\{-0\}+}|,\ |\tilde{A}^{\{-0\}+}|,\ |A^{\{+0\}-}|,\ |\tilde{A}^{\{+0\}-}|,$$
  $$\mathcal{I}m\left(\tilde{A}^{\{+0\}-}A^{\{-0\}+^*}\right),\ \mathcal{I}m\left(\tilde{A}^{\{-0\}+}A^{\{+0\}-^*}\right)$$

  and in addition the relation

  $$A^{\{-0\}+} - A^{\{+0\}-} = e^{i2(\alpha-2\beta)}\left(\tilde{A}^{\{-0\}+} - \tilde{A}^{\{+0\}-}\right)$$

  solvable
- problems:
  - discussed only one point in Dalitz plot so far
  - difficult to make projection to even momenta only

The case of $B \to 3\pi$ VI

impossible to do model independent analysis for integrated variables:

- define $\int dp T_i T_j^*$ and $\int dp T_i P^*$ as new variables from which α
- consider only neutral B's (adding charged does not help, as many new parameters as measurments)
- 6 measurements
- 9 unknowns :
  - 3 integrals over squares of moduli of T_1, $T_2 + T_3$, P
  - 3+3 integrals over $T_i T_j^*$ (also imaginary parts counted as one has a measurement $\mathcal{I}m(\bar{A}_f A_f^*)$)

:-(

Outline

- CKM triangle, where do we stand?
- determining γ from $B^\pm \to DK^\pm$
  - multibody D decays, model independent method
  - using fit to Breit-Wigner forms
- Belle results
- conclusions

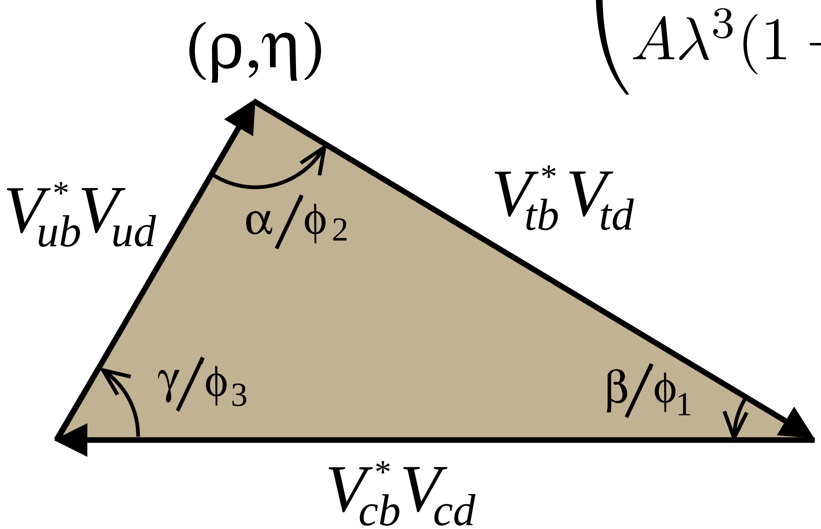
CKM triangle

CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\equiv V_{\text{CKM}}} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Wolfenstein parametr.

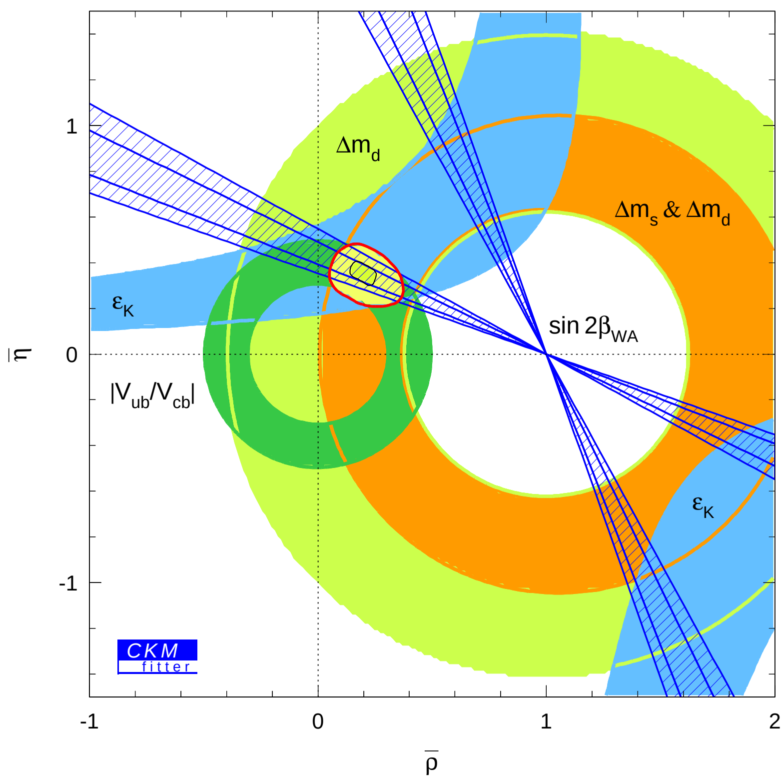
$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



“standard” unitarity triangle

CKM fit

indirect info on angles from fit to $|V_{ub}|$, $|V_{cb}|$, ϵ_K, Δm_d, Δm_s



at 95% CL :

$$\begin{cases} 19.4^\circ < \beta < 26.5^\circ \\ 77^\circ < \alpha < 122^\circ \\ 37^\circ < \gamma < 80^\circ \end{cases}$$

H. Jawahery, talk at Lepton Photon 03

from $B \to J/\Psi K_S$ system

$\sin 2\beta = 0.736 \pm 0.049$

T. Browder, talk at Lepton Photon 03

$\Rightarrow \beta = 23.7^\circ \pm 2.1^\circ$

no direct determination of α, γ yet...