

Time-dependent measurements of $\sin(2\beta + \gamma)$

Dan Pirjol

MIT

Includes work with M. Gronau and D. Wyler, Phys.Rev.Lett. 90, 051801 (2003)

Clean methods for $2\beta + \gamma = \beta - \alpha + \pi$

- $2\beta + \gamma$ from $B^0(t) \rightarrow D^{(*)\pm} \pi^\mp$
 - Hadronic uncertainties
 - Theoretical limitations
- Method using $B^0(t) \rightarrow D^{(*)\pm} \rho^\mp$
- Method using $B^0(t) \rightarrow D^{*\pm} a_1^\mp$
- Conclusions

London, Sinha, Sinha

Gronau, DP, Wyler

$$2\beta + \gamma \text{ from } B \rightarrow D^{(*)\pm} \pi^\mp$$

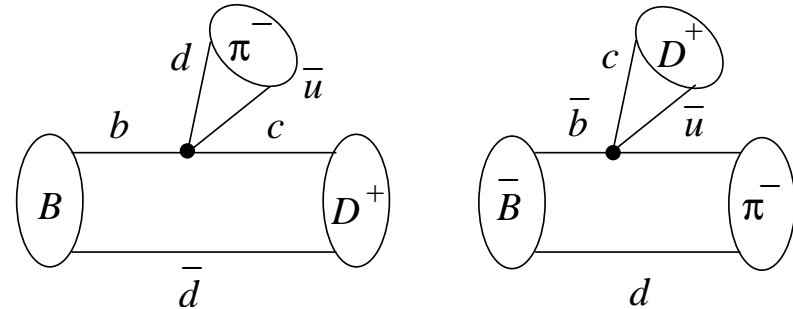
Interference of tree-mediated decays ($b \rightarrow cd\bar{u}$, $\bar{b} \rightarrow c\bar{u}d$)

Dunietz+Sachs

Aleksan, Dunietz, Kayser, Le Diberder

$$A(\bar{B}^0 \rightarrow D^{(*)+} \pi^-) = V_{cb} V_{ud}^* a$$

$$A(B^0 \rightarrow D^{(*)+} \pi^-) = V_{ub}^* V_{cd} b$$



- $B^0 - \bar{B}^0$ mixing induces a time-dependent CP asymmetry
- Analogous to $B \rightarrow J/\psi K_S$, except that the final state is non-CP eigenstate \rightarrow the asymmetry depends on hadronic parameters, besides weak phases.

$$r_{(*)} e^{i(\delta_{(*)} + \gamma)} = \frac{A(B^0 \rightarrow D^{(*)+} \pi^-)}{A(\bar{B}^0 \rightarrow D^{(*)+} \pi^-)}, \quad r \sim \lambda \frac{V_{ub}}{V_{cb}} \sim 0.02$$

Time-dependent decay rates

Dunietz

Fleischer, hep-ph/0304027

$$\Gamma(B^0(t) \rightarrow D^{(*)+} \pi^-) \sim \frac{e^{-|t|/\tau}}{2\tau} [1 - (-)^L S_+ \sin(\Delta mt) - C \cos(\Delta mt)]$$

$$\Gamma(B^0(t) \rightarrow D^{(*)-} \pi^+) \sim \frac{e^{-|t|/\tau}}{2\tau} [1 - (-)^L S_- \sin(\Delta mt) + C \cos(\Delta mt)]$$

and

$$\Gamma(\bar{B}^0(t) \rightarrow D^{(*)+} \pi^-) \sim \frac{e^{-|t|/\tau}}{2\tau} [1 + (-)^L S_+ \sin(\Delta mt) + C \cos(\Delta mt)]$$

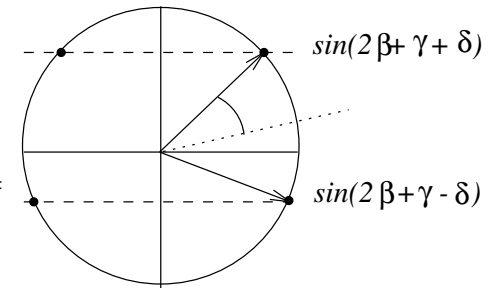
$$\Gamma(\bar{B}^0(t) \rightarrow D^{(*)-} \pi^+) \sim \frac{e^{-|t|/\tau}}{2\tau} [1 + (-)^L S_- \sin(\Delta mt) - C \cos(\Delta mt)]$$

The asymmetry parameters are

$$S_{\pm} = \frac{2r_{(*)}}{1 + r_{(*)}^2} \sin(2\beta + \gamma \pm \delta_{(*)}), \quad C = \frac{1 - r_{(*)}^2}{1 + r_{(*)}^2}$$

Assuming perfect tagging efficiency, measuring the 4 t -dependent decay rates determines $2r \sin(2\beta + \gamma \pm \delta)$

- Measure r as $r^2 = \frac{Br(B^0 \rightarrow D^+ \pi^-)}{Br(B^0 \rightarrow D^- \pi^+)} \sim 3 \times 10^{-4}$



- Assuming r known gives $2\beta + \gamma$ with a 4-fold ambiguity.

Several issues complicate this determination:

- Tagging using nonleptonic modes introduces a dependence on the hadronic parameters of the tag mode
- The Cabibbo suppressed mode $B^0 \rightarrow D^+ \pi^-$ is very hard to measure
- Indirect methods are required for inferring r (SU(3) + dynamical assumptions such as small W -exchange or weak annihilation amplitudes)

Results

- BABAR, hep-ex/0309017 (PRL) (fully reconstructed events)

$$B \rightarrow D\pi : \quad 2r \sin(2\beta + \gamma) \cos \delta = -0.022 \pm 0.038(\text{stat}) \pm 0.020(\text{sys})$$

$$2r \cos(2\beta + \gamma) \sin \delta = 0.025 \pm 0.068(\text{stat}) \pm 0.033(\text{sys})$$

$$B \rightarrow D^*\pi : \quad 2r_* \sin(2\beta + \gamma) \cos \delta_* = -0.068 \pm 0.038(\text{stat}) \pm 0.020(\text{sys})$$

$$2r_* \cos(2\beta + \gamma) \sin \delta_* = 0.031 \pm 0.070(\text{stat}) \pm 0.033(\text{sys})$$

- BELLE, hep-ex/0308048 (LP-03)

$$2r \sin(2\beta + \gamma + \delta) = 0.094 \pm 0.053(\text{stat}) \pm 0.013(\text{sys}) \pm 0.036(D^* l\nu)$$

$$2r \sin(2\beta + \gamma - \delta) = 0.022 \pm 0.054(\text{stat}) \pm 0.013(\text{sys}) \pm 0.036(D^* l\nu)$$

$$2r_* \sin(2\beta + \gamma + \delta_*) = 0.092 \pm 0.059(\text{stat}) \pm 0.016(\text{sys}) \pm 0.036(D^* l\nu)$$

$$2r_* \sin(2\beta + \gamma - \delta_*) = 0.033 \pm 0.056(\text{stat}) \pm 0.016(\text{sys}) \pm 0.036(D^* l\nu)$$

Determining r

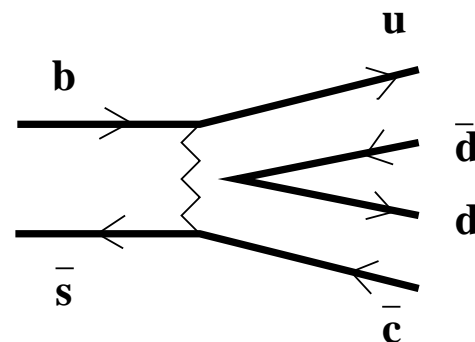
Direct measurement of r from $\frac{Br(B^0 \rightarrow D^+ \pi^-)}{Br(B^0 \rightarrow D^- \pi^+)} \sim 3 \times 10^{-4}$ is very difficult.

- The $b \rightarrow cd\bar{u}$ mode $B^0 \rightarrow D^- \pi^+$ is calculable in factorization at leading order in $1/m_b$ **Beneke, Buchalla, Neubert, Sachrajda**
Bauer, Pirjol, Stewart
- No such result has been proved for the DCS mode $B^0 \rightarrow D^+ \pi^-$. In general factorization for such modes ($b \rightarrow ud\bar{c}$) is a model-dependent statement at this time.
- The only known limit where factorization holds in $b \rightarrow ud\bar{c}$ (only color-allowed!) modes is the large N_c limit. This gives at most qualitative results ($\sim \pm 50\%$)

Alternative methods: use SU(3) and dynamical assumptions to relate it to the more accessible $\Delta S = 1$ modes **Dunietz**

Exact SU(3) relations

$$A(B^0 \rightarrow D^{(*)+} \pi^-) = \begin{cases} \frac{V_{cd}}{V_{cs}} [A(B^0 \rightarrow D_s^{(*)+} \pi^-) + A(B_s \rightarrow D^{(*)+} \pi^-)] \\ \frac{V_{cd}}{V_{cs}} A(B_s \rightarrow D_s^{(*)+} K^-) \end{cases}$$



The $B_s \rightarrow D^{(*)+} \pi^-$ mode is pure W-exchange

Assuming that it can be neglected, data on $B(B^0 \rightarrow D_s^+ \pi^-)$ could fix r

BABAR, Belle : $B(B^0 \rightarrow D_s^+ \pi^-) = (2.7_{-0.6}^{+0.7} \pm 0.8) \times 10^{-5}$

$$B(B^0 \rightarrow D_s^{*+} \pi^-) = (1.9_{-1.3}^{+1.2} \pm 0.5) \times 10^{-5}$$

$$r_{(*)} = \lambda \sqrt{\frac{Br(B^0 \rightarrow D_s^{(*)+} \pi^-)}{Br(B^0 \rightarrow D^{(*)-} \pi^+)}} = 0.019 \pm 0.004 \quad (0.017_{-0.007}^{+0.005})$$

Error estimates

The theoretical error in this r determination is related to the magnitude of the W -exchange amplitude, which can be measured in

- $\Delta S = 1$ B_s decays: $B_s \rightarrow D^+ \pi^-$, $B_s \rightarrow D^0 \pi^0$
- $\Delta S = 0$ B_d decays: $B^0 \rightarrow D_s^+ K^-$

In the absence of such data, the error on r could be as large as 100%!

Using other modes

Dunietz

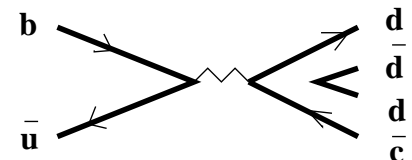
Chiang, Suprun, Rosner

- Use the $B^\pm \rightarrow D^\pm \pi^0$ mode (no $B - \bar{B}$ mixing)

$$A(B^0 \rightarrow D^+ \pi^-) \sim \sqrt{2} A(B^\pm \rightarrow D^\pm \pi^0)$$

This assumes small W -exchange *and* weak annihilation amplitudes.

The latter can be measured directly in $B^\pm \rightarrow D^\pm K^0$



Other modes: SU(3) relation

$$A(B^- \rightarrow D_s^{(*)-} \pi^0) = \frac{1}{\sqrt{2}} A(\bar{B}^0 \rightarrow D_s^{(*)-} \pi^+)$$

No new information gained by measuring this mode. Turn it around and make predictions for the as yet unobserved modes

$$Br(B^- \rightarrow D_s^- \pi^0) = (1.4 \pm 0.5) \times 10^{-5}$$

$$Br(B^- \rightarrow D_s^{*-} \pi^0) = (1.0 \pm 0.6) \times 10^{-5} .$$

Summary $B^0(t) \rightarrow D^{(*)\pm} \pi^\mp$

- $B^0(t) \rightarrow D^{(*)\pm} \pi^\mp$ (also $D^\pm \rho^\mp, D^\pm a_1^\mp$) decays offer a completely model-independent determination of $2\beta + \gamma$, provided that the small ratio r of $(b \rightarrow ud\bar{c})/(b \rightarrow cd\bar{u})$ amplitudes is known
- However, present extractions of r could have errors $\sim 100\%$

Good news: Alternative methods are available, using $B^0(t) \rightarrow D^{*+} \rho^-$ and $B^0(t) \rightarrow D^{*+} a_1^-$ decays

$B \rightarrow V_1 V_2$ modes

Described by 3 amplitudes. Two possible parameterizations

Helicity basis

(H_+, H_-, H_0)

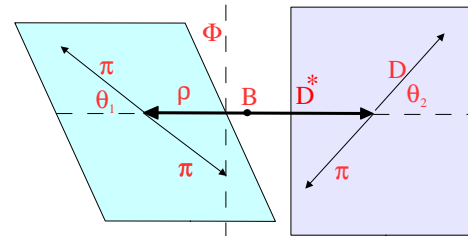
Linear basis

$(H_{\parallel}, H_{\perp}, H_0)$

$$H_{\pm} = \frac{1}{\sqrt{2}}(H_{\parallel} \pm H_{\perp})$$

Neglect time dependence. Use strong decays $D^* \rightarrow D\pi$ and $\rho \rightarrow \pi\pi$ as helicity analyzer

$$A(B \rightarrow f) = \sum_{i=\parallel, \perp, 0} H_i g_i(\theta_1, \theta_2, \phi)$$



Measuring the angular dependence of the final particles allows the extraction of $|H_{\parallel, \perp, 0}|^2$ and $\text{Im}(H_{\parallel}^* H_{\perp})$, $\text{Re}(H_0^* H_{\parallel})$, $\text{Im}(H_0^* H_{\perp})$

2-fold ambiguity: $(H_0, H_{\parallel}, H_{\perp}) \rightarrow (H_0^*, H_{\parallel}^*, -H_{\perp}^*)$

$(V \rightarrow PP$ can't distinguish left/right)

Mixing induced asymmetry with $f = D^{*+}(\rightarrow D\pi)\rho^-(\rightarrow \pi^-\pi^0)$

London, Sinha²

$$A(\bar{B}^0 \rightarrow f) = \sum_{i=\parallel,\perp,0} H_i g_i(\theta_i, \phi) \quad H_i = a_i e^{i\phi_i}$$

$$A(B^0 \rightarrow f) = \sum_{i=\parallel,\perp,0} h_i g_i(\theta_i, \phi) \quad h_i = b_i e^{i(\gamma+\bar{\phi}_i)}$$

The interference of the 3 $b \rightarrow c$ amplitudes H_i with the 3 $b \rightarrow u$ amplitudes h_j is parameterized by 3×3 matrices Λ, Σ, ρ

$$\Gamma(\bar{B}^0(t) \rightarrow f) = e^{-\Gamma t} \sum_{i,j=\parallel,\perp,0} \{ \Lambda_{ij} + \Sigma_{ij} \cos(\Delta mt) + \rho_{ij} \sin(\Delta mt) \} g_i g_j$$

$$\Gamma(B^0(t) \rightarrow \bar{f}) = e^{-\Gamma t} \sum_{i,j=\parallel,\perp,0} \{ \bar{\Lambda}_{ij} + \bar{\Sigma}_{ij} \cos(\Delta mt) + \bar{\rho}_{ij} \sin(\Delta mt) \} g_i g_j$$

Angular measurements give the coefficients Λ, Σ, ρ

$$\Lambda_{ij} = \frac{1}{2} \text{Re}(H_i H_j^* + h_i h_j^*), \quad \Sigma_{ij} = \frac{1}{2} \text{Re}(H_i H_j^* - h_i h_j^*), \quad \rho_{ij} = \text{Im} \left[\left(\frac{q}{p} \right)^* (H_i h_j^* + H_j h_i^*) \right]$$

$$2\beta + \gamma \text{ with } B^0(t) \longrightarrow D^{*+} \rho^-$$

London, Sinha, Sinha

1. Determine $a_i^2 = \Lambda_{ii} + \Sigma_{ii}$
2. Determine $a_i a_j \cos(\phi_i - \phi_j)$ from $\Lambda_{ij}, \Sigma_{ij}$
3. Determine $b_i \cos(\phi_i - \bar{\phi}_i)$ and $b_i \sin(\phi_i - \bar{\phi}_i)$ from ρ_{ii} and $\bar{\rho}_{ii}$
4. Use ρ_{ij} and $\bar{\rho}_{ij}$ to extract $\tan^2(2\beta + \gamma)$

Remarks

- No measurement of a small quantity $\sim b_i^2$ is ever required
- Choose the 2 sets of interfering amplitudes (a_i, a_j) and (b_i, b_j) ($i, j = \parallel, \perp, 0$) such as to maximize the CP asymmetry
- $2\beta + \gamma$ is determined up to a 4-fold discrete ambiguity

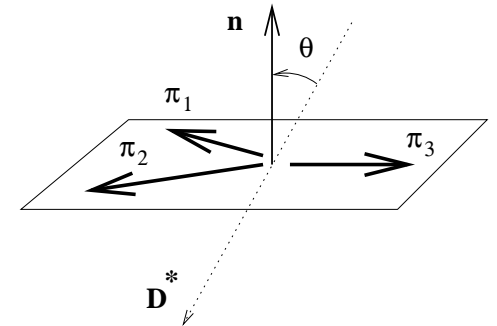
$$B^0(t) \longrightarrow D^{*+} a_1^-$$

Gronau, DP, Wyler - PRL90,051801(2003)

Consider B decays into a vector and an axial vector meson a_1, K_1, \dots

$$B(B^0 \rightarrow D^{*+} a_1^-) = (1.20 \pm 0.07(\text{stat}) \pm 0.14(\text{syst}))\% \quad \text{Babar}$$

- If the a_1^- is observed through $a_1^- \rightarrow \pi^+ \pi^- \pi^-$, the individual helicity amplitudes can be disentangled without any discrete ambiguity.
- Measuring the θ angular dependence gives $\text{Re}(H_{\parallel} H_{\perp}^*)$
- The combined (θ, ψ) angular dependence can give also $\text{Im}(H_0 H_{\parallel}^*)$, $\text{Re}(H_0 H_{\perp}^*)$



Time-dependent angular distributions in $B^0(t) \rightarrow D^*(\rightarrow D\pi)a_1(\rightarrow 3\pi)$
can give also $2\beta + \gamma$

Compared with the London, Sinha² method

- additional interference terms can be measured

$$\Lambda'_{ij} = \frac{1}{2} \text{Im}(H_i H_j^* - h_i h_j^*), \quad \Sigma'_{ij} = \frac{1}{2} \text{Im}(H_i H_j^* + h_i h_j^*), \quad \rho'_{ij} = \text{Re} \left[\left(\frac{q}{p} \right)^* (H_i h_j^* - H_j h_i^*) \right]$$

- ambiguity in $2\beta + \gamma$ resolved
- only π^\pm need to be observed in the final state

Need completely reconstructed events

Parameter count

How many parameters have to be fit?

In the SM the $B \rightarrow D^{*\pm} V^\mp$ is described by $3 + 3 + 5 + 1 = 12$ parameters ($|a_i|$, $|b_j|$, strong phases and γ). This gives relations among observable quantities.

- $B \rightarrow D^* \rho$
 - The full LSS analysis contains 36 parameters $\Lambda_{ij}, \bar{\Lambda}_{ij}, \Sigma_{ij}, \bar{\Sigma}_{ij}, \rho_{ij}, \bar{\rho}_{ij}$. Not all are independent!
 - Theoretical constraints: $\Lambda_{ij} = \bar{\Lambda}_{ij}, \Sigma_{ij} = \bar{\Sigma}_{ij}$
 - This leaves $36 - 12 = 24$ independent parameters

The LSS method can be implemented with a minimum of 12 parameters, corresponding to 2 helicity amplitudes.

- $B \rightarrow D^* a_1$
 - Parameterize the 2-fold (θ, ψ) distribution in terms of the 12 independent physical parameters, and fit for them